H.HE et al. / IU-JEEE Vol. 12(1), (2012), 1431-1435

Simple Metrics for Turbo Code Interleavers

Hao HE[1], Mao TIAN[1], Zhenghai WANG[2], Dingcheng YANG[3], Wenjian ZHANG[4]

[1]School of Electronic Information, Wuhan University, Wuhan 430072, China
[2]Southwest China Institute of Electronic Technology, Chengdu 610036, China
[3]InformationSou Engineering School, Nanchang University, Nanchang 330031, China
[4]Jiangsu Posts & Telecommunications Planning and Designing Institute Co.Ltd, Nanjing 210006, China
Email: {haohe,mtn,zhangwenjian}@whu.edu.cn zhh.wang.whu@gmail.com ydcxuanyuan@msn.com

Abstract: *In this paper, we consider the searching for good turbo code interleavers. Especially, we focus on the permutation polynomial interleavers. Three key properties (i.e. spread factor, dispersion, and edge effects) of turbo code interleavers are investigated, and two new metrics alpha and beta are proposed for turbo code interleavers. Good permutation polynomial interleavers for turbo codes of various lengths are found by these metrics. Computer simulations show that interleavers with small α or β will lead to the poor performance of turbo codes. Furthermore, turbo codes with good permutation polynomial interleavers found by these metrics always outperform the well-known s-random interleavers. Moreover, turbo codes with good cubic permutation polynomial interleavers found by these metrics are superior to the quadratic permutation polynomial interleavers adopted as turbo code interleavers in the 3rd generation partnership project long term evolution and its advance.*

Keywords: *Channel Coding, Turbo Codes, Interleavers, Permutation Polynomial Interleavers*

1. Introduction

Facing to the ever-increasing growth of user demands, practical constraints (e.g. scarce spectrum, processing delay, resource requirment, *etal.*) place more and more obstacles to the advances of wireless communication theories and practices[1, 2]. Since the adorable properties of permutation polynomial interleavers, e.g. the compact mathematical presentation and maximum contetion-free ability[2-4], which can perfectly solve the aforementioned problems, the 3rd generation partnership project long term evolution and its advance (3GPP LTE and LTE-A) [3], which aim at providing advanced services with very high speed and low lantency, have adopted turbo codes using permutation polynomial interleavers (PP interleavers).

Interleaving is a key component for the error correcting performance of turbo codes [3-5]. An interleaver is a device which reorders its input sequence, and can be always represented by a one-to-one mapping $f: S_K = \{0,1,\ldots,K-1\}$ given by $x \rightarrow f(x)$ [5-6]. Denote the set of all possible interleavers of length K by I_K. Since the cardinality of I_K is as large as $K!$, it is necessary that effective interleaver metrics are defined to rapidly search good interleavers for turbo codes.

In this paper we focus on the optimization of qudratic and cubic PP interleavers for turbo codes. The rest of the paper is organized as follows. In Section 2, the properties of interleavers relevant to the performance of turbo codes are discussed and two new metrics are proposed to seek good interleavers for turbo codes. Then, good permutation polynomial interleavers are founded by the methods based on the both new metrics in Section 3. Finally, simulations and conclusions are given in Section 4 and 5, respectively.

2. Turbo Code Interleavers

In a turbo coding scheme, interleaving is employed before the information sequence is encoded by the second component encoder [5, 6]. The first role of interleaving is to construct a long random code due to its Shannon limit approaching capability [5]. Second, a turbo code interleaver is designed to transform low-weight parity sequence of the first constituent code into high-weight parity sequence of the second constituent code [6-8]. Hence, it increases the turbo codes free distance and reduces the multiplicity of low-weight codewords. The final function of the interleaver is to spread the outputs from one decoder to provide the other with less correlated inputs [9, 10].

The bit error rate of turbo codes over an additive white Gaussian noise (AWGN) channel is upper bounded by Eq.1 [7]:

$$P_b \le \sum_{d=d_{free}}^{+\infty} \frac{A_d \omega_d}{K} Q\left(\sqrt{d\frac{2RE_b}{N_0}}\right) \qquad (1)$$

Where d_{free}, R, E_b/N_0, A_d and ω_d denote the free distance, code rate, signal to noise ratio per information bit, the multiplicity and average information weight for codewords of weight d, respectively [7]. Typically, the performance of turbo codes is dominated by the first few terms of the distance spectrum, which are produced by the low-weight self-terminating input sequences and the edge effects caused by the termination of convolution codes into block codes [5-8]. It is well-known that an interleaver for turbo codes has three important properties (i.e. spread factor, dispersion, and edge effects). The spread factor determines the effective free distance [3, 5-7], and the dispersion gives a proportional reduction in the multiplicities of low-weight codewords [3, 7]. Edge effects refer to the implications on the distance spectrum resulting from the termination of the encoder [7, 10]. Due to the termination, a low-weight parity word can be produced, even though the encoder input does not force the encoder back to the zero state.

Given an interleaver f , its dispersion D can be defined as:

$$D = \left|C(f)\right| \Big/ \binom{2}{K} \quad (2)$$

Where C is the set defined as:

$$C(f) = \left\{\left(j-i, f(j)-f(i)\right) \middle| 0 \le j < i < K\right\} \quad (3)$$

The set C is composed by the pairs of the distance between any two elements in the original input sequence and the distance of these elements interleaved by f . The dispersion D evaluates the randomness of the interleaver pattern. Obviously, a high dispersion indicates a high randomness.

The spread factor S of f is defined as:

$$S = \min\left(|i-j| \bmod(K) + |f(i)-f(j)| \bmod(K)\right) \quad (4)$$

Where $0 \le i < K$, $0 \le j < K$ and $i \ne j$. Clearly, by an interleaver with large spread factor, any two adjacent elements in the original input sequence will be separated far away in the permuted sequence.

For turbo codes using dual-termination and MAP decoding algorithm, an interleaver with large dispersion and spread factor is more suitable for turbo codes. Normally, turbo codes always use block interleavers. However, the majority of block interleavers are prone to small spread factor S or dispersion D . The asymptotic free distance of turbo codes is linear to the logarithm of K [12] and the spread factor S is upper bounded by $\sqrt{2K}$ [3]. Besides, the dispersion D gives a proportional reduction in the multiplicities of low-weight codewords. Hence, to rapidly seek block interleavers with large spread factor and dispersion simultaneously, the following metric can be defined for a turbo code interleaver f :

$$\alpha = D \ln(S) \quad (5)$$

The metric α is the product of the dispersion D and the logarithm of spread factor S . Apparently, a large enough metric α is necessary for a good turbo code interleaver. However, the performance of turbo codes depends not only on the spread factor and dispersion but also on the edge effects. For a codeword produced by an input sequence of weight 1, its weight can be upper bounded by Eq.6:

$$\omega_1 \le (K-i) + (K-f(i)) + 1 + \omega_{tail} \quad (6)$$

Where i is the index of the nonzero bit in the input sequence and ω_{tail} is the weight of the tail bits. For codewords produced by all input sequences of weight 1, their minimum weight can be estimated by Eq.7:

$$\omega_1 \le \beta + 1 + \omega_{tail} \quad (7)$$

Where $\beta = min(K - i + K - f(i))$. We define β as the edge effects factor of f to measure the minimum sum of the distances between an arbitrary bit position and its permuted bit position to the edge of the input sequences. And the definition of β can be simplified as bellow:

$$\beta = \min\left(K - i + K - f(i)\right) \Leftrightarrow \beta' = \max\left(i + f(i)\right) \quad (8)$$

Due to the edge effects, turbo codes using a block interleaver with large α does not sufficiently mean that they have a good distance spectrum. E.g. if $f(K-1) = K-1$ (i.e. $\beta = 2$), the free distance of turbo codes using two identical, parallel, 8-state, rate 1/3, recursive systematic convolutional encoders with polynomials $(13,15)_8$ is no more than 11. Clearly, a large metric β or a small metric β' is also necessary for a good turbo code interleaver.

3. Search Good PP Interleavers For Turbo Codes

Considering the memory and fast decoding requirements, PP interleavers (i.e. Eq.9) with compact formulations and maximum contention-free properties [8, 9] are preferred by practical turbo coded communication systems as the 3GPP LTE and LTE-A [13].

$$f(x) = \left(\sum_{n=0}^{n} f_n x^n\right) \bmod(K) \quad (9)$$

The sufficient and necessary conditions for the coefficients f_n of a polynomial over integer ring $\mathbb{Z}_K$ to be a PP interleaver are summarized in [14, 15]. By introducing high degree terms in the relative prime algebraic PP (i.e. $n=1$ in Eq.9) interleavers, the PP interleavers will tend to lower the spread factor somewhat and provide sufficient irregularity (i.e. randomness), and turbo codes with PP interleavers will achieve outstanding error correcting performance [3, 7]. In order to search PP interleavers with both large metrics α and β , we propose the following method:

1).Select interleavers P_K from PP_K with $\beta > d$;

2).Select interleavers with maximum α from set P_K .

The interleaver set PP_K is composed by all PP interleavers of degree n and length K , and the constant d can be set to a value near the upper bound of the minimum distance of turbo codes with length K [12]. For quadratic (i.e. $n=2$ in Eq.9) PP (QPP) and cubic (i.e. $n=3$ in Eq.9) PP (CPP) interleavers of length $K=1184$, $K=592$, and $K=104$, some good interleavers searched by the proposed method are listed in TABLE 1-3. And some interleavers with smaller α or β are also listed in these tables for comparisons.

Table 1. Interleavers for comparison, *K*=1184

f(*x*)	*S*	*D*	α	β
$19x+74x^2$	20	0.018	0.053	64
$923x+74x^2$	20	0.018	0.053	26
$441x+148x^2+74x^3$	26	0.017	0.056	48
$103x+148x^2+74x^3$	8	0.017	0.035	30

Table 2. Interleavers for comparison, *K*=592

f(*x*)	*S*	*D*	α	β
$19x+74x^2$	20	0.018	0.053	26
$93x+74x^2$	20	0.018	0.053	20
$113x+148x^2+74x^3$	20	0.019	0.057	32
$75x+148x^2+74x^3$	2	0.018	0.012	2

Table 3. Interleavers for comparison, *K*=104

f(*x*)	*S*	*D*	α	β
$7x+26x^2$	8	0.053	0.115	14
$11x+26x^2$	8	0.057	0.116	10
$33x+26x^2+26x^3$	8	0.065	0.134	24
$33x+52x^2+26x^3$	8	0.062	0.129	8

Table 4. CPPs with maximum α and large β

K	*f*(*x*)	α	β
40	$13x+10x^2+10x^3$	0.220	14
104	$33x+26x^2+26x^3$	0.134	24
160	$38+99x+20x^2+10x^3$	0.243	32
592	$113x+148x^2+74x^3$	0.057	32
640	$431x+50x^2+10x^3$	0.237	88
768	$138+138x+18x^2+18x^3$	0.274	86
1184	$441x+148x^2+74x^3$	0.056	48

In fact, when a turbo code interleaver is circularly shifted (i.e. $f_0 \neq 0$ in Eq.9), its metric α almost remains unchanged, but its metric β varies sharply. Therefore, we also can first find the PP interleaver with maximum α .Then, if the metric β of the PP interleaver is less than d , a circular shift f_0 which can maximize the metric β of the interleaver is applied. By this method, we find some good CPP interleavers as listed in TABLE 4.

Compared to the former searching method, the latter usually can find interleavers with larger metric α and β , but extra memories are needed for the storage of the circular shift constant f_0 .

4. Simulation Results

Block error rate (Bler) curves for turbo codes with the PP interleavers listed in TABLE 1-3 are shown in Fig. 1-3, respectively. We use two identical component encoders with generator polynomials $(13,15)_8$, and assume QPSK modulation and AWGN channel. The decoding is performed with the improved max-log-MAP algorithm [16] with 8 iterations.

These results show that an interleaver with small α or β will lead to the poor performance of turbo codes, and it is necessary for a turbo code interleaver to have both large metrics α and β . In Fig.1, the poor performance of turbo codes with $f(x)=103x+148x^2+74x^3 \bmod(1148)$ is incurred by the small α . And the small β leads to the high error floor of turbo codes with $f(x)=923x+74x^2 \bmod(1148)$. In Fig.2, the poor performance of turbo codes using $f(x)=75x+148x^2+74x^3 \bmod(592)$ is caused by both small metrics α and β . In Fig.3, the small β leads to the high error floor of turbo codes with $f(x)=33x+52x^2+26x^3 \bmod(104)$. Since the best CPP interleavers (i.e. the CPP interleavers in the 4th row of TABLE 1-3) always have larger metrics α than the best QPP interleavers (i.e. the QPP interleavers in the second row of TABLE 1-3), turbo codes using the best CPP interleavers always outperform the best QPP interleavers as observed in Fig.1-3. Moreover, it is worth to be noted that, the best QPP interleavers, found by the proposed method as listed in TABLE 3, have already been adopted as turbo code interleavers in LTE and LTE-A [13] which are the 4th generation standards of radio technologies designed to increase the capacity and speed of mobile telephone networks.

In Fig.1-3, the performance of turbo codes with the well-known s-random interleaver is also shown and the typical assumption for s-random interleaver $s=\sqrt{K/2}$ is assumed. We can see that turbo codes using the QPP and CPP interleavers with large metrics α and β always outperform the well-known s-random interleavers.

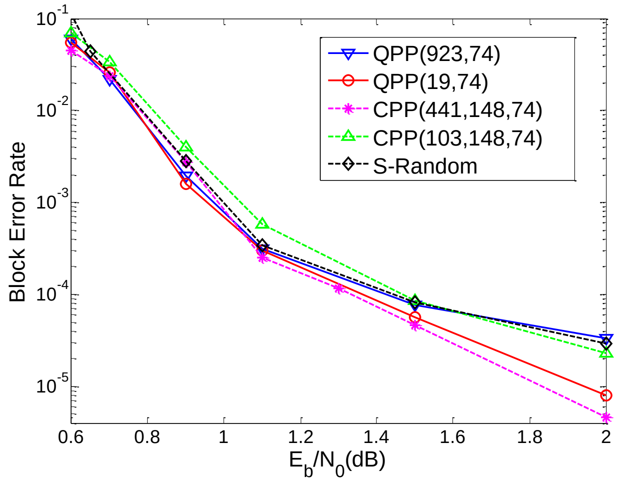

Fig 1. Bler comparison of turbo codes, *K*=1184

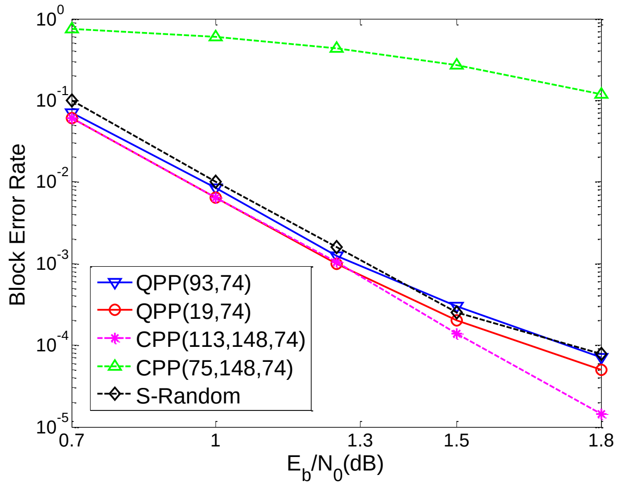

Fig 2. Bler comparison of turbo codes, *K*=592

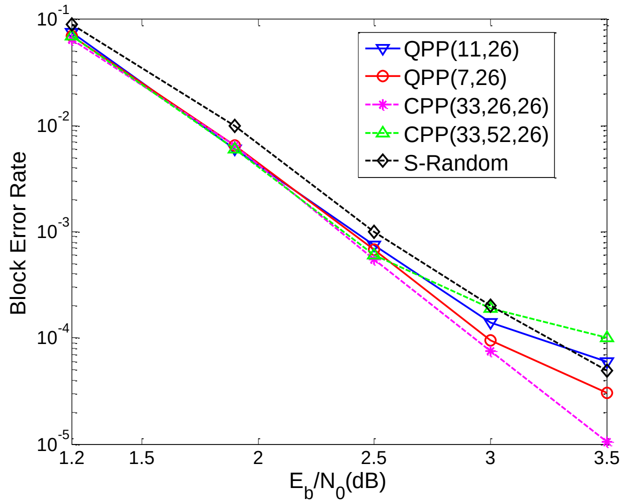

Fig 3. Bler comparison of turbo codes, *K*=104

5. Conclusions

In this paper, the spread factor, dispersion and edge effects of turbo code interleavers were studied, and we proposed two simple metrics α and β based on them to search good PP interleavers for turbo codes. Through simulation, we have proved that an interleaver with small metrics α or β will lead to the poor performance of turbo codes, and it is necessary for a turbo code interleaver to have both large metrics α and β. Furthermore, the turbo code interleavers found by the proposed metrics always perform much better than the benchmark s-random interleavers. Besides, we have also found some good CPP interleavers outperform the QPP interleavers which have been adopted as turbo code interleavers in LTE and LTE-A.

6. References

[1] S. Parkvall, A. Furuskär, and E. Dahlman, Evolution of LTE toward IMT-advanced. IEEE Commu. Mag., vol.49, no.2, pp.84-91, 2011.

[2] E. Dahlman, S. Parkvall, J. Skold and P. Beming, 3G Evolution: HSPA and LTE for Mobile Broadband, New York: Academic Press, 2008.

[3] O.Y.~Takesita, Permutation polynomial interleavers: an algebraic-geometric perspective. IEEE Trans. Inform. Theory, vol.53, no.6, pp.2116-2132, 2007.

[4] J.~Sun and O.Y.~Takesita, Interleavers for turbo codes using permutation polynomials over integer rings. IEEE Trans. Inform. Theory, vol.51, no.1, pp.101-119, 2005.

[5] C. Berrou, A. Glavieux, Near optimum error correcting coding and decoding: turbo-codes. IEEE Trans. Commu., vol.44, no.10, pp.1261-1271, 1996.

[6] B.~Vucetic, Y.~Li and L.C.~Perez, Recent advances in turbo code design and theory. Proceedings of IEEE, vol.95, no.6, pp.1323-1344, 2007.

[7] C.L.~Perez, J.~Seghers and D.J.~Costell, A distance spectrum interpretation of turbo codes. IEEE Trans. Inform. Theory, vol.42, no.6, pp.1698-1709, 1996.

[8] C.~Berrou, S.K.Y.~Saouter, and M.~Jezequel, Designing good permutations for turbo codes:towards a single model. in Proc. International Conference on Communications, vol.1, pp.341-345, Paris, France, 2004.

[9] J.~Hokfelt, O.~Edforts, and T.~Maseng, A turbo code interleaver design criterion based on the performance of iterative decoding. IEEE Commun. Lett., vol.5,no.2, pp.52-54, 2001.

[10] J.~Hokfelt, O.~Edfors and T.~Maseng, A survey on trellis termination alternatives for turbo codes. in Proc. 49th Vehicular Technology Conference, vol.3, pp.2225-2229, 1999.

[11] O.Y.~Takesita, A new metric for permutation polynomial interleavers. in Proc. 2006 International Symposium on Information Theory, pp.1983-1987, Seattle,USA, 2006.

[12] M.~Breiling, A logarithmic upper bound on the minimum distance of turbo codes. IEEE Trans. Inform. Theory, vol.50, no.8, pp.1692-1710, 2004.

[13] 3GPP: 3rd Generation Partnership Project. 3GPP TS 36.212. http://www.3gpp.org/ftp/Specs/2010-06/Rel-9/36_series/, 2010.

[14] H.~Zhao and P.~Fan, Simple method for generating mth-order permutation polynomianls over integer rings. Electron. Lett., vol.43, no.8, pp.449-451, 2007.

[15] H.~Zhao and P.~Fan, A note on 'a simple coefficient test for cubic permutation polynomials over integer rings'. IEEE Commun.Lett., vol.11 no.12, pp.991, 2007.

[16] J.~Vogt and A.~Finger, Improving the max-log-MAP turbo decoder. Electron. Lett., vol.36, no.23,pp.1937-1939, 2000.

Hao HE is a Ph.D. candidate in Wireless communication, Wuhan University. His research interests include channel coding, cooperative communication.

Mao TIAN is a professor and doctoral supervisor at School of Electronic Information in Wuhan University. His research interests include radio propagation and wireless communication.

Zhenghai WANG received his Ph.D. degrees from Wuhan University in Wireless communication. His research interest is channel coding.

Dingcheng YANG is a lecturer in Nanchang University. His research interest is cooperative communication.

Wenjian ZHANG received his Ph.D. degrees from Wuhan University. His research interests is cooperative communication.