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A fuzzy approach for determination of prostate cancer

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Abstract: *Goal of this study is a design of a fuzzy expert system, its application aspects in the medicine area and its introduction for calculation of numeric value of prostate cancer risk. For this aim it was used prostate specific antigen (PSA), age and prostate volume (PV) as system input parameters and prostate cancer risk (PCR) as output. This system gives user a range of the risk of the cancer disease and facilitates the decision of the doctor if there is a need for the biopsy. The designed system was tested by the data from the literature and the clinical data. It was compared the diagnoses data of specialists of the every disease situation and literature data and it was seen that the system can be available for every situation. It is observed that this system is rapid because it needs minimum calculation, economical, without any risk than traditional diagnostic systems, has also a high reliability than the other system and can be used as assistant system for physicians. Having used in the hospital this system was tested as decision support system and the approach used in this study can be used in difference studies and analyses, because the system is transparent and explainable to a user.*
Keywords: *Fuzzy logic, fuzzy expert system, prostate cancer, prostate specific antigen, prostate cancer risk.*

1. Introduction

In recent years, the methods of Artificial Intelligence have largely been used in the different areas including the medical applications. In the medicine area, many expert systems (ESs) were designed. ONCOCIN and ONCO-HELP are the ESs for diagnosis of the general cancer diseases (Allahverdi, 2002; Allahverdi & Yaldiz, 1998). For example, ONCO-HELP is a multimedia knowledge-based decision support system for individual tumour entities. It makes individual and prognosis-oriented treatment of patient's tumour possible (if corresponding predictor's respective prognostic factors are known). Through registration of individual patient data over tumour type, histology, metastatic type, metastasis localization and amount, as well as corresponding laboratory parameters together with a corresponding knowledge based on a patient individual prognosis-score can be determined. Using this score, a therapy concept is drafted. ONCO-HELP evaluates this concept by using therapy controls with regards to tumour progression/regression and side effects of the therapy. Consequently, a concept modification or a different therapy is proposed (Allahverdi & Yaldiz, 1998).

Computing technology and artificial intelligence are interdisciplinary research fields in computational science and it was proved that very respective area for application of these technologies has been the medicine diagnosis the last 20-25 years.

Various techniques in these areas such as ESs, neural networks, fuzzy logic, genetic algorithms, Bayesian statistics, chaos theory, etc, have been developed and applied to solve many challenging tasks in medicine and engineering design. There are some publications in the area prostate cancer prognosis or diagnosis by aid of soft computing methods (Abboad et al., 2001; Boegla et al., 2002; Lorenz et al., 2007; Nguyen et al., 2001; Ronco et al., 1999; Seker et al, 2003; Kaiser Permanente, 2007). In the study (Seker et al., 2003) a fuzzy logic based method for prognostic decision making in breast and prostate cancers is developed. In the study (Lorenz et al., 2007) five different trainable neuro-fuzzy classification algorithms based on different approaches to organize and classify biological data sets by the construction of a fuzzy interference system were investigated. The best classifier based on a mountain clustering algorithm reached recognition rates above 86 % in comparison to the Bayes classifier 79 % and the KNN classifier 78%. These results suggest that neuro-fuzzy algorithms have the potential to improve common classification methods significantly for the use in ultrasonic tissue characterization.

As seen from analysis of these studies, it is not quite possible to diagnose of prostate cancer fully based on only ultrasonography and image processing. In the prostate cancer disease except laboratory analysis of blood with aim to define the prostate specific antigen (PSA) and rectal definition of a prostate volume, here man's age plays great role. Recent modification of the PSA test is based on the observation that as man's age, the amount of PSA in the blood can normally rises without the presence of a prostate cancer. Thus, doctors can use what is referred to as an age-specific PSA, especially to evaluate borderline values. In the age-specific PSA, the normal values are adjusted for the age of the patient. Accordingly, the age-specific normal ranges are 0 to 2.5 *ng/ml* for men in their 40s, 0 to 3.5 *ng/ml* in their 50s, 0 to 4.5 *ng/ml* in their 60s, and 0 to 6.5 *ng/ml* for men 70 and over. Therefore, as an example, a PSA value of 4 *ng/ml* would be considered borderline for men in their 30s and 40s, but could be normal for men in their 50s, 60s, and 70s (Medicine Net, 2007).

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As shown from these data the borders of the age-PSA estimations are fuzzy. If one will add here the prostate volumes data, the fuzziness is seen clearly.

As known when the prostate cancer can be diagnosed earlier, the patient can be completely treated. If there is a biopsy for diagnosing, the cancer may spread to the other vital organs (Metlin et al., 1991; Saritas et al, 2003). For this reason the biopsy method is undesirable.

We have developed a rule-based fuzzy expert system (FES) that uses the laboratory and other data, and simulate an expert-doctor's behaviour and can be help doctor to determine of numerical value of the prostate cancer risk.

Prostate specific antigen (PSA), prostate volume (PV) and age of the patient are being used as laboratory data. Provided that using these data and getting help from an expert doctor the fuzzy rules which define the necessity of biopsy and the risk factor were developed. The designed system gives the user the patient's possibility ratio of the prostate cancer. The system was developed by aid of the Matlab 7.0. Comparison between the results of the developed FES and the data of 4641 patients from the literature (Brawer et al., 1999) showed that the FES gives close results. Additionally, the FES is rapid, economical, when compared to traditional diagnostic systems it has no risk and has also a high reliability and can be used as learning system for medical students, because the system is transparent and explainable to a user.

The paper is organized as bellow: In the second section, material and used methods are described. Then in the third section the developed system is discussed and a conclusion is given in the next section.

2. Materials and Methods

To develop the expert system in this study the laboratory data for the developed system were taken from the literature (Brawer et al., 1999] and (Seker et al., 2003). For the design process prostate specific antigen (PSA=A), age (B) and prostate volume (PV=C) are used as input parameters and prostate cancer risk (PCR=D) is used as output. For fuzzification of these factors the linguistic variables very small, small, middle, high, very high, very low, low and etc. were used. For the inference mechanism the Mamdani max-min inference was used. The system was developed by aid of the Matlab 7.0 fuzzy tool-box.

2.1. Fuzzy Expert System

An expert system (ES) can be viewed as fuzzy expert system (FES) if its rules are included fuzziness and parameters used by this ES can be fuzzy or can be fuzzification. Regardless the designed system it uses the amount of prostate specific antigen (PSA) in the blood of the patient, age and prostate volume (PV) of this patient for the system input parameters as crisp parameters, they are fuzzified when they input to the system. The same time prostate cancer risk (PCR) parameter viewed as output is fuzzy and because of this it has to be defuzzified. Thus in such a system with the fuzzy rules base and fuzzy inference mechanism there must be included to the system parts of fuzzification and defuzzification. General structure of the developed respective ES is shown in the Fig.1.

Amount of PSA (*ng/ml*), age (*years*) and PV (*ml*) were used as input parameters and range of PCR (*%*) was treated as output parameter in the designed system. Thus in this situation the system will have a structure as seen in Fig. 2.

Crisp values of three input parameters applied to the system input in during of the system work. As input parameters apply to the input, the system provides a firing of rule or rules in the fuzzy rules base which is/are suitable on respective fuzzy values of parameters. Then the fuzzy value(s) of PCR is (are) obtained using of Mamdani inference approach. Next, crisp value of PCR is obtained using a certain defuzzification method. The fuzzification part determines how fuzzy values are suitable to the crisp values of input parameters.

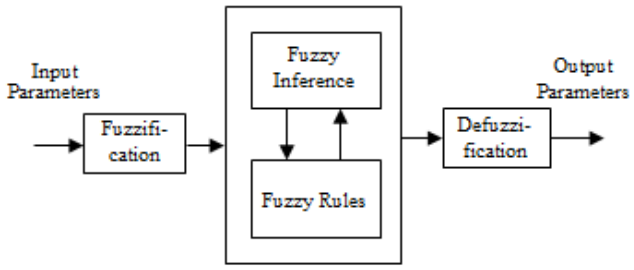


Figure 1. The Structure of FES

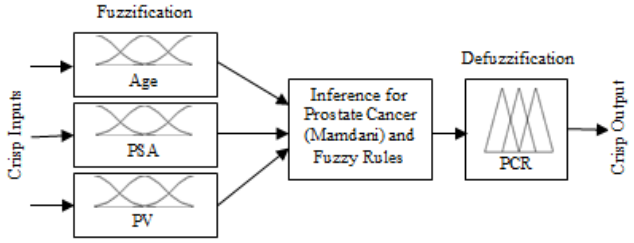


Figure 2. The Structure of the FES for Determination Prostate Cancer Risk

2.2. Fuzzification of Input and Output Parameters

In general the triangle membership function is one of the acceptable and simple methods to fuzzification of parameters applied in a fuzzy system. Therefore the triangle fuzzification method is used for the input and output parameters in the designed system. It was determined how many linguistic values will be used to present each parameter by aid of expert-doctor. These values are:

For PSA: Very Low (VL), Low (L), Middle (M), High (H) and Very High (VH);

For age: Very Young (VY), Young (Y), Middle Age (MA) and Old (O);

For prostate volume: Small (S), Middle (M), Big (B) and Very Big (VB);

For prostate cancer risk: Very Low (VL), Low (L), Middle (M), High (H) and Very High (VH).

So, these parameters are fuzzified and their membership functions are given literature (Saritas et al., 2003).

2.3. Forming of Fuzzy Rules Base and Defuzzification

Parts of the developed fuzzy rules are shown in the Table 1. Total of 80 rules are formed by aid of the expert-doctor and according to the literature data (Brawer et al., 1999). For example, Rule 1 and Rule 77 can be interpreted as follows:

Rule 1: If Age=Very Young and PSA=Very Low and PV=Very Small, then PCR=Very Low, i.e. if the patient's PSA is very low and patient is very young and patient's PV is very small, then patient's prostate cancer risk is very low.

Rule 77: If Age=Old and PSA=Very High and PV=Very Small, then PCR=Very High, i.e. if the patient's PSA is very high and patient is old and patient's PV is very small, then patient's

prostate cancer risk is high.

Table 1. Fuzzy rules (Saritas et al.,2003)

		Age		PSA		PV		PCR
1	If	VY	and	VL	and	S	then	VL
2	If	VY	and	VL	and	M	then	VL
3	If	VY	and	VL	and	B	then	VL
...		...		...		...		...
77	If	O	and	VH	and	S	then	VH
78	If	O	and	VH	and	M	then	VH
79	If	O	and	VH	and	B	then	VH
80	If	O	and	VH	and	VB	then	VH

The other rules can be interpreted by similar way. In this stage, true degrees (α) of the rules are determined for the each rule by aid of the min and then by taking max between firing rules. For example, for PSA=40 *ng/ml*, Age=55 *years*, PV=230 *ml* the rules 60 and 80 will be fired and we will obtain:

α_{60} = min(Very High PSA, Middle Age, Very Big PV)

α_{60} = min(1, 0.67, 1)=0.67

α_{80} = min(Very High PSA, Old Age, Very Big PV)

α_{80} = min(1, 0.33, 1)= 0.33

From Mamdani max-min inference we will obtain the membership function of our system as max(α_{60}, α_{80})=0.67. Then we can calculate the crisp output value. The crisp value of the PCR is calculated by the method centroid defuzzifier by the Eq.1.

$$D^* = \frac{\int D \cdot \mu_{middle}(D) dD}{\int \mu_{middle}(D) dD} \quad (1)$$

As also seen from the Fig. 3, obtained from the Matlab software, the value of PCR=78.4. This means that the patient has the prostate cancer with a possibility 78.4 %. Because this is a quite high percentage, doctor has to decide a biopsy.

3. Application aspects

The designed new system was used to the data of 119 patients in Ankara University Medicine Faculty in 2005 year. The comparing results of this system and real clinical data of the patients are shown in the Table 2. The results of the prediction of the cancer risk by the ratio of FPSA (Free PSA)/PSA, by online risk calculator (Cancer risk calculator, 2007) (Fig. 4 and 5) and the designed FES are given in this table. In the column of the "Results of Biopsy" the results are presented as "negative" and "positive", in the column of "FPSA/PSA" calculated by this division risk ratio is shown, in the columns online risk calculator and FES, calculated risk prediction ratio are presented by %.

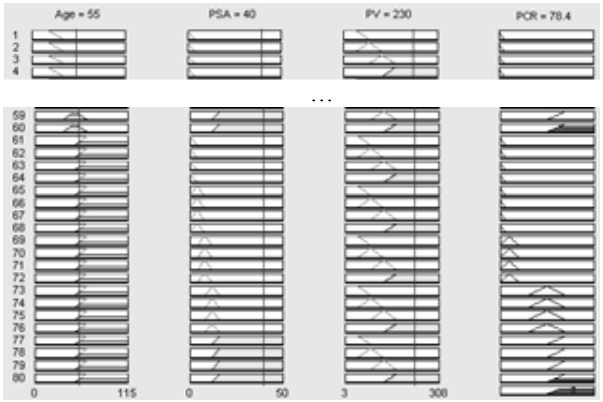


Figure 3. Calculation of the value PCR for the values PSA=40 *ng/ml*, Age=55, PV=230 *ml*

Table 2. Results of the biopsy, FPSA/PSA, risk ratios of online calculator and FES

Number of Patients	Input Variables of FES and Online Calculator			FPSA	Result of Biopsy	Risk ratio FPSA/PSA		Risk ratio of online calculator	Output Variable of FES
	Old	PSA	PV			FPSA/PSA	Predict		Risk ratio
1	44	7.60	38.00	0.80	Negative	0.11	Yes	12	10
2	51	6.76	15.00	0.28	Positive	0.04	Yes	5	9
3	51	44.00	83.00	14.00	Positive	0.32	No	61	80
4	53	4.50	39.00	0.85	Negative	0.19	No	13	6
5	53	5.83	25.00	0.40	Negative	0.07	Yes	7	8
6	53	8.34	25.00	0.62	Negative	0.07	Yes	10	20
7	54	5.62	28.00	0.84	Negative	0.15	Yes	13	8
8	54	17.30	90.00	4.75	Negative	0.27	No	38	79
9	54	17.30	45.00	1.54	Positive	0.09	Yes	19	79
10	55	10.51	54.00	2.36	Negative	0.22	No	26	40
11	56	8.90	26.00	3.04	Negative	0.34	No	30	30
12	56	9.05	39.00	0.77	Positive	0.09	Yes	12	31
13	56	16.00	146.00	1.35	Negative	0.08	Yes	18	78
14	57	12.56	52.00	8.27	Negative	0.66	No	50	58
15	58	4.48	67.50	0.72	Negative	0.16	No	11	6
16	58	4.62	48.00	0.51	Negative	0.11	Yes	9	7
17	58	5.20	58.00	1.22	Negative	0.23	No	16	8
18	58	16.39	27.00	15.09	Negative	0.92	No	62	78
19	59	0.28	168.00	0.12	Negative	0.43	No	3	2
20	59	8.36	55.00	0.63	Positive	0.08	Yes	10	30
21	59	18.20	77.00	3.23	Negative	0.18	No	31	78
22	59	19.48	79.00	4.87	Positive	0.25	No	39	78
23	59	22.51	42.00	1.58	Negative	0.07	Yes	20	78
24	59	22.65	66.00	2.45	Negative	0.11	Yes	26	80
25	60	6.58	65.00	0.97	Negative	0.15	Yes	14	9
26	60	10.60	30.00	1.78	Positive	0.17	No	21	40
27	60	11.45	46.00	2.23	Negative	0.19	No	25	45
28	60	14.79	38.00	1.02	Positive	0.07	Yes	62	72
29	60	15.51	35.00	3.26	Negative	0.21	No	31	80

30	61	4.60	37.00	0.50	Negative	0.11	Yes	8	6
31	61	10.33	62.00	2.62	Negative	0.25	No	27	37
32	61	10.36	35.00	2.05	Negative	0.20	No	23	39
33	61	10.59	56.00	1.80	Positive	0.17	No	21	40
34	61	18.30	62.00	1.28	Positive	0.07	Yes	66	79
35	62	6.12	52.00	1.48	Negative	0.24	No	19	9
36	62	6.20	25.00	0.27	Positive	0.04	Yes	44	9
37	62	8.37	43.00	0.94	Negative	0.11	Yes	14	21
38	62	8.79	45.00	0.96	Positive	0.11	Yes	14	27
39	62	20.00	53.00	1.04	Positive	0.05	Yes	68	79
40	62	51.74	29.00	3.52	Positive	0.07	Yes	33	79
41	63	8.80	31.00	1.98	Positive	0.23	No	23	27
42	64	5.70	36.00	1.70	Negative	0.30	No	21	8
43	64	6.96	45.00	0.64	Negative	0.09	Yes	10	10
44	64	8.00	40.00	0.60	Positive	0.08	Yes	49	10
45	64	11.08	26.00	1.12	Negative	0.10	Yes	15	43
46	64	16.28	21.00	1.13	Positive	0.07	Yes	16	80
47	65	4.39	30.00	0.95	Negative	0.22	No	14	5
48	65	5.15	47.00	0.81	Negative	0.16	No	12	8
49	65	7.61	23.00	0.44	Positive	0.06	Yes	48	10
50	65	7.82	75.00	1.78	Negative	0.23	No	21	10
51	65	8.33	32.00	1.21	Positive	0.15	Yes	50	20
52	66	4.38	33.00	1.03	Negative	0.24	No	15	5
53	66	6.72	61.00	0.93	Positive	0.14	Yes	13	9
54	66	7.65	89.00	1.81	Negative	0.24	No	22	10
55	66	9.00	74.00	1.70	Positive	0.19	No	21	29
56	66	9.86	49.00	2.35	Negative	0.24	No	26	36
57	67	4.39	28.00	0.04	Negative	0.01	Yes	2	5
58	67	5.65	24.00	0.58	Positive	0.10	Yes	42	8
59	67	6.24	65.00	1.37	Negative	0.22	No	18	9
60	67	8.20	36.00	1.67	Positive	0.20	No	20	27
61	67	9.68	41.00	0.72	Positive	0.07	Yes	53	35
62	67	15.93	69.00	0.97	Positive	0.06	Yes	63	79
63	67	28.00	47.00	4.20	Positive	0.15	No	36	80
64	68	5.09	47.00	0.12	Negative	0.02	Yes	3	8
65	68	5.51	45.00	0.62	Negative	0.11	Yes	10	8
66	68	7.20	33.00	0.26	Positive	0.04	Yes	47	10
67	68	9.25	91.00	0.33	Positive	0.04	Yes	52	32
68	68	12.10	61.00	1.95	Negative	0.16	No	23	50
69	68	23.70	109.00	2.38	Positive	0.10	Yes	26	80
70	68	140.00	117.00	20.00	Positive	0.14	Yes	68	80
71	68	140.00	54.00	4.60	Positive	0.03	Yes	38	80
72	69	8.80	34.00	0.79	Positive	0.09	Yes	12	27
73	69	11.06	38.00	3.30	Negative	0.30	No	31	43
74	69	15.31	74.00	4.68	Positive	0.31	No	38	75
75	69	61.00	46.00	6.06	Negative	0.10	Yes	43	80
76	69	70.56	45.00	4.25	Positive	0.06	Yes	36	80
77	69	146.00	29.00	10.70	Positive	0.07	Yes	54	80
78	70	5.39	120.00	1.03	Negative	0.19	No	15	8
79	70	5.39	42.00	0.69	Negative	0.13	Yes	11	8
80	70	13.00	40.00	2.01	Negative	0.15	No	23	60
81	70	13.95	119.00	1.92	Negative	0.14	Yes	22	67
82	70	19.20	44.00	1.94	Positive	0.10	Yes	23	80
83	70	21.94	29.00	1.56	Positive	0.07	Yes	19	80
84	70	27.70	63.00	2.49	Negative	0.09	Yes	26	80
85	71	6.08	48.00	1.30	Positive	0.21	No	43	9
86	71	12.64	50.00	1.01	Positive	0.08	Yes	59	57
87	71	22.00	57.00	2.64	Positive	0.12	Yes	27	80
88	72	6.64	32.00	1.82	Negative	0.27	No	22	9
89	72	13.31	33.00	0.51	Positive	0.04	Yes	60	63
90	72	13.31	33.00	0.50	Positive	0.04	Yes	60	63
91	72	20.00	48.00	1.58	Positive	0.08	Yes	20	80
92	72	46.00	36.00	4.92	Positive	0.11	Yes	39	80
93	72	77.00	48.00	6.40	Positive	0.08	Yes	44	80
94	73	4.65	41.00	1.95	Negative	0.42	No	23	6
95	73	7.25	19.00	0.40	Negative	0.06	Yes	47	10
96	73	7.60	74.00	2.38	Positive	0.31	No	26	10
97	73	19.00	90.00	1.30	Positive	0.07	Yes	67	80
98	73	29.52	91.00	2.90	Negative	0.10	Yes	29	80
99	73	47.40	87.00	7.53	Positive	0.16	No	48	80
100	74	12.52	27.00	1.48	Negative	0.12	Yes	19	56
101	74	150.00	54.00	25.00	Positive	0.17	No	72	80
102	75	4.61	16.00	0.81	Positive	0.18	No	12	6
103	75	10.00	34.00	0.76	Positive	0.08	Yes	12	37
104	76	9.81	56.00	3.67	Negative	0.37	No	33	36
105	76	13.61	61.00	2.71	Positive	0.20	No	28	65
106	76	13.83	54.00	2.76	Positive	0.20	No	28	67
107	76	21.00	86.00	1.14	Positive	0.05	Yes	69	80
108	77	10.00	60.00	0.60	Positive	0.06	Yes	10	37

109	77	12.05	28.00	3.26	Positive	0.27	No	31	51
110	77	56.00	51.00	4.11	Positive	0.07	Yes	36	80
111	78	4.50	180.00	0.92	Negative	0.20	No	13	6
112	78	26.10	46.00	2.25	Negative	0.09	Yes	25	80
113	78	26.13	235.00	2.16	Negative	0.08	Yes	24	80
114	78	31.60	57.00	2.80	Negative	0.09	Yes	28	80
115	79	17.10	41.00	1.30	Negative	0.08	Yes	17	80
116	80	69.51	28.00	20.00	Positive	0.29	No	68	80
117	81	4.50	28.00	0.97	Positive	0.22	No	14	6
118	81	68.36	52.00	24.11	Positive	0.35	No	71	80
119	88	10.40	32.00	0.78	Positive	0.08	Yes	12	39

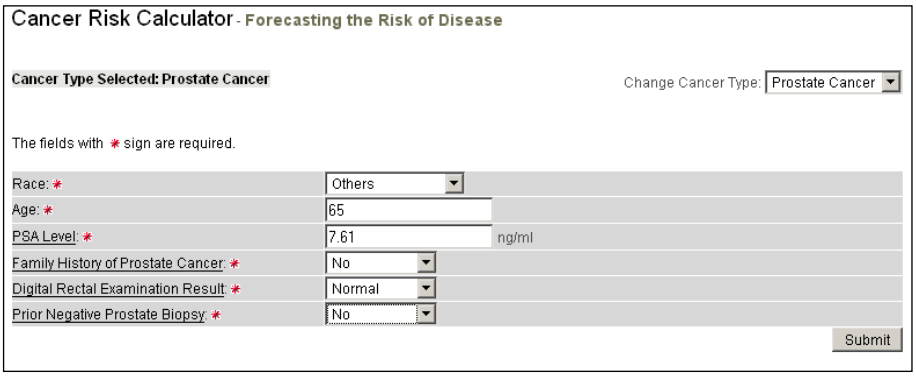


Figure 4. Online risk calculator page.

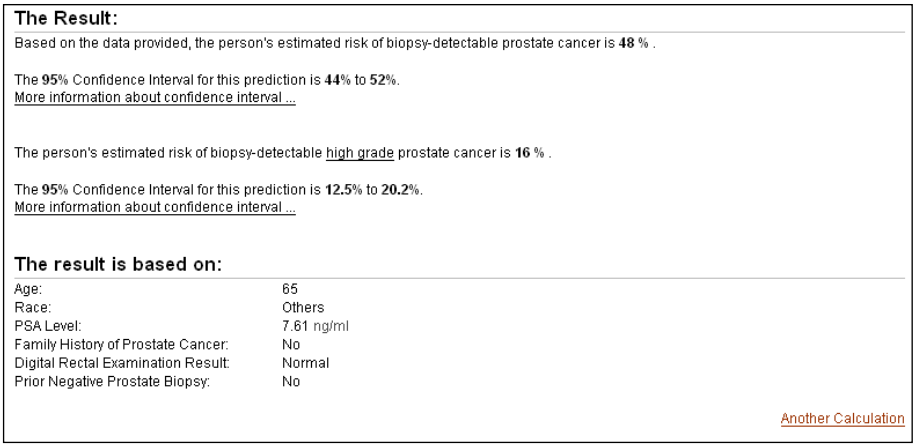


Figure 5. Online risk calculator results page.

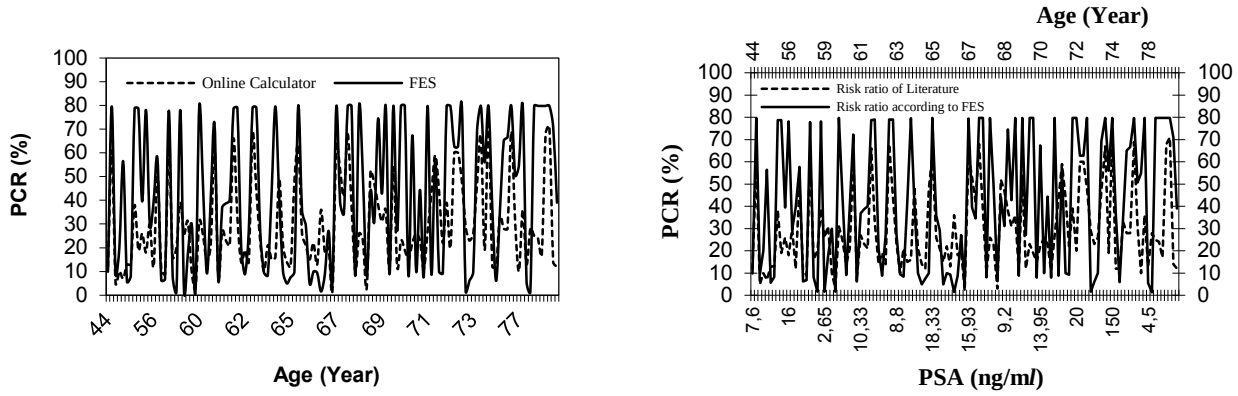
4. Discussion and conclusion

For discussion we analysed the results of biopsy of 119 patients with the results of the other methods (see Table 2). These analyses show that the true orientation ratio for 119 patients to doctor gives the methods: 60.50% FPSA/PSA, 62.18% online calculator method and 64.71% FES (Table 3). Besides, it is seen that for 56 patients from 119 patients, the using methods have different predictions and thus they have different orientation rations to the doctors. When we analysed only these patients we have seen that true bigger risk orientation ratio is fond by the FES method 75%. So, we can say that the FES method provides higher success ratio level to determine PCR than the other two methods, which have PCR ratios only 39.29% (FPSA/PSA) and 57.14% (online calculator).

The PCR ratio value's graphics are shown in the Fig. 6 and 7. In the Fig. 6.a, correlation graphic of PCR results according to age parameter of the patients calculated by the methods online calculator and FES are given. In the next graphic (Fig. 6.b) the correlation values of PCR according to the literature data and data calculated by FES are shown. In this graphic both age and PSA values are taken into account. In the Fig. 7.a, correlation graphic of PCR results according to PSA value of the patients calculated by the methods online calculator and FES are given. In the next graphic (Fig. 7.b) the correlation values of PCR according to the literature data and data calculated by FES are shown. In this graphic both age and PSA values are taken into account. From these graphics we can see that as age and PSA values are bigger, the FES predicts more correct forecasting.

Table 3. The Results of True Prediction

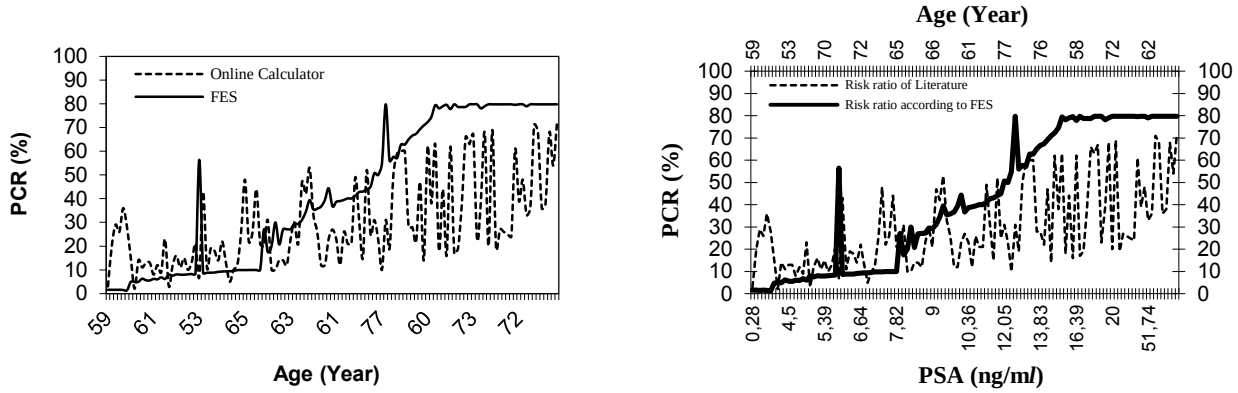
	FPSA/PSA	Online Calculator	FES
True prediction ratio of all patients (for 119 patients) (%)	60.50	62.18	64.71
True prediction ratio of the patients with different results (for 56 patients) (%)	39.29	57.14	75.00



a. PCR-Age

b. PCR-PSA-Age

Figure 6. Correlation graphic of PCR results according to ordered Age



a. PCR-Age

b. PCR-PSA

Figure 7. Graphic of PCR according to ordered PSA

Thus, the paper describes a design method and application aspects of a fuzzy expert system to define of the possibility of the determination of the prostate cancer risk, which can be used by the expert-doctors for treatment and by the students for learning scopes. The system does not answer if there is a cancer disease in the patient, but it gives a percentage of the possibility of the prostate cancer and helps the doctor to decide a biopsy or not.

This system can be developed further with increasing the knowledge rules from one side and with adding the neural network to the system from the other side. Besides, family genetic factor of a patient may be took into account. In this case the fuzzy rules have to be modified.

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Particle Swarm Optimization with Flexible Swarm for Unconstrained Optimization

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Abstract: *Particle Swarm Optimization (PSO) algorithm inspired from behaviour of bird flocking and fish schooling. It is well-known algorithm which has been used in many areas successfully. However it sometimes suffers from premature convergence. In resent year's researches have been introduced a various approaches to avoid of this problem. This paper presents the particle swarm optimization algorithm with flexible swarm (PSO-FS). The new algorithm was evaluated on 14 functions often used to benchmark the performance of optimization algorithms. PSO-FS algorithm was compared to some other modifications of PSO. The results show that PSO-FS always performed one of the better results.*
Keywords: *Particle Swarm Optimization Algorithm, Particle Swarm Optimization Algorithm with Flexible Swarm, Unconstrained Optimization.*

1. Introduction

I Particle Swarm Optimization (PSO) algorithm has been developed by Eberhart and Dr. Kennedy, inspired by behaviour of bird flocking and fish schooling (Kennedy and Eberhart, 1995). PSO is a stochastic population-based global optimization technique. In PSO context the population is called as swarm, while the members of population are called as particle. The algorithm is made of two essential steps: particle movement (dynamics) through a search space of solutions and an indirect particle interaction, which is mediated by information sharing within a social network (Blackwell and Bratton, 2008). PSO uses the physical movements of the individuals in the swarm and has a flexible and well-balanced mechanism to enhance and adapt to the global and local exploration abilities. Because of its easy implementation and inexpensive computation, its simplicity in coding and consistency in performance, the PSO has proved to be an effective and competitive algorithm for the optimization problem in continuous space (Marinakis and Marinak, 2008). PSO has many key advantages over other optimization techniques like (Alrashidi and El-Hawary, 2006):

• It is a derivative-free algorithm unlike many conventional techniques.
• It has the flexibility to be integrated with other optimization techniques to form a hybrid tool.
• It is less sensitive to the nature of the objective function, i.e., convexity or continuity.
• It has less parameter to adjust unlike many other competing evolutionary techniques.
• It has the ability to escape local minima.
• It is easy to implement and program with basic mathematical and logic operations.
• It can handle objective functions with stochastic nature.
• It does not require a good initial solution to start its iteration process.

PSO has been successfully applied in many different application areas due to its robustness and simplicity. In comparison with other stochastic optimization techniques, PSO have fewer complicated operations and fewer defining parameters, and can be coded in just a few lines (Wanget al., 2007). But this algorithm sometimes may suffer from premature convergence problem. There have been many researchs to improve the performance of the original PSO algorithm.

Bergh and Engelbrecht (2004) proposed a cooperative particle swarm frame (CPSO), the cooperative particle swarm optimizer. The authors used multiple swarms to optimize different components of the solution vector cooperatively. They tested CPSO over five benchmark optimization problems. The tests showed that CPSO reaches significantly better solution than PSO. Pena, Upegui and Sanchez (2006) presented a hybrid bio-inspired optimization technique that introduces the concept of discrete recombination in a particle swarm optimizer (PSODR), obtaining a simple and powerful algorithm, well suited for embedded applications. Proposed algorithm validated over four optimization problems and noted that PSODR shows a better performance than the standard PSO algorithm. Baskar and.Suganthan (2004) present PSO (CONPS0) algorithm to alleviate the premature convergence problem of PSO algorithm. CONPSO is a type of parallel algorithm in which modified PSO and FDR-PSO algorithms are simulated concurrently with frequent message passing between them. To demonstrate the effectiveness of the proposed algorithm experiments were conducted on six benchmark optimization problems. Results clearly demonstrate the superior performance of the proposed algorithm. Kok and Snyman (2008) proposed dynamic interacting particle swarm optimization algorithm (DYN-PSO). In this method, the

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minimization of a function is achieved through the dynamic motion of a strongly interacting particle swarm, where each particle in the swarm is simultaneously attracted by all other particles located at positions of lower function value. The force of attraction experienced by a particle at higher function value due to a particle at a lower function value is equal to the difference between the respective function values divided by their stochastically perturbed position difference. The resultant motion of the particles under the influence of the attracting forces is computed by solving the associated equations of motion numerically. An energy dissipation strategy is applied to each particle. The specific chosen force law and the dissipation strategy result in the rapid collapse (convergence) of the swarm to a stationary point. Obtained results of seven benchmark optimization problem show that, in comparison to the standard particle swarm algorithm, the proposed DYN-PSO algorithm is promising. Yan Jiang, Tiesong Hu, ChongChao Huang, Xianing Wu (2007) proposed an improved particle swarm optimization (IPSO). In IPSO, a particle sampled randomly from the feasible space. Then the population is partitioned into several sub-swarms, each of which is made to evolve based on particle swarm optimization (PSO) algorithm. At periodic stages in the evolution, the entire population is shuffled, and then points are reassigned to sub-swarms to ensure information sharing. Simulations for three benchmark test functions show that IPSO possesses better ability to find the global optimum than that of the standard PSO algorithm. Ali and Kaelo (2008) proposed some modifications in the position update rule of particle swarm optimization algorithm in order to make the convergence faster. These modifications result in two new versions of the particle swarm optimization algorithm. A numerical study is carried out using a set of 54 test problems some of which are inspired by practical applications. Results show that the new algorithms are much more robust and efficient than some existing particle swarm optimization algorithms. Yuxin Zhao, Wei Zu, Haitao Zeng (2009) proposed a new modified algorithm to ensure the rational flight of every particle's dimensional component. Meanwhile, two parameters of particle-distribution-degree and particle-dimension-distance are introduced into the proposed algorithm in order to avoid premature convergence. Simulation results of the new PSO algorithm show that it has a better ability of finding the global optimum, and still keeps a rapid convergence as with the standard PSO. S. He, Q.H. Wu, J.Y. Wen, J.R. Saunders, R.C. Paton (2004) presented a particle swarm optimizer with passive congregation to improve the performance of standard PSO. By introducing passive congregation to PSO, information can be transferred among individuals of the swarm. A particle swarm optimizer with passive congregation (PSOPC) is tested with a set of 10 benchmark functions with 30. Experimental results indicate that the PSO with passive congregation improves the search performance on the benchmark functions significantly. Qi Kang, Lei Wang, Qi-di Wu (2008) presented a particle swarm optimization algorithm from the angle of ecological population evolution, called the ecological particle swarm optimization (EPSO). From the basis of the ecological population competition model, the EPSO algorithm and its general framework are proposed; in which particle swarm system with ecological hierarchy and competition model is defined and two collocating strategies of inertia weight factor are considered. The convergence performance and population dynamics including population aggregation and population diversity of the proposed approach are discussed separately through empirical simulations with four benchmarks optimization problems. Chen (2011) proposed a two-layer particle swarm optimization (TLPSO) to increase the diversity of the particles so that the drawback of trapping in a local optimum is avoided. In order to design the TLPSO, a structure with two layers (top layer and bottom layer) is proposed so that M swarms of particles and one swarm of particles are generated in the bottom layer and the top layer, respectively. Each global best position in each swarm of the bottom layer is set to be the position of the particle in the swarm of the top layer. Therefore, the global best position in the swarm of the top layer influences indirectly the particles of each swarm in the bottom layer so that the diversity of the particles increases to avoid trapping into a local optimum. Besides, a mutation operation is added into the particles of each swarm in the bottom layer so that the particles leap the local optimum to find the global optimum. Nine optimization problems were used to illustrate the efficiency of the proposed method. The author noted that proposed TLPSO approach is superior to the other evolutionary algorithms in the ability to finding the global optimum solution. Bratton and Blackwell (2008) introduced PSO with discrete recombination model (PSO-DR). They have noted that the PSO-DR variant is important not only because of its improved performance on several benchmark functions, but also because its simplified state allows us to examine what happens to the standard algorithm when pieces are modified or removed. PSO-DR has been tested over 14 benchmark problem. Bratton and Kennedy (2007) have compared three PSO algorithms in the interest of demonstrating the performance gains granted by improvements to the technique: the original algorithm, a constricted swarm using a global topology, and a constricted swarm using a local topology. The algorithms have been tested over 14 benchmark problem. Chen and Chi (2010) have tuned some of the parameters and added mechanisms to the PSO algorithm in order to improve its robustness in finding the global solution. This approach has been tested over 10 problems. Abd-El-Wahed,Mousa, and El-Shorbagy (2011) introduced a hybrid approach combining two heuristic optimization techniques, particle swarm optimization (PSO) and genetic algorithms (GA). The method integrates the merits of both GA and PSO and it has two characteristic features. Firstly, the algorithm is initialized by a set of random particles which travel through the search space. During this travel an evolution of these particles is performed by integrating PSO and GA. Secondly, to restrict velocity of the particles and control it, the authors introduce a modified constriction factor. Finally, the results of various experimental studies using a suite of multimodal test functions taken from the literature have demonstrated the superiority of the proposed approach to finding the global optimal solution. Akbari and Ziarati (2011) presented a variation on the standard PSO algorithm called the rank based particle swarm optimizer (PSOrank). In this method, in order to efficiently control the local search and convergence to global optimum solution, the γ best particles are taken to contribute to the updating of the position of a candidate particle. The contribution of each particle is proportional to its strength. The strength is a function of three parameters: Strivness, immediacy and number of contributed particles. All particles are sorted according to their fitness values, and only the γ best particles will be selected. The value of γ decreases linearly as the iteration increases. A time-varying inertia weight decreasing non-linearly is introduced to improve the performance. PSO rank is tested on five commonly used optimization problems.

In this study PSO-FS model has been proposed. The work is organized as follows: In Section 2 and Section 3 classical PSO

algorithm and new PSO-FS algorithms are presented respectively. In Section 4, Experimental Results and Comparison has been explained. In Section 5 this paper is concluded.

2. Overview of the standard PSO

Since PSO is a population based optimization method, first a population of random particles must be generated. Suppose that the search space is D-dimensional and the swarm consists of N particles. The ith particle is represented as $X_i = (x_{i1}, x_{i2}, \dots, x_{id})$, i=1,2,…N. A current velocity of i^{th} particle is represented as $V_i = (v_{i1}, v_{i2}, \dots, v_{id})$, i=1,2,…N. The personal best position achieved by i^{th} particle is represented by $P_{best} = (p_{i1}, p_{i2}, \dots, p_{id})$ and the best position achieved by swarm is represented by $G_{best} = (g_{i1}, g_{i2}, \dots, g_{id})$. The velocity of particle and its next position is updated according to the following equations (Bratton and Kennedy, 2007):

$$V_{ij} = \chi[v_{ij} + c_1 r_1(p_{ij} - x_{ij}) + c_2 r_2(p_{gj} - x_{gj})] \quad (1)$$

$$x_{ij} = x_{ij} + v_{ij} \quad (2)$$

where i=1,2,…N, j=1, 2,…D. χ is a parameter called as constriction factor which is used to limit the maximum velocity; c_1 and c_2 are two positive constants called cognitive and social parameter, respectively; and r_1, r_2, are random numbers in the range [0, 1].

The execution of the PSO algorithm is as follows:

1. Initialization: randomly initialize a population of particles, set parameters c_1, c_2, r_1, r_2, χ .

2. Population loop: for each particle x_i do:

2.1. Goodness evaluation and update: evaluate the goodness $f(x_i)$ using corresponding fitness function of the particle. If $f(x_i) > Pbest_i$, namely its goodness is greater than its best goodness so far, then this particle becomes the best particle found so far.

2.2. Neighbourhood evaluation: if $f(x_i) > Gbest_i$, namely the goodness of this particle is the best among all its neighbors, then this particle becomes the best particle of the whole neighbourhood.

2.3. Determine v_i: Update the velocity v_i using Equation 1.

2.4. Particle update: Update the position x_i using Equation 2.

3. Cycle: repeat Step 2 until a given termination criterion is met.

3. PSO with flexible swarm

In this method, natural selection has been applied to swarm. For this after each iteration particles were scored between *1*-*N*, where successful particle has *N*, unsuccessful particle has *1* point. After *m* iteration higher score maxs and lower score mins has been found. Using Equation 3 valid score vs has been calculated.

$$vs = \sqrt{\max s^* \min s} \quad (3)$$

The particles, which score less than vs were removed and a new particles has been generated instead it.

The execution of the algorithm is as follows:

1. Initialization: randomly initialize a population of particles, set parameters c_1, c_2, r_1, r_2, χ , m. Set parameter *iter*=0.

2. Iteration loop: Set *iter*=*iter*+1.

3. Population loop: for each particle x_i , do:

3.1. Goodness evaluation and update: evaluate the goodness $f(x_i)$ using corresponding fitness function of the particle. If $f(x_i) > Pbest_i$, namely its goodness is greater than its best goodness so far, then this particle becomes the best particle found so far.

3.2. Neighbourhood evaluation: if $f(x_i) > Gbest_i$, namely the goodness of this particle is the best among all its neighbours, then this particle becomes the best particle of the whole neighbourhood.

3.3. Determine v_i: Update the velocity v_i: using Equation 1.

3.4. Particle update: Update the position *x* using Equation 2.

3.5. Determine s_i: Determine the scor s_i .

3.6. Revision of swarm: If *iter*=*m*, then set *iter*=0, determine *vs* using Equation 3, wipe out particles, which score less than *vs* and compose new particle instead of they.

4. Cycle: Go to step 2 until a given termination criterion is met.

4. Experimental Results and Comparison

Algorithms have been tested over of 14 benchmark functions for 300000 iterations for each function. Testing functions, feasible bounds, optimum points and dimension of functions are shown in Table 1 and Table 2. Functions f_1-f_3 are functions with a single minimum, f_4-f_9 are complex problems, with a number of local minimas and a single global minimum. Functions f_1-f_9 are high-dimensional problems. Function f_{10} is symmetric around the origin, so it has two global minimum and a several local minimas. f_{11}–f_{14} are problems with a several local minimas and a single global minimum point.

Some parameters of the algorithm in this paper are set as follow: the number of particles in the population space is set as *ps*=50; The maximum iteration number is set as 300000 in each running. Each algorithm for each function has been run 30 times. Dimension is set as d=30 for functions f_1-f_9, *d*=2 for functions $f_{10\text{-}11}$ and d=4 for functions $f_{12\text{-}14}$. These dimensions are widely used in literature, so for objective comparison in this study they have been preferred. During the optimization process, the particles are limited to move in the region defined by $[x_{min}, x_{max}]$. An inertia weight of 0.5, cognitive and social parameters of 2 were used. A fully connected topology is used in all cases. Performance was measured as the minimum error $|f(x) - f(x^*)|$ found over the trial, where $f(x^*)$ is the optimum fitness for the problem.

Table 2. Results of the biopsy, FPSA/PSA, risk ratios of online calculator and FES

Equation	Name
$f_1 = \sum_{i=1}^{D} x_i^2$	Sphere / Parabola
$f_2 = \sum_{i=1}^{D} \left(\sum_{j=1}^{i} x_j \right)^2$	Schwefel 1.2
$f_3 = \sum_{i=1}^{D-1} \left\{ 100\left(x_{i+1} - x_i^2\right)^2 + \left(x_i - 1\right)^2 \right\}$	Generalized Rosenbrock
$f_4 = -\sum_{i=1}^{D} x_i \sin\left(\sqrt{x_i}\right)$	Generalized Schwefel 2.6
$f_5 = \sum_{i=1}^{D} \left\{ x_i^2 - 10\cos(2\pi x_i) + 10 \right\}$	Generalized Rastrigin
$f_6 = -20\exp\left\{ -0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2} \right\} - \exp\left\{ \frac{1}{D}\sum_{i=1}^{D} \cos(2\pi x_i) \right\} + 20 + e$	Ackley
$f_7 = \frac{1}{4000}\sum_{i=1}^{D} x_i^2 - \prod_{i=1}^{D} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	Generalized Griewank
$f_8 = \frac{\pi}{D}\left\{ 10\sin^2(\pi y_i)\left\{ 1 + 10\sin^2(\pi y_{i+1}) \right\} + \left(y_D - 1\right)^2 \right\} +$ $+ \sum_{i=1}^{D} \mu(x_i, 10, 100, 4)$ $y_i = 1 + \frac{1}{4}(x_i + 1)$ $\mu(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{cases}$	Penalized Function P8
$f_9 = 0.1\left\{ \sin^2(3\pi x_i) + \sum_{i=1}^{D-1} (x_i - 1)^2 \left\{ 1 + \sin^2(3\pi x_{i+1}) \right\} + \left(x_D - 1\right)^2 \times \left\{ 1 + \sin^2(2\pi x_D) \right\} \right\} + \sum_{i=1}^{D} \mu(x_i, 5, 100, 4)$	Penalized Function P16
$f_{10} = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	Six-hump Camel-back
$f_{11} = \left\{ 1 + (x_1 + x_2 + 1)^2 \left(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2\right) \right\} \times \left\{ 30 + (2x_1 - 3x_2)^2 \left(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2\right) \right\}$	Goldstein-Price
$f_{12} = -\sum_{i=1}^{5} \left\{ \sum_{j=1}^{4} (x_j - a_{ij})^2 + c_i \right\}^{-1}$	Shekel 5
$f_{13} = -\sum_{i=1}^{7} \left\{ \sum_{j=1}^{4} (x_j - a_{ij})^2 + c_i \right\}^{-1}$	Shekel 7
$f_{14} = -\sum_{i=1}^{10} \left\{ \sum_{j=1}^{4} (x_j - a_{ij})^2 + c_i \right\}^{-1}$	Shekel 10

Table 2. The feasible bounds, optimas and dimensions of benchmark function

Equation	Feasible Bounds	Optimum	Dimension
f_1	$(-100,100)^D$	0.0^D	30
f_2	$(-100,100)^D$	0.0^D	30
f_3	$(-30,30)^D$	1.0^D	30
f_4	$(-500,500)^D$	420.9687^D	30
f_5	$(-5.12,5.12)^D$	0.0^D	30
f_6	$(-32,32)^D$	0.0^D	30
f_7	$(-600,600)^D$	0.0^D	30
f_8	$(-50,50)^D$	-1.0^D	30
f_9	$(-50,50)^D$	1.0^D	30
f_{10}	$(-5,5)^D$	$(-0.0898 \quad ,0.7126), (0.0898 \quad ,-0.7126)$	2
f_{11}	$(-2,2)^D$	$(0 \quad ,-1)$	2
f_{12}	$(0,10)^D$	4.0^D	4
f_{13}	$(0,10)^D$	4.0^D	4
f_{14}	$(0,10)^D$	4.0^D	4

Table 3. Mean error after 30 trials

Eq.	PSO	PSO-FS	Const. Gbest	Const. Lbest	PSO-DR M1 Ring	PSO-DR M1 Global	PSO-DR M2	PSO-DR M3
f_1	0	0	0	0	0	0	0	0
f_2	173.5927	0	0	0.1259	0.01	0	3.7E-7	5.14
f_3	36.4739	0	8.1579	12.6648	16.79	0.8	34.57	18.64
f_4	2112	0.0134	3508	3360	2697	3754	2418	1830
f_5	62.7733	0.1990	140.4876	144.8155	44.64	115.51	35.21	9.88
f_6	$1.8608e^{-009}$	$2.1574e^{-011}$	17.6628	17.5891	0.68	18.51	0	0
f_7	0.2269	0.0245	0.0308	0.0009	0	0.008	0	0
f_8	3	0	0.1627	0	0	0.005	0	0
f_9	0,0179	0	0.0040	0	0	0.002	0	0
f_{10}	0	0	0	0	0	0	0	0
f_{11}	0	0	0	0	0	0	0	0
f_{12}	5.1753	1.1789	4.5882	2.5342	0.17	4.34	0	0
f_{13}	2.6457	0.7046	4.4747	1.0630	0	2.55	$8.1e^{-11}$	0
f_{14}	4.7298	0.3574	3.8286	0.5409	0	3.13	$6.6e^{-11}$	0

In many studies researches showed the results on the few functions. Hence we couldn't compare all of our result with a number of others. Table 3 shows the averaged results of 30 independent trials relative to the works Bratton & Blackwell (2008) and Bratton & Kennedy (2007).

5. Conclusions

The PSO algorithm was inspired by the social behaviour of birds and fishes flocking to a food source. There are numerous researches in modifications of original PSO algorithm. In this study the particle swarm optimization algorithm with flexible swarm has been presented. Proposed algorithm was evaluated on fourteen functions, which often used to benchmark the performance of optimization algorithms. PSO-FS algorithm was compared to some other modifications and classical PSO. The results show that PSO-FS always performed one of the better results than the others.

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