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# ERROR PERFORMANCE OF TRELLIS CODED QUANTIZATION/ CONTINUOUS PHASE MODULATION SYSTEMS (TCQ/CPM) ON AWGN CHANNELS

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## *ABSTRACT*

*In this paper, Trellis Coded Quantization/Continuous Phase Modulation systems have been introduced and error performance analysis has been investigated on AWGN channels. This system benefits fully from low coding and quantization coplexity as well as spectral and power efficiency. As an example, an eight state 4-ary TCQ/CPM system is designed and its error performance is evaluated for AWGN channel for effective signal to noise ratios.*

.**Keywords:** *Trellis Coded Quantization, Continuous Phase Modulation, Trellis Coded Quantization/ Continuous Phase Modulation.*

## I. Introduction

Trellis Coded Modulation (TCM) [1] is a very effective modulation scheme for band limited channels. Motivated by trellis coded modulation, Trellis Coded Quantization (TCQ) was developed as a computationally efficient scheme for source coding. The main feature of TCQ is the utilization of a structured codebook with an expanded set of quantization levels based on Ungerboeck's nation of set partitioning. The design of trellis coded scalar quantization systems for memoryless and Gaussian/Markov sources is explained in detail in [2] Signal set expansion and partitioning ideas of trellis coded modulation have been applied to quantization problem to develop a source coding technique known as trellis coded quantization. For the AWGN channel, Marcellin and Fisher [2] [3] constructed a joint source/channel coding system by using trellis coding quantization and modulation. They used identical trellis for TCQ and TCM and a consistent labeling between quantization levels and modulation symbols.

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Since using two identical trellis separately for TCQ and TCM became unnecessarily complex, the combined form of TCQ/TCM was introduced in [4] and thus coding steps and the number of memories are reduced.
Continuous phase modulation (CPM) [5] is widely used on radio channels because of its good spectral properties.The phase continuity of CPM signals improves spectral properties, and introduces memory which induces error control capability and therefore power efficiency.
CPM can be decomposed into a continuous phase encoder (CPE) and a memoryless modulator (MM) [5] The CPE is a convolutional encoder which produces codeword sequences that are mapped onto waveforms by the MM, creating a continuous phase signal. Once the memory of CPM was made explict, it became possible to design trellis and convolitionally coded CPM systems which allowed the trellis code and the CPE to be combined into a single joint convolutional code. In this paper we have derived the BER –SNR curves of TCQ/CPM system in AWGN channel using Lloyd-Max and Optimum quantization levels [2].

## 2. TRELLIS CODED QUANTIZATION/TRELLIS CODED MODULATION

General approach to the selection of a combined TCQ/TCM system is to assume that TCQ and TCM bit and symbol rates are equal so that the squared distance between channel sequences is commensurate with squared error in quantization. The mapping from quantization level within a TCQ subset to modulation level within a TCM subset is selected in the obvious way, so that the level/symbol order is consistent. Since the probability of a TCM error is related to the squared Euclidean distance between the allowable paths through the trellis, consistent labeling guarantees that the squared Euclidean distance between the symbols is in line with mean square error in quantization.
The reproduction codebook size (i.e. number of quantization levels) is selected as $N=2^{R+CEF}$ There are totally $N_1=2^{r+CEF}$ subset. $N$ is chosen such that it can be properly divided by $N_1$, so each subset has exactly $N_2=N/N_1=2^{R-r}$ codewords. Here $R\geq 1$ is the encoding rate in bits/sample, r and *CEF* are positive integers satisfying $1\leq r\leq R$ and $CEF\geq 0$. The parameter *CEF* stands for "codebook expansion factor" , since the codebook size is $2^{CEF}$ times of a normal *R* bit/sample scalar quantizer. On the branch of the trellis, both quantization levels $q_{k,l}$ (k=0,1,......., $N_1-1$; l=1,2,........., $N_2$) and signal set $s_j$ (j=1,2,.........,*N*-1) are placed using Ungerboeck's rules [1]

## 3. CONTINUOUS PHASE MODULATION

In [5] Rimoldi derived the tilted–phase representation of CPM, with the information bearing phase given by

$$\Phi(t,\alpha)=4\pi h\sum_{i=0}^{\infty}\alpha_i q(t-iT) \quad (1)$$

The modulation index *h* is equal to *K/P*, where *K* and *P* are relatively prime integers. $\alpha$ is an input sequence of *M*-ary symbols, $\alpha_i\in\{0,1,2,\ldots,M-1\}$. T is the channel symbol period.
The phase response function *q(t)* is a continuous and monotonically increasing function subject to the constraints

$$q(t)=\begin{cases}0 & t\leq 0\\ 1/2 & t\geq LT\end{cases} \quad (2)$$

where *L* is an integer. The phase response is usually defined in terms of the integral of a frequency pulse *g(t)* of duration *LT*, i.e., $q(t)=\int_{-\infty}^{t}g(\tau)d\tau$ for full response signaling *L*=1, while for partial response systems L>1.

Finally the transmitted signal s(t) is given by

$$s(t,\alpha)=(2E_s/T)^{1/2}Cos(2\pi f_c t+\Phi(t,\alpha)+\Phi_0) \quad (3)$$

Here $f_c$ is the carrier frequency, $E_s$ is the energy per channel symbol and $\Phi_0$ is the initial carrier phase.

# 4. TCQ/CPM SYSTEM MODEL

As shown in Fig.1 the transmitter of the TCQ/CPM system consist of a scalar quantizer, convolutional encoder (CE), continuous phase encoder (CPE) and a memoryless mapper (MM). In this system CE encoder realizes external coding.

In the every signalling interval, analog signal that is coming to TCQ/CPM system input is once quantized by quantizer then coding by joint coder. n output bits of the CE are input to the CPE. CPE output bits are then mapped to TCQ/CPM signals by memoryless mapper (MM), after than these signals are sent to channel. In the channel, noise is added to the signals. Thus, with only one TCQ/CPM structure, both quantization and coding and continuous phase modulation have been realized. Besides, the complexity of the system has been decreased and the decoding process has been done in one step**.** At the receiver side, original information data are obtained by demodulating, decoding and dequantizing of received noisy signals via combined TCQ/CPM receiver.

**An Example:**

***4-ary TCQ/CPM (h=1/2) System***

4-ary TCQ/CPM (h=1/2) system consist of rate 1/2 having two memory units convolutional encoder, rate 2/3 having one memory unit continuous phase encoder and memoryless mapper. Joint code has three memories and it's rate is 1/3. Encoder structure and encoder's trellis structure is given consequently in figure 2 and figure 3.

In this system, modulation scheme is chosen as full response CPFSK.

Quantization levels that mapped one by one and signal set is shown in trellis structure's branches. From every state to next state thare is only one transition. In these transitions quantization levels are chosen as $q_{k,l} \in Q_k, k = 0,1,2,3; l = 1,2$. Table 1 shows the next phase state ($\theta_{n+1}$), output signal ($s_i$) and quantization level (Q) according to recent phase state ($\theta_n$) and CPE input $\beta_n$.

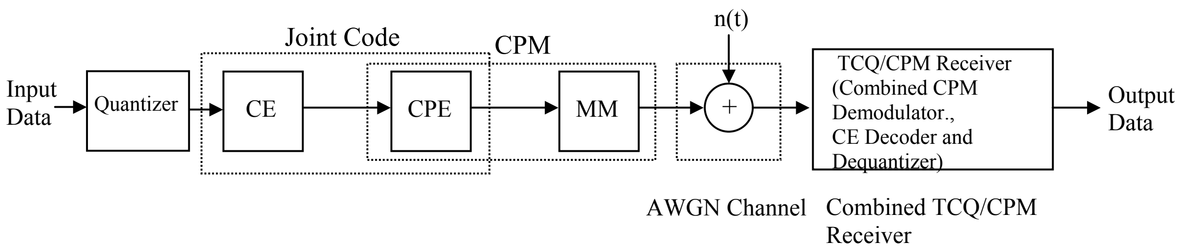


Figure 1 TCQ/CPM Block Diagram

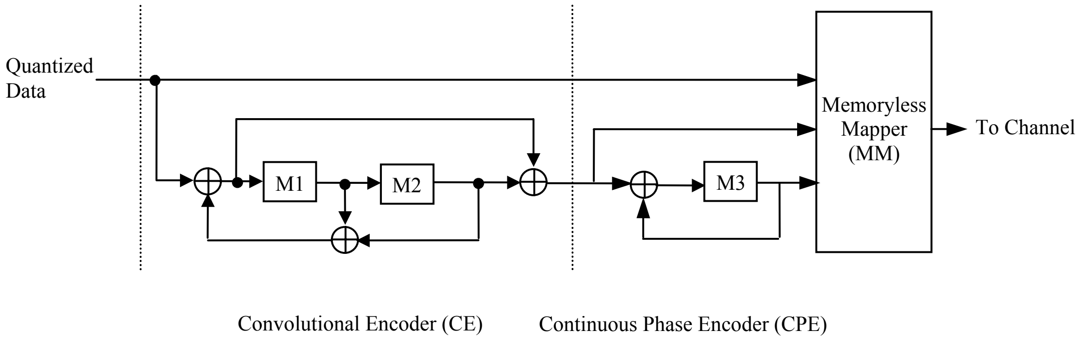


Figure 2 4-ary TCQ/TCCPM (h=1/2) System Encoder Structure

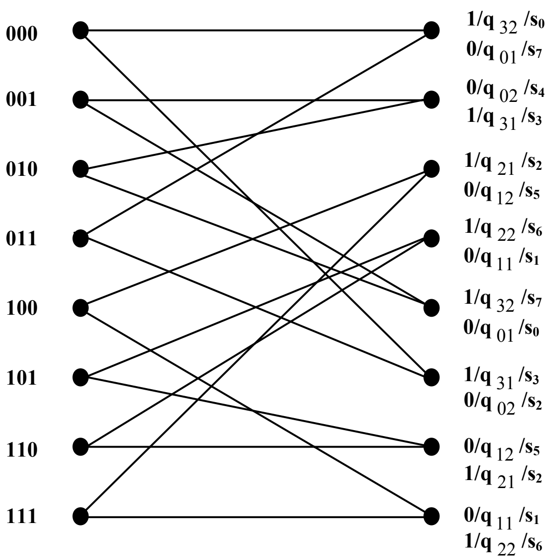


Figure 3 4-ary TCQ/CPM (h=1/2) System Trellis Diagram

Table 1 4-ary TCQ/CPM (h=1/2) System's Phase values and signal map informations

| $\theta_n$ | $\beta_n$ | Q | $s_i$ | $\theta_{n+1}$ |
|---|---|---|---|---|
| 0 | 0 | $q_{01}$ | $S_0$ | 0 |
| | 1 | $q_{02}$ | $S_1$ | $\pi$ |
| | 2 | $q_{11}$ | $S_2$ | 0 |
| | 3 | $q_{12}$ | $S_3$ | $\pi$ |
| $\pi$ | 0 | $q_{21}$ | $S_4$ | $\pi$ |
| | 1 | $q_{22}$ | $S_5$ | 0 |
| | 2 | $q_{31}$ | $S_6$ | $\pi$ |
| | 3 | $q_{32}$ | $S_7$ | 0 |

## 5. SIMULATION RESULTS

Simulation results are shown in figure 4 for AWGN channels for optimum and Llyod-max quantization levels. Here CPFSK modulation index h is 1/2 and the modulation input level, M, is 4. In this simulation the decode algotihm was chosen as Viterbi algorthm and it's decision depth of the VA was chosen to be equal 10. Our proposed TCQ/CPM method has better BER performance than other memoryless modulation techniques such as PSK,QAM etc. For h=1/2 and 4-ary TCQ/CPM system we have approximately 4.3dB coding gain for compare to 8-PSK TCQ/TCM system.

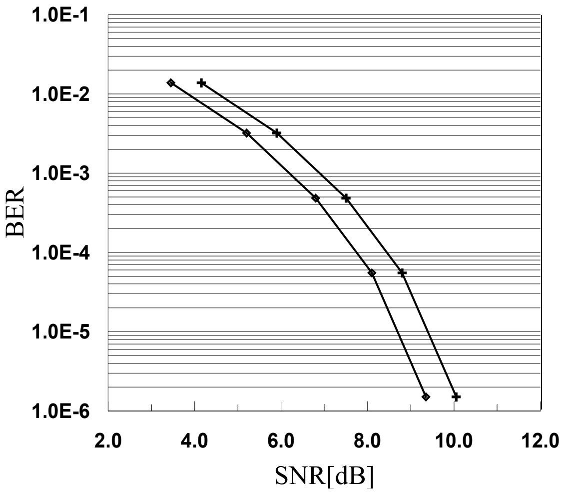


Figure 4 BER-SNR curves of 4-ary TCQ/CPM (h=1/2 ) system for K=∞

## 6. CONCLUSION

In this paper, TCQ/TCM signals are mapped to continuous phase frequency shift keying (CPFSK) signals which is a special type of continuous phase modulation (CPM). For this reason the system will have the continuous phase structure between the signals and it will have all the advantages of the continuous phase system had. Furthermore coding, quantization and modulation have been realized in one joint trellis structure. Consequently error performance of the system will give very good results, besides the band and power efficiency will be obtained.

As an example 4-ary TCQ/CPM (h=1/2) system have been designed, the error performance curves of this system in AWGN channels have been obtained with the aid of computer simulations and the error performance superiorities of the TCQ/CPM systems have been demonstrated.

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