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DECOMPOSABLE SEC-DED CODES OF ENHANCED AUGMENTED PRODUCT CODE CONSTRUCTION

Gökmen ALTAY

University of Bahçeşehir, Faculty of Engineering, Department of Electrical Electronic Engineering, Bahçeşehir 34900 İstanbul – Turkey

E-mail:galtay@bahcesehir.edu.tr

ABSTRACT

Enhanced augmented product code construction (EAPCC) and its decomposable SEC-DED (single-error-correcting/double-error-detecting) codes, which are composed by augmenting the simple component codes (2,1,2) and (2,1,1) on to a two dimensional general product code structure, are presented by this paper. Some of the best-known codes with minimum distance four that have been attained using this method have also been presented as a result of simulations.

Keywords: *Decomposable SEC-DED codes, product codes.*

1. INTRODUCTION

By combining simple component codes, decomposable codes can be constructed if it can be decomposed into component codes of smaller dimensions or shorter lengths so that less complexity can be provided in terms of encoding and decoding processes. Some of the decomposable codes are cubing construction [2] and $|a_1+x| \ldots |a_m+x|a_1+\ldots..+a_m+x+y|$ construction [3]. This study presents a new group of decomposable single-error-correcting/double-error-detecting (SEC-DED) codes that are constructed on to the structure of two–dimensioanal array codes and can be formed as shown in figure (4) and (5), which are described below in detail.

2. CODE CONSTRUCTION

A decomposable general product code, *C*, is formed by a direct product of two component codes that are $C_1 = (n_1,k_1,d_1)$ and $C_2 = (n_2,k_2,d_2)$. The generator matrix, *G*, of *C* is represented in the form of a *Kronecker product* of generator matrixes of its component codes, G_1 and G_2, as following [3]:

$G = G_1 \otimes G_2 = (g^{(1)}_{i,j}\, G_2)$, for $G_1 = (g^{(1)}_{i,j})$, $G_2 = (g^{(2)}_{i,j})$.

When G_2 is a single-parity-check (SPC) code as in figure (1), then G becomes as in figure (2) as a result of the operation $G = G_1 \otimes G_2$.

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$$G_2 = \begin{vmatrix} 1 & & & & & 1 \\ & 1 & & & & 1 \\ & & \cdot & & & \cdot \\ & & & \cdot & & \cdot \\ & & & & \cdot & \cdot \\ & & & & 1 & 1 \end{vmatrix}$$

Fig. (1)

$$G = \begin{vmatrix} G_1 & & & & & G_1 \\ & G_1 & & & & G_1 \\ & & \cdot & & & \cdot \\ & & & \cdot & & \cdot \\ & & & & \cdot & \cdot \\ & & & & G_1 & G_1 \end{vmatrix}$$

Fig.(2)

The new decomposable SEC-DED code C′ has been developed by augmenting simple component codes C_{Zi} =(2,1,1) onto the structure that is shown in figure (2). C′ is constructed by employing the following generator matrix, which is presented by figure (3) that is called enhanced augmented product code construction (EAPCC).

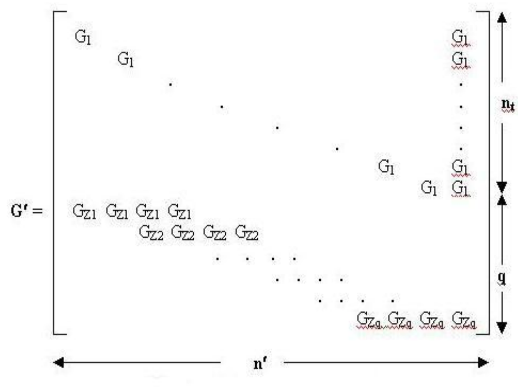

Fig. (3) EAPCC generator matrix [1].

Lets describe the parameters and set the basic criteria of EAPCC generator matrix to clarify the construction. $G_{Zi} = G_{Z1}, G_{Z2}, \ldots, G_{Zq}$ are the generator matrixes of augmenting component codes C_{Zi} where i = 1, 2, 3, … , q and the value of q is defined as the number of rows on which augmenting component codes are placed. Maximum value of q is specified as below.

$$q_{max} = \left[(n_2 - 2) / 2 \right] \quad , \; n_2 \geq 4 .$$

The parameter n refers to the length of the code C, n_2 refers to the length of C_2, n′ refers to the length of C′ and in figure (3), $n_t = n_2 - 1$. The parameter k refers to the dimension of the code C that is also expressed as the length of message in a C codeword; k′ refers to the dimension of C′ and maximum value of k′ is that $k'_{max} = n_2 - 1 + q_{max}$. The parameter d refers to the minimum distance of the code C and d′ refers to the minimum distance of C′.

Then, the final code, C′, is defined as C′ = (n′,k′,d′) = ($2n_2$, k′, 4) and can be formed as figure (4) for the most optimum case when $q = (n_2 - 2) / 2$. As the value of q decreases, a slight change occurs in the formation of figure (4) in such a way that is demonstrated in figure (5) as an example of how it is formed depending on the value of q. When $q = (n_2 - 2) / 2$, C can be formed as figure (4),

$| a_1+z_1 | a_2+z_1 | a_3+z_1+z_2 | a_4+z_1+z_2 | \ldots | a_{(n2-2)} + z_{q-1}+z_q | a_{(n2-1)}+z_q | a_1+a_2+\ldots+a_{(n2-1)} + z_q |$

Fig. (4).

And, when $q < (n_2 - 2) / 2$,

$| a_1+z_1 | a_2+z_1 | a_3+z_1+z_2 | a_4+z_1+z_2 | \ldots\ldots | a_{(n2-2)} + z_q | a_{(n2-1)} + z_q | a_1+a_2+\ldots+ a_{(n2-1)} |$

Fig. (5).

Where $a \in C_1$, $a = G_1 = [\ 1\ 1\ ]$, and $z_i \in C_{Zi}$, $z_i = G_{Zi} = [\ 1\ 0\ ]$, i = 1,2,…,q , are specified as component generator matrices and their values are fixed just as given so that optimum results can be obtained. All the basic conditions and limitations have been given above to construct a code utilizing EAPCC. Figure (4) is considered to be a default formation of EAPCC that gives the highest performance to produce codes with minimum distance four. The key aspect of this argument is that the size of the code, C′, barely depends on the value of n_2, as all the component codes have been specified. Above all, a condition must be set up on how to place augmenting component codes in the generator matrix G′ in terms of their position over rows and their values. Since the values are fixed as $a \in C_1$, $a = G_1 = [\ 1\ 1\ ]$, and $z_i \in C_{Zi}$, $z_i = G_{Zi} = [\ 1\ 0\ ]$ and G_2 is a single-parity-check (SPC) code, fig. (1), then the minimum distance of the final augmented product code always equal to four, for as long as the following criteria is provided:

i. The number of generator matrixes of augmenting component codes, G_{Zi}, on a row is equal to four and $z \in C_{Zi}$, $z = G_{Zi} = [\ 1\ 0\ ]$.
ii. The base product code structure, in figure (2), is fixed and is employed as shown in the generator matrix G′, in figure (3). Its size can only be changed by the value of n_2.
iii. When placing generator matrixes of augmenting component codes G_{Zi}, a crucial point is that, more than two G_{Zi} cannot be placed one on top of another among different augmenting rows regardless of which row and order. To attain optimum structure the generator matrix should be constructed as shown in figure (3).

EAPCC could produce (28, 19, 4) code that is considered to be the upper limit of its capability in terms of producing the best-known codes of minimum distance four, [4]. Lets see the implementation of EAPCC on (28, 19, 4) code. G is the base matrix that is built as in figure (2) in the size of 13x28 and written in G′ just as letter for simplicity.

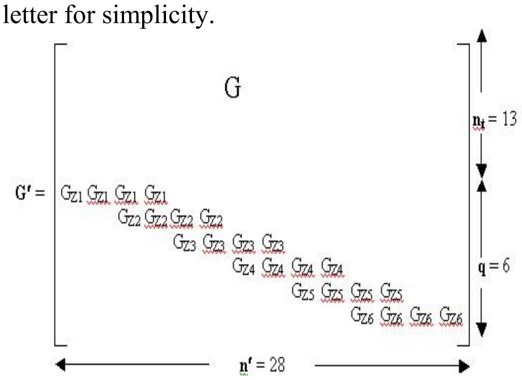

Fig. (6). EAPCC generator matrix for (28, 19, 4) code.

3. SIMULATION RESULTS

Finally, the simulation results are presented below by figure (7) in which all the best-known codes [4] that have been attained utilizing EAPCC are included. Beyond these codes, we can still obtain bigger size of SEC-DED codes using EAPCC but they will no longer be the best-known codes. Using similar approach, codes that have the minimum distance bigger than four is likely to be attained and a study on that is currently being conducted.

When, $C = (n,k,d)$, $C_1=(n_1,k_1,d_1)$, $C_{Zi} = (n_{Zi},k_{Zi},d_{Zi})$ and $C'=(n',k',d')$,

C	C_1	n_2	C_{Zi}	q	$C'=(n',k',d')$
(28,13,4)	(2,1,2)	14	(2,1,1)	6	**(28,19,4)**
(26,12,4)	(2,1,2)	13	(2,1,1)	5	**(26,17,4)**
(24,11,4)	(2,1,2)	12	(2,1,1)	5	**(24,16,4)**
(24,11,4)	(2,1,2)	12	(2,1,1)	4	**(24,15,4)**
(22,10,4)	(2,1,2)	11	(2,1,1)	4	**(22,14,4)**
(20, 9, 4)	(2,1,2)	10	(2,1,1)	4	**(20,13,4)**
(20, 9, 4)	(2,1,2)	10	(2,1,1)	3	**(20,12,4)**
(18, 8, 4)	(2,1,2)	9	(2,1,1)	3	**(18,11,4)**
(18, 8, 4)	(2,1,2)	9	(2,1,1)	2	**(18,10,4)**
(16, 7, 4)	(2,1,2)	8	(2,1,1)	3	**(16,10,4)**
(16, 7, 4)	(2,1,2)	8	(2,1,1)	2	**(16, 9, 4)**
(14, 6, 4)	(2,1,2)	7	(2,1,1)	2	**(14, 8, 4)**
(14, 6, 4)	(2,1,2)	7	(2,1,1)	1	**(14, 7, 4)**
(12, 5, 4)	(2,1,2)	6	(2,1,1)	2	**(12, 7, 4)**
(12, 5, 4)	(2,1,2)	6	(2,1,1)	1	**(12, 6, 4)**
(10, 4, 4)	(2,1,2)	5	(2,1,1)	1	**(10, 5, 4)**
(8, 3, 4)	(2,1,2)	4	(2,1,1)	1	**(8 , 4 , 4)**

Fig. (7)

4. CONCLUSION

This study has presented a sort of decomposable product code construction method that is newly developed and named as EAPCC method, in fig. (3), which is capable of producing codes with minimum distance four some of which have been demonstrated by fig. (7). An attractive part of the method, EAPCC, may be said that it is very systematic and quite easy to built codes with minimum distance four and needs easier decoding effort utilising multistage decoding since it consists of the most simple component codes (2,1,2) with generator matrix [1 1] and (2,1,1) with generator matrix [1 0].

5. REFERENCES

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