

---

## HACETTEPE JOURNAL OF MATHEMATICS AND STATISTICS

Volume 52 Issue 2 (2023)

---

E-ISSN: 2651-477X

### **Honorary Editor**

Lawrence Michael Brown

### **Editor in Chief**

#### **Mathematics**

Murat Diker (Hacettepe University, Mathematics - hjms@hacettepe.edu.tr)

#### **Statistics**

Nursel Koyuncu (Hacettepe University, Statistics - statisticshjms@gmail.com)

### **Associate Editors**

Derya Ersel (Hacettepe University, Statistics - dtektas@hacettepe.edu.tr)

Yasemin Kayhan Atilgan (Hacettepe University, Statistics - ykayhan@hacettepe.edu.tr)

### **Production Editors**

Talha Arıkan (Hacettepe University, Mathematics - tarikan@hacettepe.edu.tr)

İsmail Aslan (Hacettepe University, Mathematics - ismail-aslan@hacettepe.edu.tr)

Berke Kaleboğaz (Hacettepe University, Mathematics - bkuru@hacettepe.edu.tr)

Özge Karadağ Ataş (Hacettepe University, Statistics - ozgekaradag@hacettepe.edu.tr)

Esra Korkmaz (Hacettepe University, Mathematics - esrakaratas@hacettepe.edu.tr)

Uğur Sert (Hacettepe University, Mathematics - usert@hacettepe.edu.tr)

O. Oğulcan Tuncer (Hacettepe University, Mathematics - otuncer@hacettepe.edu.tr)

Ayfer Ezgi Yılmaz (Hacettepe University, Statistics - ezgiyilmaz@hacettepe.edu.tr)

### **Area Editors**

#### **Mathematics**

Emre Alkan (Number Theory - ealkan@ku.edu.tr)

Angel Del Rio (Algebra, Group Theory - adelrio@um.es)

Naoki Endo (Commutative Rings and Algebras - nendo@rs.tus.ac.jp)

Aysel Erey (Graph Theory and Combinatorics - aysel.erey@gtu.edu.tr)

Sergio Estrada (Homological Algebra - sestrada@um.es)

Rodrigo Hernandez Gutierrez (General Topology - rod@xanum.uam.mx)

Varga Kalantarov (Partial Differential Equations - vkalantarov@ku.edu.tr)

Serap Öztop Kaptanoğlu (Abstract Harmonic Analysis - oztops@istanbul.edu.tr )

Adam Lecko (Complex Analysis - alecko@matman.uwm.edu.pl)

Emre Mengi (Numerical Analysis - emengi@ku.edu.tr)

Cihan Orhan (Analysis, Summability - orhan@science.ankara.edu.tr)

Abdullah Özbekler (Ordinary Differential Equations - aozbekler@gmail.com)

Ekin Özman (Number Theory, Algebraic Geometry - ekin.ozman@boun.edu.tr)

Mehmetçik Pamuk (Topology, Manifolds and Cell Complexes - mpamuk@metu.edu.tr)

Changguo Shao (Group Theory - shaoguozi@163.com)

Bülent Ünal (Differential Geometry - bulent@fen.bilkent.edu.tr)

Luca Vitagliano (Differential Geometry - lvitagliano@unisa.it)

Dingguo Wang (Hopf Algebras, Tensor Categories - dgwang@qfnu.edu.cn)

Sibei Yang (Harmonic Analysis and Functional Analysis - yangsb@lzu.edu.cn)

Aslı Yıldız (Partial Differential Equations, Integrable systems, Soliton theory - aslipekcan@hacettepe.edu.tr)

Xin Zhang (Graph Theory and Combinatorics - xzhang@xidian.edu.cn)

Shengda Zeng (Partial Differential Equations, Nonlinear Analysis, Control Theory - zengshengda@163.com)  
Yunus E. Zeytuncu (Functions of Several Complex Variables - zeytuncu@umich.edu)

## Statistics

Ali Allahverdi (Operational Research & Statistics - ali.allahverdi@ku.edu.kw)  
Olcay Arslan (Robust Statistics - oarslan@ankara.edu.tr)  
Narayanaswamy Balakrishnan (Applied Statistics, Theory of Statistics - bala@mcmaster.ca)  
Adil Baykasoglu (Operational Research - adil.baykasoglu@deu.edu.tr)  
Haydar Demirhan (Categorical Data Analysis, Monte Carlo Simulation - haydar.demirhan@rmit.edu.au)  
Sat Gupta (Sampling, Time Series - sngupta@uncg.edu)  
Tahir Hanalioğlu (Stochastic Processes Theory, Probability Theory - tahirkhaniyev@etu.edu.tr)  
Burcu Hüdaverdi (Time Series Analysis, Multivariate Statistics - burcu.hudaverdi@deu.edu.tr)  
Zeynep İşl Kalayhoğlu (Bayesian Inference, Model Selection - kzeynep@metu.edu.tr)  
Hasan Örkçü (Data Envelopment Analysis, Operation Research - hhorkcu@gazi.edu.tr)  
Birdal Şenoğlu (Experimental Design, Statistical Distributions - senoglu@science.ankara.edu.tr)

# Contents

## Mathematics

### Research Articles

1	Asymptotic equivalence of impulsive dynamic equations on time scales <i>by Sibel Doğru Akgöl</i>	277
2	Numerical solutions of Troesch and Duffing equations by Taylor wavelets <i>by Gökçe Özaltun Şimşek and Sevin Gümgüm</i>	292
3	Some structures on the coframe bundle with Cheeger-Gromoll metric <i>by Arif Salimov and Habil Fattayev</i>	303
4	Orlicz dual of log-Aleksandrov–Fenchel inequality <i>by Chang-Jian Zhao</i>	317
5	An efficient numerical method for a singularly perturbed Volterra-Fredholm integro-differential equation <i>by Muhammet Enes Durmaz, Ömer Yapman, Mustafa Kudu and Gabil M. Amiraliyev</i>	326
6	Statistical $\rho$ -commutative algebras <i>by Zahra Bagheri and Esmaeil Peyghan</i>	340
7	Smallest maximal matchings of graphs <i>by Mostafa Tavakoli and Tomislav Došlić</i>	356
8	On some second maximal subgroups of non-solvable groups <i>by Yuyun Wang, Long Miao and Wei Liu</i>	367
9	Relative Buchweitz-Happel theorem respect to a self-orthogonal class <i>by Haiyu Liu, Yuxian Geng and Rongmin Zhu</i>	374
10	Transformation formulae for terminating balanced ${}_4F_3$ -series and implications <i>by Wenchang Chu</i>	391
11	On $T_1$ -reflection of topological spaces <i>by Sami Lazaar, Abdelwaheb Mhemdi, Tareq M. Al-shami and Hadjer Okbani</i>	398
12	$S$ -cotorsion modules and dimensions <i>by Refat Abdelmawla Khaled Assaad and Xiaolei Zhang</i>	410

13	A constructive approach: From local subgroups to new classes of finite groups <i>by Zhencai Shen, Baoyu Zhang and Haonan Jiang</i>	420
14	Completeness of fuzzy quasi-pseudometric spaces <i>by Yi Shi and Wei Yao</i>	426
15	Some congruences with $q$ -binomial coefficients and $q$ -harmonic numbers <i>by Sibel Koparal, Neşe Ömür and Laid Elkhiri</i>	445

## Statistics

### *Research Articles*

16	Robust correlation scaled principal component regression <i>by Aiman Tahir and Maryam Ilyas</i>	459
17	Significantly improved dominance relation for no-wait flowshop scheduling problems with uncertain setup times <i>by Muberra Allahverdi</i>	487
18	Estimation of $Pr(X < Y)$ for exponential power records <i>by Caner Tanış, Buğra Saraçoğlu, Akbar Asgharzadeh and Meraj Abdi</i>	499
19	kNN robustification equivariant nonparametric regression estimators for functional ergodic data <i>by Somia Guenani, Wahiba Bouabsa, Mohammed Kadi Attouch and Omar Fetitah</i>	512
20	Comparing discrete Pareto populations under a fixed effects model <i>by M. Baratnia, A.H. Rezaei Roknabadi and M. Doostparast</i>	529