An activity on fixed point

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Abstract

In this study, Newton methot;

$$x_{n+1}\ f'(x_n) = x_n\ .f'(x_n) -- f(x_n)$$

is also an example of fixed point iteration, for the equation, We begin by asking whether the equation x = f(x) has a solution. We have proved the importance of the Newton method once again.

Key words

Functions, fixed point iterations ,convergence speed , Newton methot and fixed point.

1. Introduction

The solution of some problems in mathematics is finding a solution to an equation that can be written as

$$f(x) = x$$

for an appropriate h function. I know that solution of such equations are called fixed -point and the theorems which examine the existence of these fixed points are called fixed point theorems. Fixed-point and fixed point theorems play an important role in solving the problems of existence and uniqueness in the analysis, geometry , topology, which are the subdivisions of mathematics**.**

S.Elmas , examined the Newton-Rapson method from fixed point iterations. With a few examples. I proved the validity of the method again. In particular, fixed point techniques have been applied in such diverse fields as Physics,Biology, Chemistry, Economics and Engineerig.

Theorem 1.1 Let f : [a; b] → [a; b] be a differentiable function such that

$$|f'(x)| \leq \alpha < 1 \text{ for all } x \in [a; b]$$

Then f has exactly one fixed point $q_0 \in [a; b]$ and the sequence (x_n) defined by the process,with a starting point $x_0 \in [a; b]$, converges to q_0.

Proof : By the intermediate value property f has a fixed point, say p_0. The convergence of (x_n)) to q_0 follows from the following inequalitie

$$|x_n - q_0| = |f(x_{n-1}) - f(q_0)| \leq \alpha|x_{n-1} - q_0|$$
$$\leq \alpha^2|x_{n-2} - q_0|$$
$$\ldots\ldots\ldots\ldots$$

$\leq \alpha^n |x_0 - q_0| \to 0$ If q_1 is a fixed point then

$$|q_1 - q_0| = |f(q_1) - f(q_0)| \leq \alpha |q_1 - q_0| < |q_1 - q_0|$$

This implies that $q_1 = q_0$.

2. Fixed Point Iteration Method

The idea of fixed point iteration methods is to first reformulate an equation into an equivalent fixed point problem:

$$g(x) = 0 \quad \leftrightarrow \quad x = f(x)$$

and then to use the iteration: with an initial guess x_0 chosen, compute a sequence

$$x_{n+1} = f(x_0); \qquad n \geq 0$$

in the hope that $x_{n \to}$ a ∈ R

There are several ways to present an equivalent fixed point problem for a given equation.; for example, for any function F (x) with the property

$$F(x) = 0 \quad \leftrightarrow \quad x = 0$$

we can take

$$f(x) = x + F(g(x)).$$

The iteration method found may or may not converge.

Example:

1. g(x) = 1 + 5 sin x
2. g(x) = 3 + 2 sin x

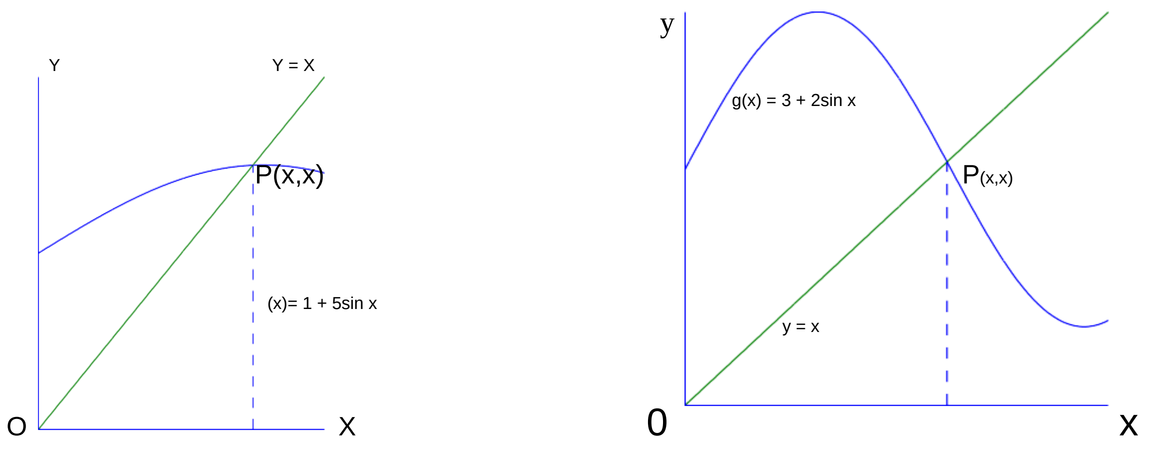

Figure 1.1

The solutions are

1. σ = 1:37860002406754
2. σ = 3:08327230203817

I aim to use a numerical scheme called fixed point iteration. It amounts to making an initial guess of x_0 and substituting this into the right side of the equation. Let's show the results in this way:

$\boldsymbol{x_1}$ and then the process is repeated, this time substituting x_1 into the right side. This work is repeated with convergence occurs. The process is repeated until the end

1) $x_{n+1} = 1 + 5 \sin x_n$

2) $x_{n+1} = 3 + 2 \sin x_n$, for n = 0; 1; 2; :::

	R1	R2
	x_n	x_n
0	0.00000000000000	3.00000000000000
1	1.00000000000000	3.11213000508861
2	1.31062438130280	2.60852066070444
3	1.38327088145320	2.70800014377402
4	1.38743077328805	1.63514278540540
5	1.38758424441078	3.85816846103650
6	1.38760081420602	1.06563065299215
7	1. 38760001501132	3.75018861639464
8	1.38760022213676	1.00142864236515
9	1.38760002240701	3.68448404916096
10	1.38760002240638	1.00066752354758

Table 1.1

I show the results of the first ten iterations in the table R1. Clearly convergence is occurring with R1, but not with R2.
In general, we are interested in solving the equation

$$x = f(x)$$

by means of fixed point iteration:

$$x_{n+1} = f(x_n) \quad , \qquad n = 0, 1, 2, \ldots$$

It is called 'fixed point iteration' because the root σ of the equation $x - f(x) = 0$ is a fixed point of the function $f(x)$, meaning that σ is a number for which $f(\sigma) = \sigma$.
Newton methot;

$$x_{n+1} \cdot g(x_n)' = x_n \cdot g(x_n)' -- g(x_n)$$

is also an example of fixed point iteration, for the equation

$$x \cdot g(x)' = x \cdot g(x)' -- g(x)$$

I begin by asking whether the equation x = f(x) has a solution. Graphics for this to occur

Of y = x and y = f(x) should be cut as shown in the graphics above. (**Figure 1.1**)

Lemma:2.1
Let $f(x)$ be a continuous function on the interval $[a, b]$ and suppose it satisfies the property

$$a \le x \le b \quad \Rightarrow \quad a \le f(x) \le b$$

So the equation x = g (x) has at least one solution α in the interval [a, b].

The proof of this is very intuitive. See function

$$g(x) = x - f(x), \quad a \le x \le b$$

Evaluating at the endpoints,

$$g(a) \le 0, \qquad g(b) \ge 0$$

The function g(x) is continuous on [a, b], and zero converges in this interval..

Example 1. Similarly, the equation

$$x = 3 + 2 \sin x$$

Here

$$f(x) = 3 + 2 \sin x$$

has a solution in $[a, b]$ with $1 \le a$ and $b \le 5$.

Example 2. Consider the equation

$$x = 1 + 0.3 \sin x.$$

Here

$$f(x) = 1 + 0.3 \sin x.$$

Note that $0.7 \le f(x) \le 1.3$ for any $x \in \mathrm{R}$. Also, $f(x)$ is a continuous function. Applying the existence lemma, we conclude that the equation $x = 1 + 0.3 \sin x$ has a solution in $[a, b]$ with $a \le 0.7$ and $b \le 1.3$.

Theorem 2.1

Assume f(x) and f'(x) exist and are continuous on the interval [a; b]; and further, assume

$$a \le x \le b \quad \to \quad a \le f(x) \le b$$

$$\beta = max[f'(x)] < 1 \quad , \quad a \le x \le b$$

Then:

Q1. The equation x = f(x) has a unique solution $q \epsilon$[a; b].

Q2. For any initial guess $x_0 \in$ [a; b], the iteration

$$x_{n+1} = f(x_n) \quad , \quad n = 0, 1, 2, \ldots$$

will converge to .

3.Result

Aitken extrapolation can greatly speed up the convergence of an object.linear convergent iteration

$$x_{n+1} = f(x_n)$$

This shows the strength of understanding the error's behavior.in a numerical process. According to this understanding, usually increasing accuracy through extrapolation or another procedure.

This is a rationale for using mathematical analysis..

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