

---

## HACETTEPE JOURNAL OF MATHEMATICS AND STATISTICS

Volume 49 Issue 3 (2020)

---

E-ISSN: 2651-477X

### **Honorary Editor**

Lawrence Michael Brown

### **Editor in Chief**

#### **Mathematics**

Ayşe Çiğdem Özcan (Hacettepe University, Mathematics - hjms@hacettepe.edu.tr)

#### **Statistics**

Çağdaş Hakan Aladağ (Hacettepe University, Statistics - statisticshjms@gmail.com)

### **Associate Editors**

Bülent Sarac (Hacettepe University, Mathematics - bsarac@hacettepe.edu.tr)

Aslı Yıldız (Hacettepe University, Mathematics - aslipekcan@hacettepe.edu.tr)

Ceren Eda Can (Hacettepe University, Statistics - cerencan@hacettepe.edu.tr)

Derya Ersel (Hacettepe University, Statistics - dtektaş@hacettepe.edu.tr)

### **Production Editors**

Gülbünu Tekbulut (Hacettepe University - dergilerhacettepe@gmail.com)

O. Oğulcan Tuncer (Hacettepe University, Mathematics - otuncer@hacettepe.edu.tr)

Talha Arıkan (Hacettepe University, Mathematics - tarikan@hacettepe.edu.tr)

Nurbanu Bursa (Hacettepe University, Statistics - nurbanubursa@hacettepe.edu.tr)

Erhan Pişirir (Hacettepe University, Statistics - erhanpisirir@hacettepe.edu.tr)

Derya Turfan (Hacettepe University, Statistics - deryaturfan@hacettepe.edu.tr)

### **Area Editors**

#### **Mathematics**

Evrim Akalan (Associative Rings and Algebras - eakalan@hacettepe.edu.tr)

Okay Çelebi (Partial Differential Equations - acelebi@yeditepe.edu.tr)

Olgür Çelikbaş (Commutative Rings and Algebras - olgur.celikbas@math.wvu.edu)

Angel Del Rio (Algebra, Group Theory - adelrio@um.es)

Gülin Ercan (Algebra, Group Theory - ercan@metu.edu.tr)

Sergio Estrada (Homological Algebra - sestrada@um.es)

Rodrigo Hernandez Gutierrez (General Topology - rod@xanum.uam.mx)

Varga Kalantarov (Partial Differential Equations - vkalantarov@ku.edu.tr)

Emre Mengi (Numerical Analysis - emengi@ku.edu.tr)

Cihan Orhan (Analysis, Summability - orhan@science.ankara.edu.tr)

Murad Özaydin (Differential Geometry, Global Analysis - mozaydin@math.ou.edu)

Abdullah Özbekler (Ordinary Differential Equations - aozbekler@gmail.com)

Ekin Özman (Number Theory, Algebraic Geometry - ekin.ozman@boun.edu.tr)

Serap Öztop Kaptanoğlu (Abstract Harmonic Analysis - oztops@istanbul.edu.tr )

Mehmetçik Pamuk (Topology, Manifolds and Cell Complexes - mpamuk@metu.edu.tr)

Bülent Ünal (Differential Geometry - bulent@fen.bilkent.edu.tr)

Yunus E. Zeytuncu (Functions of Several Complex Variables - zeytuncu@umich.edu)

## Statistics

- Ali Allahverdi (Operational Research & Statistics - ali.allahverdi@ku.edu.kw)  
Olcay Arslan (Robust Statistics - oarslan@ankara.edu.tr)  
Narayanaswamy Balakrishnan (Applied Statistics, Theory of Statistics - bala@mcmaster.ca)  
Adil Baykasoglu (Operational Research - adil.baykasoglu@deu.edu.tr)  
Sat Gupta (Sampling, Time Series - sngupta@uncg.edu)  
Yasemin Kayhan Atilgan (Computational Statistics & Mathematical Statistics - ykayhan@hacettepe.edu.tr)  
Birdal Senoglu (Experimental Design, Statistical Distributions - senoglu@science.ankara.edu.tr)  
Tahir Hanalioglu (Stochastic Processes Theory, Probability Theory - tahirkhaniyev@etu.edu.tr)  
Zeynep Işıl Kalaylıoglu (Bayesian Inference, Model Selection - kzeynep@metu.edu.tr)

## Contents

### Mathematics

#### Research Articles

1	D-extending modules <i>by Yilmaz Durđun</i>	914
2	Analysis of an epidemic model for transmitted diseases in a group of adults and an extension to two age classes <i>by Meltem Gölgelei and Fatihcan M. Atay</i>	921
3	On subgroup topologies on fundamental groups <i>by M. Abdulla Rashid, Noorollah Jamali, Behrooz Mashayekhy, Seyyed Z. Pashaei and Hamid Torabi</i>	935
4	Relative bi-ideals and relative quasi ideals in ordered semigroups <i>by Noor Mohammad Khan and Md. Firoj Ali</i>	950
5	On approximation of hexagonal Fourier series in the generalized Hölder metric <i>by Hatice Aslan and Ali Guven</i>	962
6	A numerical algorithm for solving the Cauchy singular integral equation based on Hermite polynomials <i>by B. Parsa Moghaddam, J.A. Tenreiro Machado, Parisa Sattari Shajari and Zeynab S. Mostaghim</i>	974
7	Asymptotically isometric copies of $\ell^{1\oplus 0}$ <i>by Veysel Nezir</i>	984
8	Multiplicative order convergence in $f$ -algebras <i>by Abdullah Aydin</i>	998
9	On anti-Kähler manifolds with complex semi-symmetric metric $F$ -connection <i>by Cagri Karaman and Aydin Gezer</i>	1006
10	Covering morphisms of topological internal groupoids <i>by Osman Mucuk and H. Fulya Akiz</i>	1020
11	Linear positive operators constructed by using Beta-type bases <i>by Octavian Agratini</i>	1030
12	Complex and Kähler structures on hom-Lie algebras <i>by E. Peyghan and L. Nourmohammadifar</i>	1039
13	On $K$ -pseudoframes for subspaces <i>by Hamide Azarmi, Mohammad Janfada and Rajab Ali Kamyabi-Gol</i>	1057

14	Parity numerical semigroups by María Ángeles Moreno-Frías and José Carlos Rosales . . . . .	1067
15	New generalizations of modular spaces by Tayebe Lal Shateri . . . . .	1076
16	Traveling wave solutions in a higher dimensional lattice competition-cooperation system with stage structure by Yanli He and Kun Li . . . . .	1084
17	Ladders and fan graphs are cycle-antimagic by Martin Bača, Pon Jeyanthi, Narayananperumal Thillaiammal Muthuraja, Pothukutti Nadar Selvagopal and Andrea Semaničová-Feňovčíková . . . . .	1093
18	New collocation scheme for solving fractional partial differential equations by Afshan Kanwal Chang Phang and Jian Rong Lo . . . . .	1107
19	Additive maps on prime and semiprime rings with involution by A. Alahmadi, H. Alhazmi, S. Ali, N.A. Dar and A.N. Khan . . . . .	1126
20	The generalized Drazin inverse of operator matrices by Li Guo, Honglin Zou and Jianlong Chen . . . . .	1134
21	Automorphisms of a certain subalgebra of the upper triangular matrix algebra by Özkay Özkan and Mustafa Akkurt . . . . .	1150
22	Weakly prime ideals issued from an amalgamated algebra by Najib Mahdou, Moutu Abdou Salam Moutui and Youssef Zahir . . . . .	1159
23	An integrated vendor-buyer model with quadratic demand under inspection policy and preservation technology by Magfura Pervin, Sankar Kumar Roy and Gerhard Wilhelm Weber . . . . .	1168
24	The Bogomolov multiplier of Lie algebras by Z. Araghi Rostami, M. Parvizi and P. Niroomand . . . . .	1190
25	Generalization of functions of bounded Mocanu variation with respect to 2k-symmetric conjugate points by Rasoul Aghalary and Jafar Kazemzadeh . . . . .	1206