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MULTIVARIABLE CONTROL IN HELICOPTERS

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ABSTRACT

In this paper, general control approaches are evaluated for helicopter system. Control of a helicopter is its ability to respond to control inputs and achieve the desired condition of flight. There is an important contradiction between stability and controllability, since decent controllability does not necessarily exist with decent stability. In fact, a high degree of stability may tend to reduce the controllability of the aircraft.

Key Words: *Multvariable control, helicopter systems*

ÖZET

Bu makale de helicopter sistemlerin genel control yaklaþýmlarý ele alýnmýþtýr. Helikopterin kontrolu, giriþlere sistemin verdiði tepkinin kontrolu ve istenen uçuþ ortamýnýn oluþturulmasý ile tanýmlanabilir. Kontrol edilebilirlik ile kararlýlýk arasýnda önemli farklýlýklar mevcuttur. Yüksek dereceli kararlýlýk, helikopterin control edilebilirliðini azaltabilir.

Anahtar Kelimeler: *Helikopter, Çok deðiþkenli kontrol sistemleri*

1. INTRODUCTION

The purpose of the engine control is to make the pilot's job easier. The pilot's job is to fly the helicopter smoothly by managing the forces and the moments of the helicopter. An engine control contributes to this by maintaining a steady rotor speed at all times and in particular during rotor load changes. For a given forward air speed, rotor thrust is primarily a function of rotor speed and collective control angle; by maintaining steady rotor speed the pilot can manage rotor thrust directly with the collective pitch control.

Rotor speed changes are produced by imbalances between the rotor torque required and the torque produced by the engine. If the control balances rotor load changes, the rotor speed will remain steady. Base, in the "steady running line " engine model, the control technique is straightforward: modulate fuel flow to maintain rotor speed near its constant reference speed.

2. CONTROL THEORY

The most important aspects of design and analysis of helicopters are stability and control. As with the fixed wing airplane, the problem of controlling the aircraft was one of the major stumbling blocks in the development of a successful air vehicle. Today, designing for adequate handling qualities still remains a major concern in the development of helicopters, with new requirements continually demanding improved performance.

The flying or handling qualities of an aircraft can be defined as stability and control characteristics that have a major bearing on the safety of flight and on the pilot's impressions of ease of flying and maneuvering an aircraft.

2.1 Stability

An aircraft is in a state of equilibrium, or trimmed in steady flight, when the vector sum of all forces and all moments is equal to zero. In equilibrium, there is no acceleration in either rotational or translation motion and the aircraft remains in its trimmed condition. The term stability is used to describe the behavior of an aircraft after it has been disturbed from trimmed condition.

Maneuver stability or angle of attack stability describes the level of a characteristic produced by combined effects of the flapping back of rotor due to an increase in angle of attack

2.2. Control Sensitivity and Control Power

Two important control parameters are control power and control sensitivity. Control power or control authority refers to the maximum moment that can be generated on the aircraft by application of the cockpit controls. Control sensitivity refers to the moment generation capability per unit of control displacement, for instance per inch of cyclic stick movement. These two parameters are related and can be formulated in there different ways as follows[1]:

$$\text{Control Sensitivity} = \frac{control\_power}{rotor\_damping}$$

$$= \frac{\dfrac{control\_power}{stick\_displacement}}{\dfrac{damping\_moment}{angular\_velocity}}$$

$$= \frac{angular\_velocity}{stick\_displacement} \qquad (1)$$

Helicopters with conventional control systems are subject to high control sensitivity. The maximum roll rate achieved by a small helicopter may be as great as those of some modern fighter airplanes at the speeds for their maximum roll rates. This is not true because of high power control, but because of low damping, which, for the helicopters, is a fraction of that for airplanes. Also, high control sensitivity can lead to over controlling, which results in a short-period, pilot-induced lateral oscillation.

3. GOALS OF DESIGN

A typical set of design goals has been selected for control:

*Maintaining steady state rotor speed within 0.5% of reference speed. (pilot detection threshold)

*Constrain the control to minimum and maximum engine torque limits.

*For rotor load changes, return rotor speed smoothly to reference speed within one second, without oscillation and minimal overshoot.

Especially in this paper, the goal of designing the helicopter against the effects of gusts is to reduce the effects of atmospheric turbulence on helicopters. The reduction of the effects of gusts is very outstanding;

*To reduce pilot's workload.

*To enables aggressive maneuvers to be carried out in poor weather conditions.

*To reduce buffeting the airframe and components lives.

*To increase the passenger comfort.

*To robust stability and performance.

Several designs have used frequency information about the disturbance to limit the system sensitivity, but in general there has been no explicit consideration of the effect of atmospheric turbulence; Therefore by incorporating practical knowledge about the disturbance characteristics, and how they affect real helicopter, improvements to the overall performance should be possible.

We will use nonlinear helicopter model for simulation purposes, which was developed at the Defense Research Agency (DRA), Bedford and is known as the Rationalized Helicopter Model (RHM)[2]. A turbulence generator module has recently been included in the RHM and this enables controller designs to be tested on line for their disturbance rejection properties. In this paper, we design of a controller to diminish the atmospheric turbulence effects on helicopters and reduce the effects of gusts by using H-infinity mixed-sensitivity design and disturbance rejection design.

4. THE HELICOPTER MODEL

The aircraft model used in our work is representative of the Westland Lynx, a twin-engine multi-purpose military helicopter, approximately 9000 lbs gross weight, with a four-blade semi-rigid main rotor. The un-augmented aircraft is unstable, and exhibits many of the cross-couplings characteristics of a single main rotor helicopter. In addition to the basic rigid body, engine and actuator components, the model also has second order rotor flapping and coning modes for off-line use, the model has an advantage that the same model can be used for a real-time piloted simulations as for a work station-based off-line handling qualities assessment.

In first step, we will obtain an eight-order differential equation modeling the small-perturbation rigid motion of the aircraft about however [3].

Table 1. Helicopter's Eight State Rigid Body Vector

STATE	DESCRIPTION
θ	Pitch attitude
ϕ	Roll attitude
P	Roll rate (body-axis)
q	Pitch altitude (body-axis)
ξ	Yaw rate
V_x	Forward velocity
V_y	Lateral velocity
V_z	Vertical velocity

For forward flight, the rotor thrust axis is considered to tilt from the vertical axis by α_v, and the rotor is mounted on an aircraft moving at a constant speed V_x, along an inclined path. V_x is broken down into a vertical component V_z and a lateral component V_y.

$$V_x = V_y \cos \alpha_v + V_z \sin \alpha_v \quad (1)$$

The corresponding state-space model is

$$\dot{x} = Ax + Bu \quad (2)$$

$$y = Cx \quad (3)$$

where the matrices A, B, and C for the approximately scaled system.

The standard helicopter configuration being considered consists of: one main rotor, a tail rotor, a horizontal stabilizer, a collective stick, a latitudinal and longitudinal cyclic stick, and foot pedals for tail rotor collective. One system for main-rotor control, considered the basic configuration and the most common, uses a rotating swash plate as part of the main rotor cyclic system. An alternative to the swash plate is the spider system used on some westland designs. Lastly, on some Kaman helicopters both cyclic and collective controls are achieved by twisting the flexible blades by the forces from controllable servo flaps fixed to the training edge.

Our helicopter model (RHM)'s main rotor collective changes all the blades of the main rotor by an equal amount and so roughly speaking control lift. The longitudinal and lateral cyclic inputs change the main rotor blade angles differently thereby tilting the lift vector to give longitudinal and lateral motion, respectively. The tail rotor is used to balance the torque generated by the main rotor so that it stops the helicopter spinning around. It is also used to give lateral motion.

Ideally a helicopter should be designed to have such a good inherent flying qualities that the pilot requires no extra help. Many helicopters today are flyable without additional controls. Stability Augmentation Systems (SAS) improve upon the design by reducing the pilot workload and improving performance. SAS equipment improves flying qualities by damping the pitch and roll motions caused due to gusts and by installing small gyros to generate electrical signals proportional to pitch and roll rates. These signals are then used to control hydraulic or electrical actuators that tilt the swash plate in the right direction to resist the helicopter motion.

We are interested in the design of full-authority controllers, which means that the controller has total control over the blade angles of the main and tail rotors, and is positioned between the pilot and the actuator system. So the pilot has limited authority and only provides the reference commands.

Auto-plot type devices can increase stability and control. These systems can hold course, altitude and speed while flying by using signals from attitude gyros, altimeters, and air speed systems. The outputs of our helicopter model consists of four controlled outputs, which are;

*Heave velocity $\dot{H}$ $\Rightarrow$
*Pitch attitude θ $\Rightarrow$
*Roll attitude ϕ $\Rightarrow$ y_1
*Heading rate ψ $\Rightarrow$

Together with two additional (body-axis) measurements
*Roll rate P $\Rightarrow$ y_2
*Pitch rate q $\Rightarrow$

The four blade angles demands are helicopter inputs
*Main rotor collective $\Rightarrow$
*Longitudinal cyclic $\Rightarrow$ u
*Lateral cyclic $\Rightarrow$
*Tail rotor collective $\Rightarrow$

One Degree Of Freedom Controllers

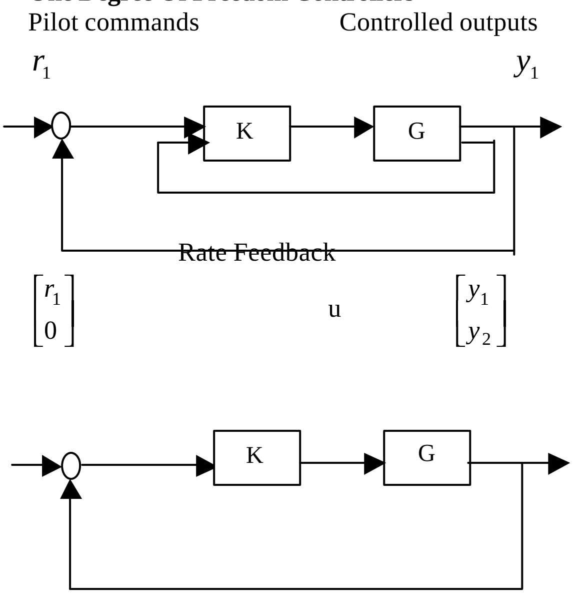

Figure 1. Helicopter Control Structure

In standard one degree of freedom configuration, a zero vector due to the rate feedback signals augments the pilot reference commands r_1. These zeros indicate that there are no a priori performance specifications on
$y_2 = [p \quad q]^T$.

H-Infinity Mixed-Sensitivity Design

The following assumptions are typically made in H_∞ mixed sensitivity design. With a state-space realization of generated plant P given by

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \quad (4)$$

1. $(A, B_2, C_2,)$ is stabilizable and detectable.

2. D_{12} and D_{21} have full rank.

3. $\begin{bmatrix} A - jwI & B_2 \\ C_1 & D_{12} \end{bmatrix}$
has full column rank for all w.

4. $\begin{bmatrix} A - jwI & B_2 \\ C_1 & D_{12} \end{bmatrix}$
has full row rank for all w.

5. $D_{11} = 0$ and $D_{22} = 0$

6. $D_{12} = \begin{bmatrix} 0 \\ I \end{bmatrix}$ and $D_{21} = [0 \quad I]$.

7. $D^T{}_{12} C_1 = 0$ and $B_1 D^T{}_{21} = 0$

8. (A, B_1) is stable and (A, C_1) is detectable.

Assumption 1 is required for existence of stabilizing controllers K.

Assumption 2 is needed to ensure the controllers are proper and realizable.

Assumption 3 and 4 ensure that the optimal controller does not try to cancel poles and zeros on the imaginary axis, which would result in closed loop instability.

Assumption 5 is required to make p_{11} and p_{22} strictly proper and to simplify the formulas in the algorithms.

Assumption 6 is used for simplicity.

Assumption 7 is used where there is no cross terms in the cost function and the process noise and measurement noise are uncorrelated.

Assumption 8 is used when if assumption 7 holds then assumption 3 and 4 maybe replaced by assumption 8.

Mixed sensitivity is name given to transfer function shaping problems in which the sensitivity function $S=(I+GK)^{-1}$ is shaped along with one or more other closed loop transfer functions such as KS or the complementary sensitivity function T=I-S

We have a regulation problem in which we want to reject a disturbance d entering at the plant output and it is assumed that the measurement noise is relatively insignificant. But it is important to include KS as a mechanism for limiting the size and bandwidth of the controller and hence the control energy used. The size of KS is also important for robust stability with respect to uncertainty modeled as additive plant perturbations.

S/KS Mixed-Sensitivity Minimization

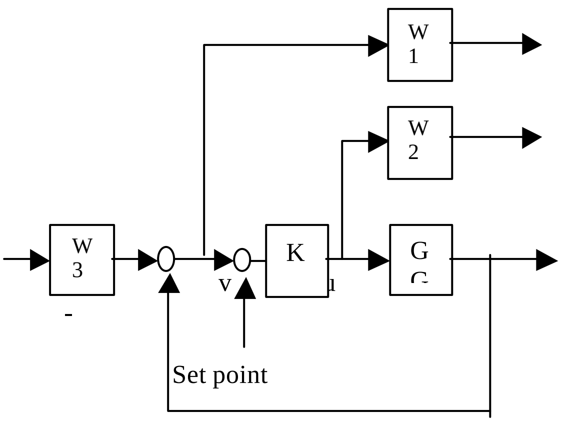

Figure 2. Input-Output Relation of S/KS Mixed Sensitivity Minimization Model

We can formulate general setting of mixed sensitivity problem by imagining the disturbance r as a single exogenous input and define an error signal $z=\left[z_1^{\,T} \quad z^{T}{}_2\right]^T$, where $z_1 = W_1\, y_e$ and $z_2 = W_2\, u$ as illustrated above [4].
$z_1 = W_1SW_3w$ and $z_2 = W_2KSw$ are required to determine the elements of the generated plant P as

$$P_{11}=\begin{bmatrix} W_1W_3 \\ 0 \end{bmatrix} \qquad P_{12}=\begin{bmatrix} -W_1G \\ W_2 \end{bmatrix} \tag{5}$$

$$P_{21}=-W_3 \quad P_{22}=G \tag{6}$$

Where the partitioning is such that

$$\begin{bmatrix} z_1 \\ z_2 \\ v \end{bmatrix}=\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}\begin{bmatrix} w \\ u \end{bmatrix} \tag{7}$$

$$F_1(P,K)=\begin{bmatrix} W_1SW_3 \\ W_2KSW_3 \end{bmatrix} \tag{8}$$

Here we consider a tracking problem. The exogenous input is a reference command d, and the error signals are $z_1 = W_1e = W_1(d-y_e)$ and $z_2 = W_2\, u$

We have in this tracking problem $z_1 = W_1SW_3\, w$ and $z_2 = W_2KS\, W_3w$. The cost function is as;

$$\left\| \begin{bmatrix} W_1SW_3 \\ W_2KSW_3 \end{bmatrix} \right\|_\infty \tag{9}$$

W_1 and W_2 are selected as loop shaping weights whereas W_3 is signal based. The disturbance r is typically a low frequency signal and therefore it will be successfully rejected if the maximum singular value of S is made small over the same low frequencies. To materialize this, we could pick a scalar low pass filter $W_1(s)$ with a bandwidth equal to that of the disturbance and then find a stabilizing controller that minimizes

$\|w_1 S w_3\|_\infty$ [5]. The cost function focuses on just one closed loop transfer function and it is useful in practice to minimize.

$$\left\| \begin{bmatrix} W_1 S W_3 \\ W_2 K S W_3 \end{bmatrix} \right\|_\infty \quad (10)$$

Where w_2 is a scalar high pass filter with a crossover frequency approximately equal to that of the desired closed loop bandwidth. In general, matrixes $W_1(S)$ and $W_2(S)$ can replace the scalar weighting functions $w_1(s)$ and $w_2(s)$. This can be useful for systems with channels of quite different bandwidths when diagonal weights are recommended.

Yue and Postlethwaite's controller [3] was successfully tested on a piloted flight simulator at DRA Bedford so we propose to use the same weight. The design weights were selected as

$$W_1 = diag\begin{cases} 0.5\dfrac{s+12}{s+0.012}, \\ 0.89\dfrac{s+2.81}{s+0.005}, \\ 0.89\dfrac{s+2.81}{s+0.005}, \\ 0.5\dfrac{s+10}{s+0.01}, \\ \dfrac{2s}{(s+4)(s+4.5)}, \\ \dfrac{2s}{(s+4)(s+4.5)} \end{cases} \quad (11)$$

To be precision in each of the controlled outputs the sensitivity function should be small. This recommends forcing integral action into the controller by selecting an s^{-1} shape in the weights incorporated to the controlled outputs.

These weights were given a finite gain of 500 at low frequencies. It is found that a finite reduction at high was useful in reducing overshoot; therefore high gain low pass filters were used in the primarily channels to give accurate tracking up to 6 rad/s.

Unmodelled rotor dynamics limits the bandwidth of W_1 about 10 rad/s. These channels are to improve the disturbance rejection properties around crossover 4 to 7 rad/s. This is possible by using second order band pass filters in the rate channels of W_1.

$$W_2 = 0.5\frac{s+0.0001}{s+10} I_4 \quad (12)$$

The same first order high pass filter is used in each channel with a corner frequency of 10 rad/s to limit input magnitudes at high frequencies and thereby limit the closed loop bandwidth.

The high frequency gain of W_2 can be augmented to limit fast actuator movement. The low frequency gain of W_2 was set to –100db to provide that the cost function is dominated by W_1 at low frequencies.

$$W_3 = diag\ \{1,1,1,1,0.1,0.1\} \quad (13)$$

W_3 is a weighting on the reference input r. The main goal of W_3 is to force equally good tracking of each of the primary signals [4].

The reduced weighting on the rates provides some disturbance rejection on these outputs, without them significantly affecting the cost function as in Figure 3.

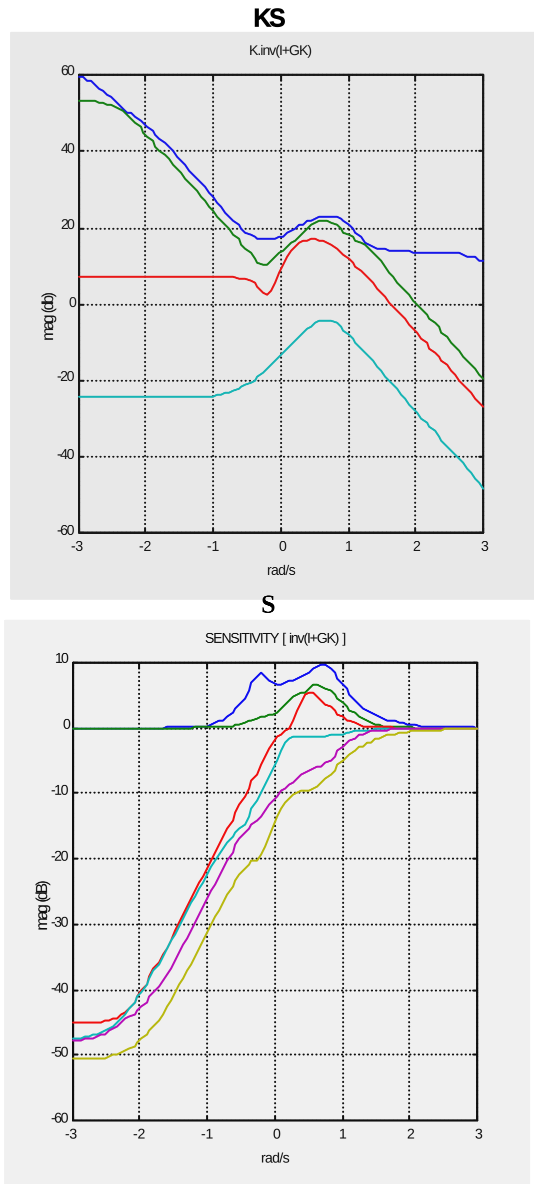

Figure 3. Singular Values of S and KS

Disturbance Rejection Design

In this design, we will assume that the atmospheric turbulence can be modeled as gust velocity components that perturb the helicopter's helicopter velocity states V_x, V_y and V_z by $d = [d_1 \quad d_2 \quad d_3]^T$ as in the equation below; therefore the disturbed system is

$$\dot{x} = Ax + A\begin{bmatrix} o \\ d \end{bmatrix} + Bu \qquad (14)$$

$$y = Cx \qquad (15)$$

B_d is assigned as the columns 6, 7, and 8 of A matrix and placed in the state-space model.

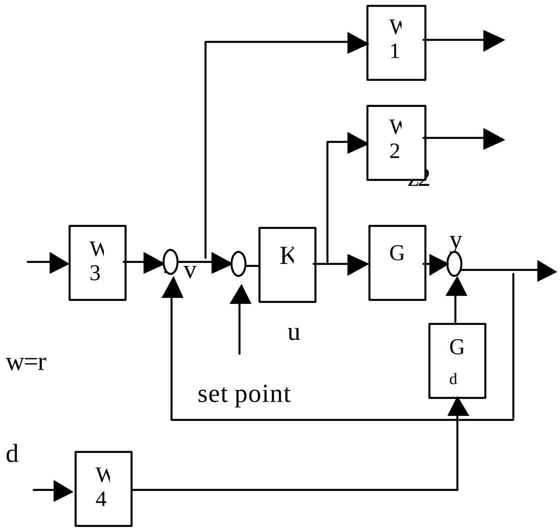

Figure 4. Disturbance Rejection Design

$$\dot{x} = Ax + Bu + B_d d \qquad (16)$$

The transfer function is;

$$y = G(s)u + G_d(s)d \qquad (17)$$

where

$$G(s) = C(sI - A)^{-1}B, \qquad (18)$$

$$G_d(s) = C(sI - A)^{-1}B_d \qquad (19)$$

We can formulate general setting of disturbance rejection design by imagining the disturbance r, the atmospheric turbulence disturbance d as exogenous inputs, and define an error signal

$$z = \left[z_1^{\ T} \quad z^T{}_2 \right]^T \tag{20}$$

The elements of the generated plant P:

$$P_{11} = \begin{bmatrix} W_1 W_3 \\ 0 \end{bmatrix}$$

$$P_{12} = \begin{bmatrix} -W_1 G & -W_1 W_4 G_d \\ W_2 & 0 \end{bmatrix}$$

$$P_{21} = -W_3$$

$$P_{22} = \begin{bmatrix} G & W_4 G_d \end{bmatrix} \tag{21}$$

where the partitioning is such that

$$\begin{bmatrix} z_1 \\ z_2 \\ v \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} r \\ d \\ u \end{bmatrix} \tag{22}$$

$$F_1(P,K) = \begin{bmatrix} W_1 S W_3 & -W_1 S G_d W_4 \\ W_2 K S W_3 & -W_2 K S G_d W_4 \end{bmatrix} \tag{23}$$

The optimization problem is to find a stabilizing controller K that minimizes the cost function,

$$\left\| \begin{bmatrix} W_1 S W_3 & -W_1 S G_d W_4 \\ W_2 K S W_3 & -W_2 K S G_d W_4 \end{bmatrix} \right\|_\infty \tag{24}$$

which is the H-infinity norm of the transfer function from $\begin{bmatrix} r \\ d \end{bmatrix}$ to z. If we adjust W_4 to zero, the problem reverts to the S/KS mixed sensitivity design [6].

We will use the same weights W_1, W_2 and W_3 as in the S/KS design. $W_4 = \alpha I$, with α a scalar parameter used to emphasize disturbance rejection. After a few iterations we finalized $\alpha=30$.

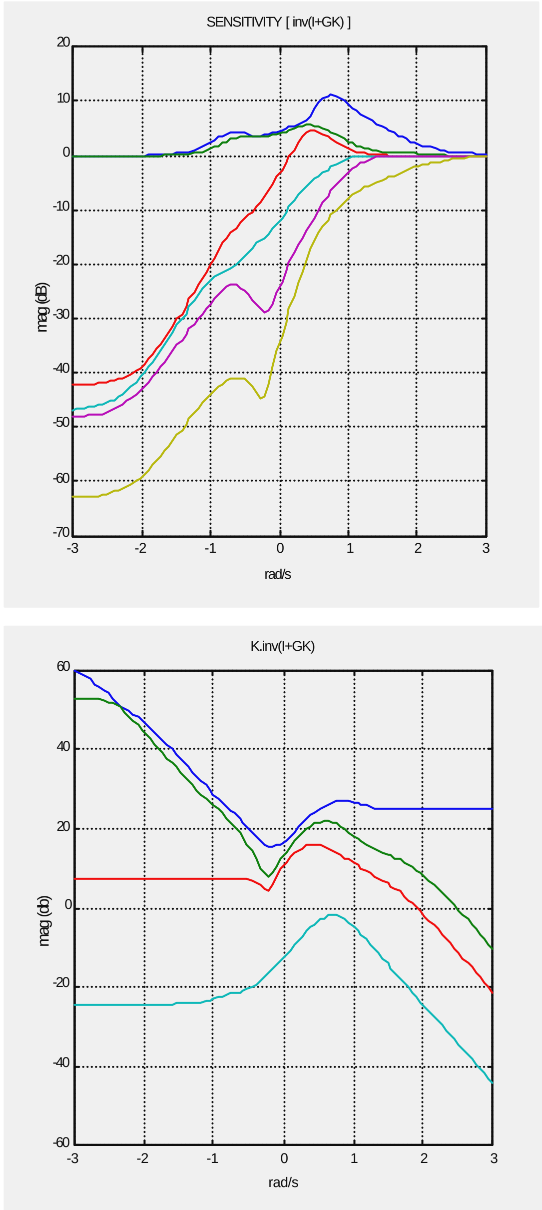

***Figure 5.** S/KS design output*

Comparison of Two Designs

We compared the disturbance rejection properties of two designs and simulated both controllers on our helicopter model with a discrete gust model for atmospheric turbulence. With our simulations help, gusts cannot be generated at hover and so the RHM nonlinear helicopter model was trimmed at a forward speed of 20 knots [1]. And we examined the effects of the gust on the four controlled outputs. If you examine the simulation test results, you will see the robustness of the controllers. While doing these tests, we applied different violent gusts to our helicopter model for two designs. In all cases the disturbance rejection design had strongly better results than the S/KS design.

The effects of the gust on the controlled outputs

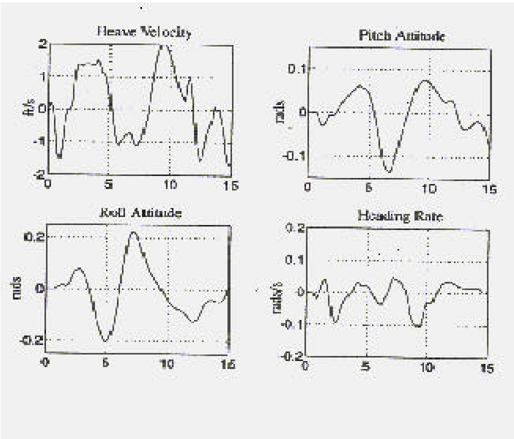

Figure 6. Response to turbulence of the S/KS design

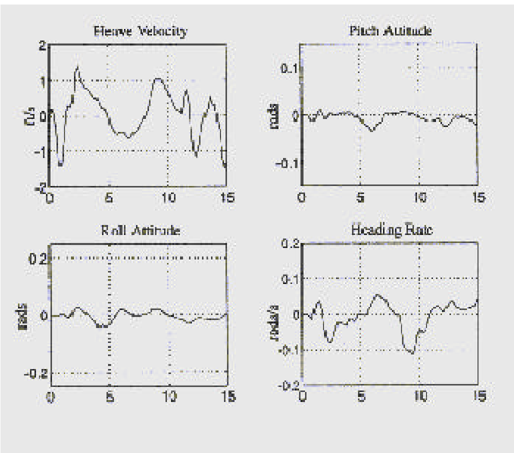

Figure 7. Response to turbulence of the disturbance rejection design

The disturbance rejection controller effectively reduces the turbulence effect on pitch and roll attitude if you compare with the S/KS design. The difference of pitch and roll attitude values between two designs are about %85-90. The disturbance rejection controller approximately 50% reduces the effects of gust on heavy velocity, but the heading rate variation between two designs is less than the other controlled outputs.

CONCLUSION

Problems for the helicopter control engineers are numerous. Helicopters are basically unstable and poorly damped. As aircraft have become increasingly more complicated, aids in design and development have become crucial. Today, engineers can analyze the model using one of the commercially available programs with a graphics package such as Matrix, CTRL C, and MATLAB for acceptable handling qualities. The next step is to integrate the model in a full up motion based simulator for validation. Finally, data collection techniques and system identification methodology help to minimize cost in the development of an aircraft design. These designs used to demonstrate and improve the effect of parametric changes in operating conditions. This clearly demonstrated the utility of computer modeling in the design and development of helicopter engine control systems.

By using designing methods we want to maintain steady state rotor speed within 0.5% of reference speed, to Constrain the control to minimum and maximum engine torque limits and to return rotor speed smoothly to reference speed within one second, without oscillation and minimal overshoot for rotor load changes. In this project, we designed two controller, which had the similar frequency domain properties and the same degree, but we can reduce the turbulence effects effectively on heave velocity, pitch and roll attitude by using disturbance rejection design, which integrates the knowledge about turbulence activity. This enables the reduction in a pilot's workload, more aggressive maneuvers to be carried out accurately and to augment the passenger and safety.

REFERENCES

[1] Francis, B. A. and Zames, G. (1984). On optimal sensitivity theory for systems, *IEEE Transactions on Automatic Control* **AC-2**9(1): 9–16.

[2] Dahl, H. J. and Faulkner, A. J. (1979). Helicopter simulation in atmospheric turbulence, *Vertica* pp. 65–78.

[3] Yue, A. and Postlethwaite, I. (1990). Improvement of helicopter handling qualities using H-infinity optimization, *IEEE Proceedings-D Control Theory and Applications* 137: 115–129.

[4] Anderson, B. D. O. (1986). Weighted Hankel-norm approximation: Calculation of bounds, *Systems & Control Letters* 7(4): 247–255.

[5] Anderson, B. D. O. and Liu, Y. (1989). Controller reduction: Concepts and approaches, *IEEE Transactions on Automatic Control* **34**(8): 802–812.

[6] Pernebo, L. and Silverman, L. M. (1982). Model reduction by balanced state space representation, *IEEE Transactions on Automatic Control* **AC-27**(2): 382–387.

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