

Developing a Fuzzy Logic Decision Support System for Strategic Planning in Industrial Organizations

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Abstract: Internal – External (IE), Strategic Position and Action Evaluation (SPACE), Boston Consulting Group (BCG), and Grand Strategy matrices are important tools in generating and evaluating alternative output strategies which is the most important step in the formulation stage in strategic management process. Managers need to make difficult and important decisions on the basis of imprecise information and incomplete knowledge and select the right strategic position or alternative output strategies. Managers found it difficult to make the right decision when the values that resulted from the internal and external audits of an organization's environment fall in the middle or on the sharp edges in these matrices. It is found that each matrix depends on some factors that represent the internal and external environment of an organization and neglect the others. A systematic approach that incorporates fuzzy set theory in conjunction with strategic planning is proposed to assist managers in reaching the best alternative output set of strategies. The four matrices are combined by relations in order to integrate more than two fuzzy factors in selecting the best alternative output strategies. This make the suggested strategic plan robust and help organizations with limited resources to neglect alternative output strategies that do not depend on the four matrices jointly. The implementation of the developed Fuzzy Logic Decision Support System (FLDSS) is done using MATLAB fuzzy logic tools and the relations between these matrices are entered as rules in the Graphical User Interface (GUI) screens. The model was validated by a case study on a Jordanian company, and the results were satisfactory.

Keywords: Fuzzy Logic, Decision Support System, Strategic Planning, Industrial Organization.

1. Introduction

Strategic management allows an organization to be more proactive than reactive in shaping its own future. It allows an organization to initiate an influence (rather than respond to) activities - and thus to exert control over its own destiny. Small business owners, chief executive officers, presidents, and managers of many for-profit and non-profit organizations have recognized and realized the benefits of strategic management.

In the process of generating and selecting alternative output strategies in the formulation stage of the development of a strategic plan, there are mainly four matrices that are commonly used in formulating a strategy; these are: Strategic Position & Action Evaluation (SPACE) Matrix, Boston Consulting Group (BCG) Matrix, Internal – External (IE) Matrix, and the last one is the Grand strategy Matrix.

In BCG Matrix, viewing every product as a Star, Cash Cow, Dog, or Question Mark is an oversimplification. Many products fall right in the middle of the BCG Matrix and thus are not easily classified [1]. In most decision making process, decision-makers have to make decisions with incomplete information and under uncertain circumstances. These situations have been recognized by many researchers as a suitable field to use fuzzy set theory. Fuzzy logic is used in system control and analysis design because it shortens the time for engineering development and sometimes, in

the case of highly complex systems, it is the only way to solve the problem. Fuzzy logic can apply also to economics, psychology, marketing, weather forecasting, biology, and politics [2]. Therefore, based on the concepts of decision support system (DSS), an integrated framework was developed that incorporates fuzzy theory in the strategic position selection. This framework provides managers with a flexible, expandable and interactive DSS to select and evaluate the alternative output strategies in the formulation stage.

Strategists must decide which alternative strategies will benefit the organization most, and therefore, a manageable set of the most attractive alternative strategies must be developed [1]. Strategists never consider all feasible alternatives that could benefit the organization because there is a large number of possible strategies and a large number of ways to implement them, as well as, because no organization has unlimited resources to pursue all alternative strategies.

This work integrated the process of generating and selecting the best alternative output strategy or set of strategies using fuzzy logic decision support system. Therefore, it ensures defining a well formulated, implemented, and evaluated strategic plan in the organization to reach its destination.

2. Literature review

The strategic management process consists of three stages: strategy formulation, strategy implementation, and strategy evaluation. Strategy formulation includes developing a vision and mission statements, identifying an organization's external opportunities and threats, determining internal strengths and weaknesses,

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establishing long-term objectives, generating alternative strategies, and choosing particular strategy to pursue. Strategy formulation issues include deciding what new business to enter, what businesses to abandon, how to allocate resources, whether to expand operations or diversify, or whether to enter international markets. Alternative strategies that an organization could pursue can be categorized into 11 main strategies which are: Forward integration, Backward integration, Horizontal integration, Market penetration, Market development, Product development, Related diversification, Unrelated diversification, Retrenchment, Divestiture, and Liquidation.

2.1. Decision Support System

Some researchers defined a Decision Support System (DSS) broadly as a computer - based system that aids the process of decision making, while others define it as an interactive, flexible, and adaptable computer-based information system, especially developed supporting the solution of a non-structured management problem for improved decision making [3]. DSS combines the use of models and analytical techniques with conventional data access and retrieval functions and has features (including interactive features) which make its use by non-computer people easier. A Decision Support System provides one form of information technology that are capable of storing, retrieving, presenting, and manipulating data and models in an on-line, real time, and interactive manner to help decision makers to solve semi-structures problems [4].

2.2. Fuzzy Logic

In today's rapidly changing and highly uncertain environment, the strategic decisions have an extremely complex and fuzzy nature [5]. Generally, decisions are made with limited information because decision makers do not have full knowledge of the problem they face and generally cannot even determine a reasonable probability for alternative outcomes; thus, they must make their decisions under conditions of high uncertainty. In addition, many decisions in organizations, especially important one that have far-reaching effects on organizational activities and personnel, are made in groups. One problem with group decision making is that each member in the decision group has not the same knowledge of the problem as the others have. This means that decision makers will face a decision-making situation with various peers possessing different confidence levels regarding the problem to be handled. Thus, the domain of strategic management has already been recognized as a field appropriate for the application of a fuzzy set theory [6].

However, linguistic variables per se contain ambiguity and multiplicity of meanings and therefore, the information obtained can be expressed as a range of fuzzy sets, instead of a single value in traditional methods. Applying fuzzy logic seems to be the most appropriate method for strategic decision making [7]. The field of strategic management has been recognized as an appropriate field for the application of the fuzzy set theory because of the fuzziness of the main concepts and terms, since the contexts of strategic management belong to the area of uncertainty and vagueness [8]. Lin and Hsieh [8] developed an integrated framework that incorporates fuzzy theory into strategic portfolio selection based on the concepts of decision support system (DSS). This framework provides managers with a flexible, expandable and interactive DSS to select projects for portfolio management. They used the GE Multifactor Portfolio matrix (that developed jointly by General Electric and McKinsey and company) to express the competitive position of the organization which is based primarily on two variables: industry attractiveness (IA) and business strength (BS),

and used the 3Cs model (which concerned with the business' customer relations, process capabilities, and functional competencies) to evaluate the feasibility of the strategic plans.

Ghazinoory et al., [5] attempted to solve certain structural problems of the SWOT matrix by following the fuzzy approach to the internal and external factors (in the form of fuzzy membership functions). They recommended in their research to combine more than two fuzzy factors for extracting a single strategy. Ghazinoory et al., [9] evaluated both internal and external factors in linguistic terms and in terms of fuzzy triangular numbers. The fuzzy numbers are fed into an industry attractiveness-business analysis matrix. The matrix is composed of zones, which represent pre-defined sets of strategies. The evaluation of an organization's internal and external conditions results in a position (usually a point) in one of the strategy zones. Keropyan and Gil-Lafuente [10] demonstrated the effects of different decision styles on strategic decisions and likewise, on an organization. The technique that was presented in the study is based on the transformation of linguistic variables to numerical value intervals.

This fuzzy methodology approach allows examining the relations between decision making styles and strategic management processes when there is uncertainty. The purpose of this work is to provide results to companies that may help them to exercise the most appropriate decision making style for its different strategic management processes. The research is leaving more research topics for further studies that may be applied to other decision making areas within the strategic management process.

As a conclusion there is a limited number of researches that use fuzzy logic decision support system to formulate the organization's strategy. Also many researchers rely in their researches on two fuzzy factors to extract the best alternative output strategy that the organization must pursue to achieve its long term objective. In this research and based on the recommendations of previous researchers [5], more than two factors are used to reach for the best alternative output strategy or set of strategies. Eight factors or variables in the four matching stage matrices used jointly to evaluate the alternative output strategies, i.e., in the SPACE Matrix (Financial Position, Stability Position, Competitive Position, and Industry Position), in the BCG Matrix (Industry sales growth rate, and Relative market share position), in the IE Matrix (External Factor Evaluation, and Internal Factor Evaluation), and in the Grand strategy Matrix (Competitive position and Market growth rate) as shown in Table 1.

3. Methodology

3.1. Integrating the four matrices

In this stage the most frequent alternative output strategies among the four matrices (SPACE, BCG, IE, and Grand) will be determined directly using Fuzzy Logic Decision Support System (FLDSS) without generating and evaluating the alternative output strategies for each matrix separately. On the basis of outcomes emanating from the four matrices in the matching stage of a strategic development process, the alternative output strategies that an organization must pursue should be selected to achieve its long term objectives. The criterion for the selection of alternative output strategies is the frequency of choice.

Table 1. The possible value of the inputs that come in the middle of each matrix

Matrix	Inputs	The possible values of the inputs
IE Matrix	IFE (Internal Factor Evaluation)	1, 2, 3, 4
	EFE (External Factor Evaluation)	1, 2, 3, 4
BCG Matrix	R.M.S.P (Relative Market Share Position)	0, 0.5, 1
	I.S.G.R (Industry Sales Growth Rate)	-20, 0, 20
SPACE Matrix	(FP+SP)*	-6, 0, 6
	(IP+CP)**	-6, 0, 6
Grand strategy matrix	Market growth	0.05
	Competitive position	1, 2.5, 4

* (FP+SP): The resultant of (Y – Axis) in SPACE Matrix.

** (IP+CP): The resultant of (X – Axis) in SPACE Matrix.

The four matrices will be integrated or linked by relations between these matrices to reach for the most frequent alternative output strategy or set of strategies in the different strategic positions using fuzzy logic decision support system model (FLDSS) to ensure formulating the strategy with lower cost in the organization that have limited resources. For example, Forward Integration Strategy will be found twice in SPACE Matrix: in the Aggressive position

and Competitive position, once in BCG Matrix in Stars position, once in Grand strategy Matrix in Quadrant one position, and once in IE Matrix in Grow and Build region as shown in Table 2.

Figure 2 shows how each of the eleven alternative output strategies resulted from the combination of different strategic positions in the four matching stage matrices. These relations among the four matrices will help in setting the rules for each alternative output strategy in the model. The first column represents the three strategic positions of the IE Matrix (Grow and build, Hold and Maintain, Harvest or Divest), the second column represents the SPACE Matrix which has four strategic positions (Aggressive, Competitive, Conservative, and Defensive), the third column represents the BCG Matrix which has four strategic positions (Stars, Question marks, Cash cows, and Dogs), the fourth column represents the Grand Strategy Matrix that has four strategic positions (Quadrant 1, Quadrant 2, Quadrant 3, and Quadrant 4), and the fifth column represents the eleven main alternative output strategies (Forward integration, Backward integration, Horizontal integration, Market development, Market penetration, Market development, Retrenchment, Divesture, and Liquidation).

The relations among the four matrices for each alternative output strategy consist of the strategy's positions in the different strategic positions or regions in these matrices depending on the values of the inputs that are derived from the internal and external audit of the organization's environment. On contrary, to other matrices which have two ranges or levels in their variables, each strategic position (Grow and build, Hold and maintain, Harvest or divest)

Table 2. The repetition or frequency of the alternative output strategies in the different strategic positions of the four matrices

No.	Strategy	1. SPACE Matrix				2. BCG Matrix				3. IE Matrix			4. Grand strategy Matrix				Count
		Aggressive	Competitive	Conservative	Defensive	Stars	Cash cows	Question marks	Dogs	Grow and build	Hold and maintain	Harvest or divest	Q1	Q2	Q3	Q4	
1	Forward Integration	X	X			X				X			X				5
2	Backward Integration	X	X			X				X			X				5
3	Horizontal Integration	X	X			X				X			X	X			6
4	Market development	X	X	X		X		X		X			X	X			8
5	Market penetration	X	X	X		X		X		X	X		X	X			9
6	Product development	X	X	X		X	X	X		X			X	X			9
7	Related diversification	X		X			X				X		X		X	X	6
8	Unrelated diversification	X					X								X	X	4
9	Retrenchment				X		X		X		X				X		5
10	Divesture				X		X	X	X		X				X	X	7
11	Liquidation				X				X					X	X		4

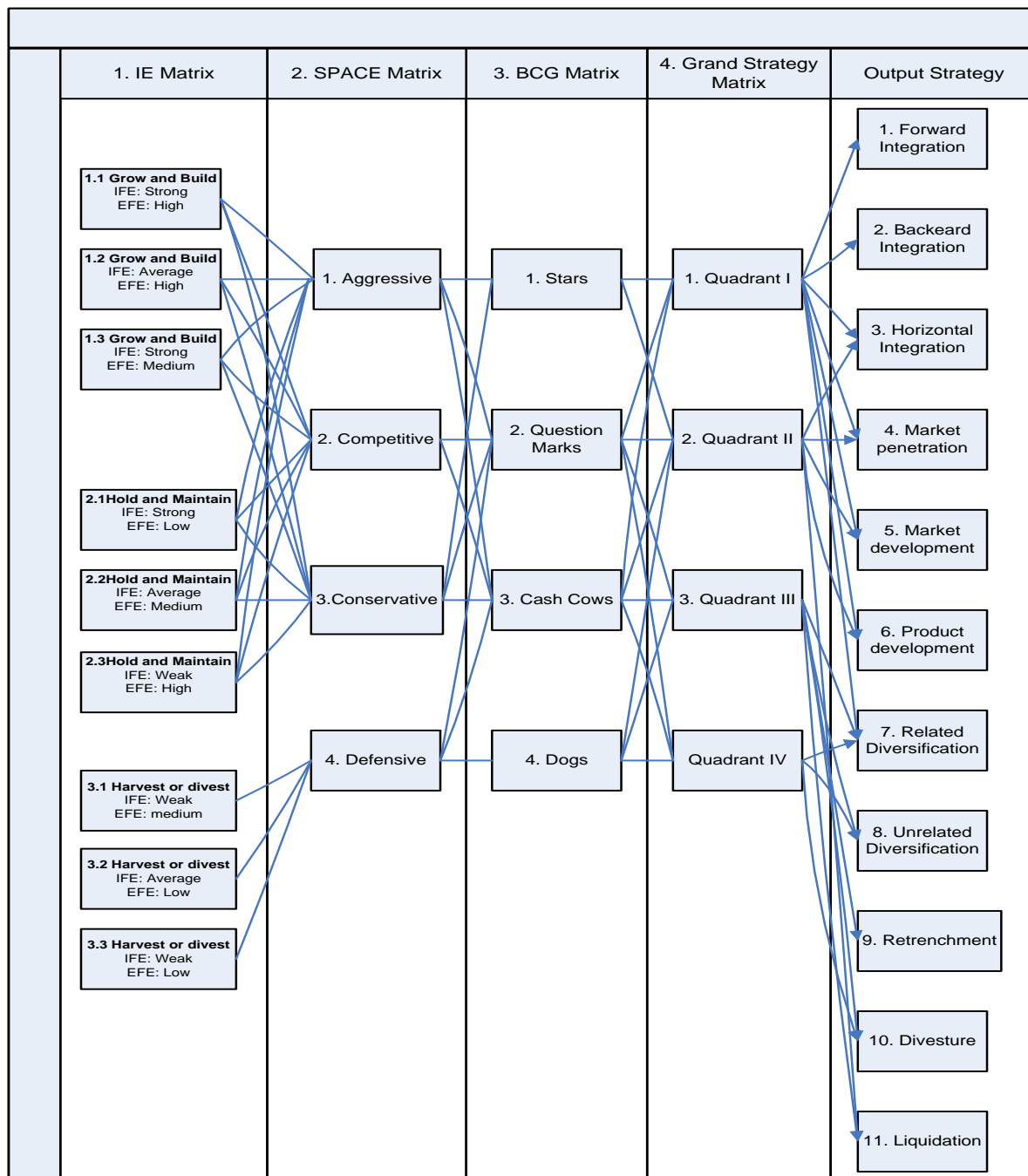


Figure 2. Overview of the integrated four matrices and the output strategies

in IE Matrix is resulted from three ranges or levels of the Internal Factors Evaluation (IFE) and External Factors Evaluation (EFE) matrices' variables. For example Grow and Build region resulted when:

1. IFE ranges between (3– 4) and EFE ranges between (3– 4).
2. IFE ranges between (2– 3) and EFE ranges between (3– 4).
3. IFE ranges between (3– 4) and EFE ranges between (2– 3).

There are eleven tables that represent the relations among the four matrices, in which each table represents an alternative output strategy, and each row in these tables represents a relation that combines the different strategic positions of the four matrices. For example, Table 3 shows the 6 combinations or relations between the four matrices that will lead to the Forward Integration strategy, and each row in the table indicates a relation or a condition that might be achieved.

There are 274 relations that lead to the eleven alternative output strategies, and each table is formed from five columns; the first four columns represent the strategic positions or regions of the four matrices that the alternative output strategy may fall in, and the fifth column represents the eleven alternative output strategies.

There are duplications or similarities in many relations or rows that lead to different alternative output strategies. This means that there is a conflict because different alternative output strategies have the same relation. For example, Table 4 represents how the same relation between the four strategic positions (Grow and Build (IFE: 3 - 4, EFE: 3 - 4), Aggressive, Stars, and Quadrant 2) of the four matrices will lead to the different alternative output strategies (Horizontal Integration, Market penetration, Market development, Product development).

Table 3. Forward Integration Strategy's rules in all strategic position of the four matrices

1. IE Matrix	2. SPACE Matrix	3. BCG Matrix	4. Grand Strategy Matrix	Output Strategy
Grow and build (IFE:3 - 4,EFE:3 - 4)	Aggressive	Star	Quadrant 1	Forward Integration
Grow and build (IFE:3 - 4,EFE:3 - 4)	Competitive	Star	Quadrant 1	Forward Integration
Grow and build (IFE:2 - 3,EFE:3 - 4)	Aggressive	Star	Quadrant 1	Forward Integration
Grow and build (IFE:2 - 3,EFE:3 - 4)	Competitive	Star	Quadrant 1	Forward Integration
Grow and build (IFE:3 - 4,EFE:2 - 3)	Aggressive	Star	Quadrant 1	Forward Integration
Grow and build (IFE:3 - 4,EFE:2 - 3)	Competitive	Star	Quadrant 1	Forward Integration

Table 4. The duplication of same rule for different outputs

1. IE Matrix	2. SPACE Matrix	3. BCG Matrix	4. Grand Strategy Matrix	Output Strategy
Grow and build (IFE:3 - 4,EFE:3 - 4)	Aggressive	Stars	Quadrant 2	Horizontal integration
Grow and build (IFE:3 - 4,EFE:3 - 4)	Aggressive	Stars	Quadrant 2	Market penetration
Grow and build (IFE:3 - 4,EFE:3 - 4)	Aggressive	Stars	Quadrant 2	Market development
Grow and build (IFE:3 - 4,EFE:3 - 4)	Aggressive	Stars	Quadrant 2	Product development

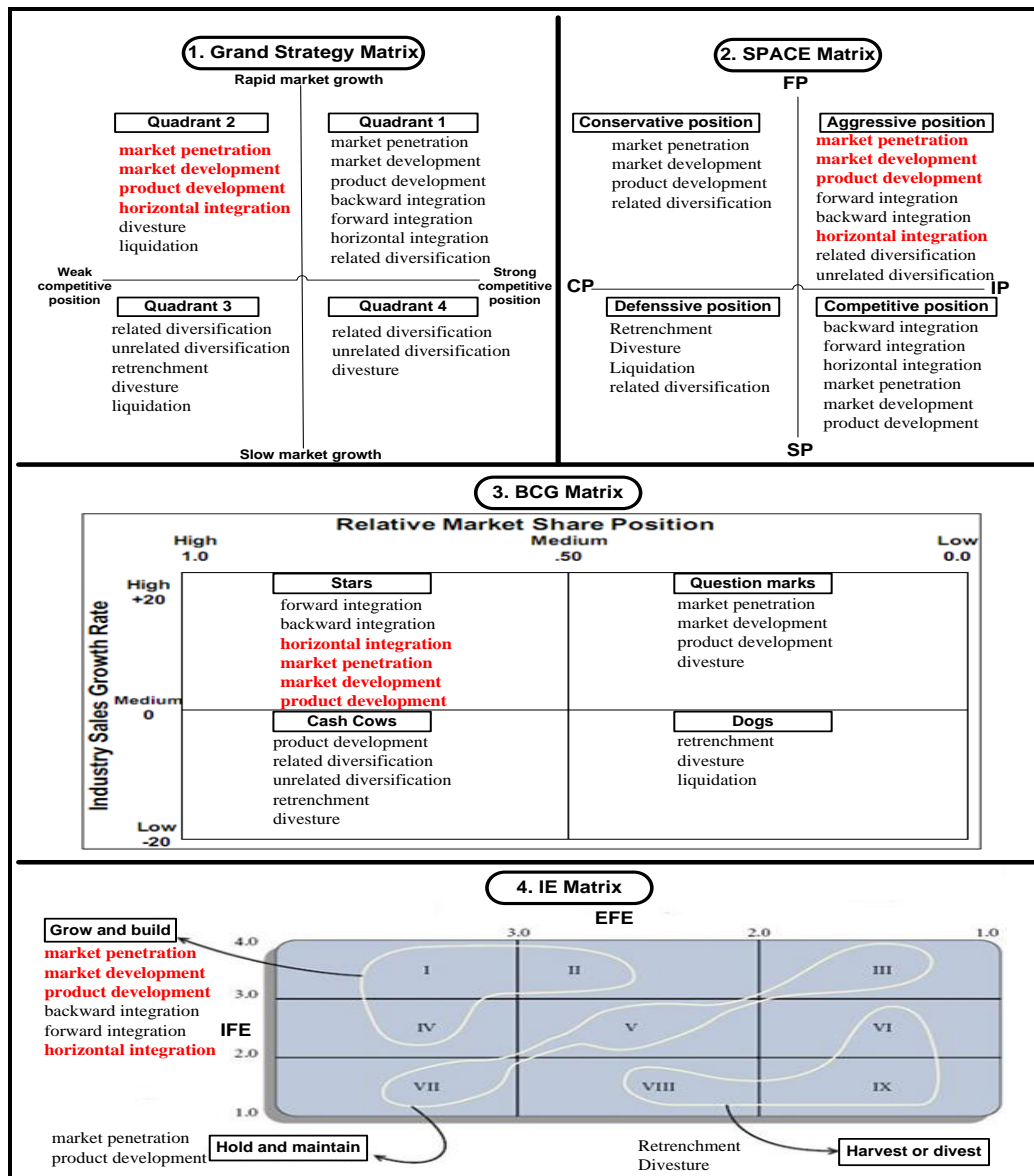


Figure 3. How the group of alternative strategies (Horizontal Integration, Market development, Market penetration, Product development) fall in different strategic position.

Figure 3 shows how this set of alternative output strategies (Horizontal Integration, Market Penetration, Market Development, Product Development) which are shown in Table 4 falls in the different strategic positions of the four matrices and should be pursued congruently when the internal and external audit of an organization lead to Quadrant 2 in Grand Strategy Matrix, Aggressive position in SPACE Matrix, Stars position in BCG Matrix, and Grow and Build region in IE Matrix. Also, it is noted that the alternative output strategies that are not similar to or frequent between these matrices will be neglected and these alternative output strategies are Divesture and Liquidation in Grand Strategy Matrix, Forward Integration, Backward Integration, Related Diversification, and Unrelated Diversification in SPACE Matrix, Forward and Backward Integration in BCG Matrix, and Forward and Backward Integration in IE Matrix).

So, there is a conflict because the same relation or condition leads to multi or different alternative output strategies. To solve this problem the duplications must be eliminated by combining the alternative output strategies which have the same relations or combination between the different strategic positions of the four matrices.

After combining the alternative output strategies that have the same relations, 134 are obtained relations and each one leads to one or group of alternative output strategies, ten outputs (strategy or set of strategies) were resulted from the different 134 relations, these ten outputs are:

1. Forward Integration, Backward Integration, Horizontal Integration, Market penetration, Market Development, Product Development.
2. Horizontal Integration, Market penetration, Market Development, Product Development.
3. Market penetration, Market Development, Product Development.
4. Market penetration and Product Development.
5. Product Development.
6. Related Diversification.
7. Related Diversification, Unrelated Diversification.
8. Retrenchment, Divesture.
9. Divesture.
10. Liquidation.

3.2. Building (FLDSS) Model

The process of generating and evaluating alternative output strategies is called matching stage, and this is the second and important stage in strategy formulation step. Four primary GUI tools were used for building, editing, and observing fuzzy inference systems in the toolbox:

1. Fuzzy Inference System (FIS) Editor.
2. Membership Function Editor.
3. Rule Editor.
4. Rule Viewer.

Each tool of the four primary GUI tools will be implemented in details to build the Fuzzy Logic Decision Support System model as in the following:

3.2.1. Fuzzy Inference System (FIS) Editor

There are 8 inputs or variables and 1 output in the four matrices and in this screen these variables will be entered as the following:

First: Inputs

We have 8 inputs which are:

1. IFE: ranges from 1.0 to 4.0
2. EFE: ranges from 1.0 to 4.0
3. Relative Market Share Position (R.M.S.P.): ranges from 0 to 1

4. Industry Sales Growth Rate (I.S.G.R.): ranges from -20 to +20
5. (IP+CP): It is the resultant of the two variables: Industry position (IP) & Competitive position (CP) in X - axis of the SPACE Matrix, (IP) ranges from 0 to +6, and (CP) ranges from -6 to 0, So, (IP+CP) ranges from -6 to +6
6. (FP+SP): It is the resultant of the two variables: Financial position (FP) & Stability position (SP) in Y - axis of the SPACE Matrix, (FP) ranges from 0 to +6, and (SP) ranges from -6 to 0, So, (FP+SP) which ranges from -6 to +6. Figure 4 shows how the (FP+SP) variable ranges from -6 to +6 on the Y - axis and how the variable (IP+CP) ranges from -6 to +6 on the X-axis
7. Market growth: the input of this variable is whether it exceeds or less than 0.05
8. Competitive position: ranges from 1.0 to 4.0

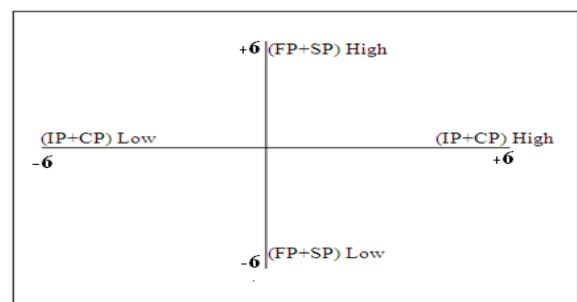


Figure 4. Explanation of (FP+SP) and (IP+CP) on SPACE Matrix

Second: Output

We have one output which is the alternative output strategy or set of strategies that the organization should pursue. Alternative output strategies: range from 0 to 1. Figure 5 shows the eight inputs (IFE, EFE, R.M.S.P., I.S.G.R, IP+CP, FP+SP, Market growth, and Competitive position) to the left and the output (alternative output strategy) to the right in the Fuzzy Inference System Editor screen.

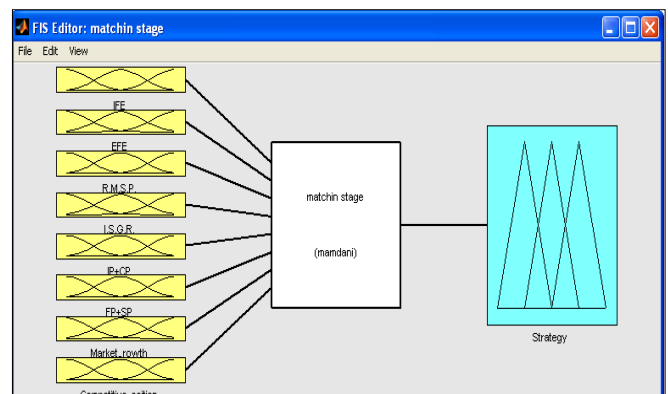


Figure 5. Fuzzy Inference System Editor screen

3.2.2. Membership Function Editor

The Membership Function Editor is a tool that let users display and edit all of the membership functions associated with all inputs and outputs variables for the entire fuzzy inference system. Each input has a number of membership functions equal to the number of the conditions or states for it. For example, if the input has two ranges (1 – 2) and (2 – 3) then there are two membership functions (high and low). The model uses the Gaussian curve built-in membership function for the eight inputs and the Triangular-shaped built-in membership functions for the output which are the most and commonly used functions:

Gaussian curve built-in membership function:

The symmetric Gaussian function depends on two parameters σ (standard deviation) and c (mean) and given by the following equation:

$$f(x; c, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-c)^2}{2\sigma^2}} \quad (1)$$

Triangular-shaped built-in membership function:

The triangular curve is a function of a vector, x , and depends on three scalar parameters a , b , and c , as given by:

$$f(x; a, b, c) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & c \leq x \end{cases} \quad (2)$$

The parameters a and c locate the "feet" of the triangle and the parameter b locates the peak.

The membership functions for the inputs and output in this model are as the following:

First: Inputs

1. *IFE*: The membership Functions are (Figure 6):

(From 3 to 4): Strong.

(From 2 to 3): Average.

(From 1 to 2): Weak.

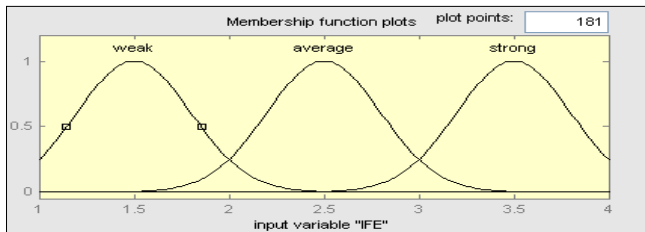


Figure 6. Membership Function of IFE

2. *EFE*:

The membership Functions are:

(From 3 to 4): High.

(From 2 to 3): Medium.

(From 1 to 2): Low.

3. *(IP+CP)*:

The Membership Functions are:

(From -6 to 0): low.

(From 0 to +6): High.

4. *(FP+SP)*:

The Membership Functions are:

(From -6 to 0): low.

(From 0 to +6): High.

5. *Relative Market Share Position (R.M.S.P.)*:

The Membership Functions are:

(From 0.5 to 1): High

(From 0 to 0.5): Low

6. *Industry Sales Growth Rate*:

The Membership Functions are:

(From 0 to +20): High

(From -20 to 0): Low

7. *Competitive Position*:

The Membership Functions are:

(From 2.5 to 4): Strong.

(From 1 to 2.5): Weak.

8. *Market Growth*:

The Membership Functions are:

(Greater than 0.05): Rapid.

(Less than 0.05): Slow.

Second: The Output

The output is the ten alternative output strategies, so there are ten Membership Functions in this model which are:

1. (From 0 to 0.1) the Membership Function is: (1)* which indicates Liquidation.
2. (From 0.1 to 0.2) the Membership Function is: (2) which indicates Divesture.
3. (From 0.2 to 0.3) the Membership Function is: (3) which indicates Retrenchment, and Divesture.
4. (From 0.3 to 0.4) the Membership Function is: (4) which indicates Related Diversification.
5. (From 0.4 to 0.5) the Membership Function is: (5) which indicates Related Diversification, and Unrelated Diversification.
6. (From 0.5 to 0.6) the Membership Function is: (6) which indicates Product Development.
7. (From 0.6 to 0.7) the Membership Function is: (7) which indicates Market Penetration, and Product Development.
8. (From 0.7 to 0.8) the Membership Function is: (8) which indicates Market penetration, Market Development, and Product Development.
9. (From 0.8 to 0.9) the Membership Function is: (9) which indicates Horizontal Integration, Market penetration, Market Development, and Product Development.
10. (From 0.9 to 1) the Membership Function is: (10) which indicates Forward Integration, Backward Integration, Horizontal Integration, Market penetration, Market Development, and Product Development.

Figure 7 indicates the Membership Function's numbers from 1 to 10 for the alternative output strategies which mentioned above, respectively.

Because there is no space in the Membership function plots screen to write the membership functions' names over each one; the numbers from 1 to 10 indicate the names of these membership functions.

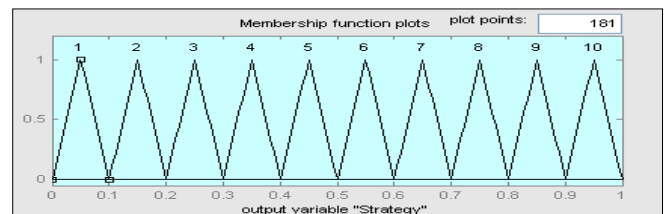


Figure 7. Membership Functions of the output (Strategy)

3.2.3. Rule Editor

There are 134 rules that resulted from the combination of the four matrices and installed in the rule editor screen, for example the first rule that was installed in this screen is as in the following:

"If (IFE is Average) and (EFE is High) and (R.M.S.P. is High) and (I.S.G.R. is High) and (FP+SP is High) and (IP+CP is High) and (Market growth is Rapid) and (Competitive Position is Strong) Then the Output is 10". This rule means: if (IFE) ranges between (2 - 3), (EFE) ranges between (3 - 4), Relative Market Share Position ranges between (0.5 to 1), Industry Sales Growth Rate ranges between (0 to +20), the resultant of Financial Position and Stability Position (FP + SP) ranges between (0 to +6), the resultant of Industry Position and Competitive Position (IP + CP) ranges between (0 to +6), Market growth exceeds 0.05, and Competitive Position ranges between (2.5 to 4), then the alternative output strategies that the organization should pursue are: Forward Integration, Backward Integration, Horizontal Integration, Market penetration, Market Development, and Product Development. Figure 8 shows the Rule Editor screen for the first 13 rules.

3.2.4. Rule viewer

This screen displays a roadmap of the whole fuzzy inference process. It is based on the fuzzy inference diagram. The nine plots across the top of Figure 9 represent the antecedent and consequent of the first rule. Each rule is a row of plots, and each column is a variable. The rule numbers are displayed on the left of each row. The first eight columns of plots show the membership functions referenced by the antecedent, or the "if-part" of each rule. The ninth column of plots shows the membership functions referenced by the consequent, or "then-part" of each rule. The 135th plot in the ninth column of plots represents the aggregate weighted decision for the given inference system. This decision

will depend on the input values for the system. The defuzzified output is displayed as a bold vertical line on this plot, and the defuzzified value has been achieved directly through the model, whereas, for the calculating the aggregate weighted decision for the given inference system manually, the common and useful defuzzification technique which is the center of gravity (COG) or center of area (COA) will be used. This method determines the centre of the area of the combined membership functions through the following equation:

$$COA = \frac{\int_{x_{min}}^{x_{max}} f(x)x dx}{\int_{x_{min}}^{x_{max}} f(x) dx} \tag{3}$$

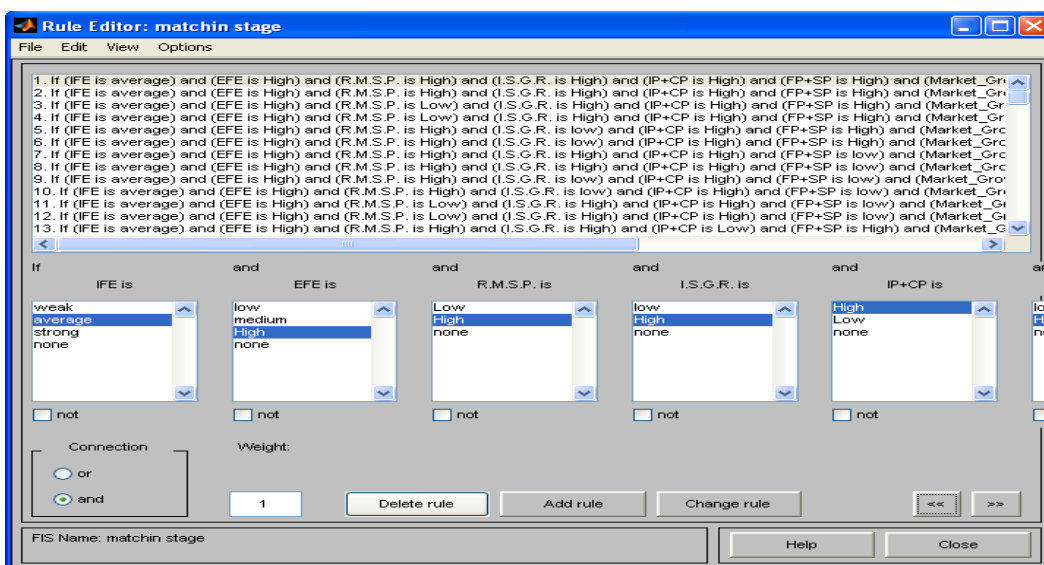


Figure 8. Rule Editor Screen

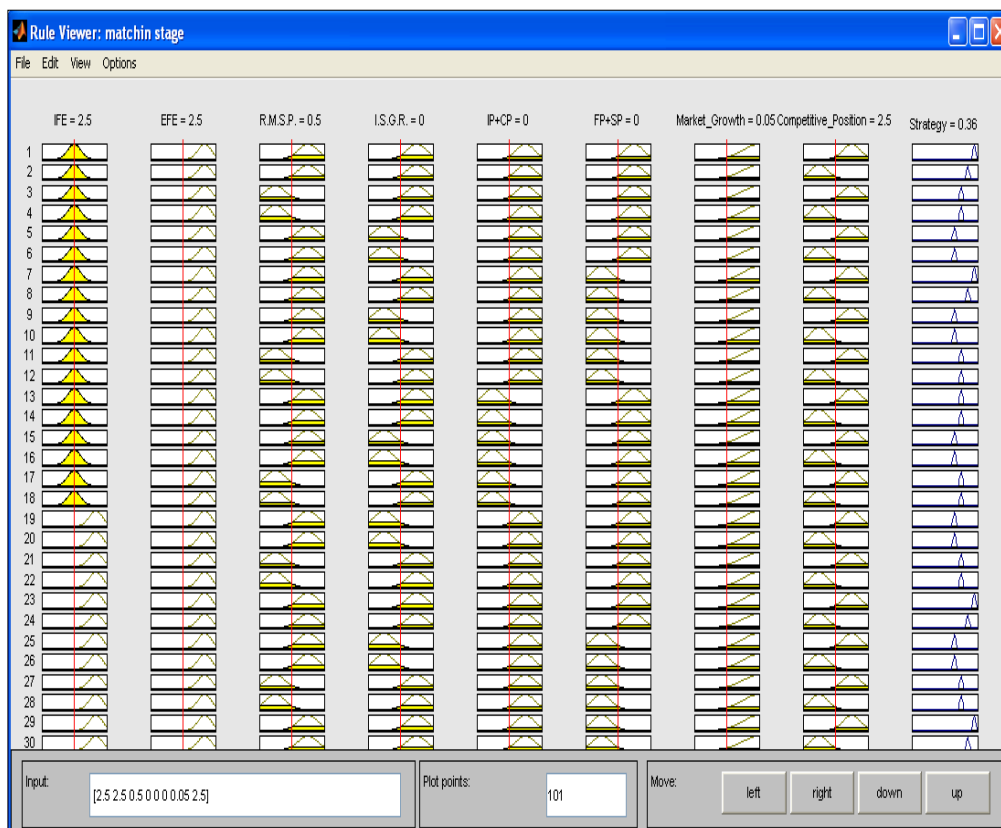


Figure 9. Rule viewer screen

4. Case Study

A case study on a Jordanian company called Alliance Chocolate Manufacturing Company (ACMC) was implemented to validate the model and to generate and evaluate the alternative output strategies that the company should pursue to achieve its long term objectives.

Inputs

The company has been visited and a thorough audit was performed to gather data and information regarding the current internally and externally status of the company. As a result, the variables or inputs for each matrix was obtained and summarized in Table 5 to start generating and evaluating the alternative output strategies using fuzzy logic decision support system model.

Table 5. The eight variables of the four matrices in ACMC Company

Matrix	Input	Value
IE	IFE	2.55
	EFE	2.7
BCG	(R.M.S.P.)	10%
	(L.S.G.R)	0.2
SPACE	(FP+SP)	0.34
	(IP+CP)	1
Grand strategy	Market growth	0.2
	Competitive position	3

Rule viewer

The variables and their current values are displayed on top of the columns of the Rule viewer screen. In the lower left rectangle which marked in circle form in Figure 10, there is a text field Input in which the values from Table 5 is entered.

The Defuzzified alternative output strategy is 0.643 which is shown on the top of the 9th column of the Rule viewer and marked in the circle in Figure 4.2. The value of 0.643 falls in the membership function or in the range of (0.6 to 0.7) which corresponds to the Market Penetration and Product Development alternative output strategies. So, Market Penetration and Product Development strategies are the common and most frequent strategies between the four matrices. If the organization has limited resources then these alternative output strategies are the first option for the organization to pursue.

Market Penetration and Product Development are the achieved alternative output strategies using our FLDSS Model. It depends on the eight fuzzy factors or variables of the four matrices together on the contrary of taking the alternative output strategies in each matrix separately which depends on two fuzzy factors only. By pursuing these two alternative output strategies instead of pursuing all the alternative output strategies that resulted from the four matrices, the cost of implementing the strategic plan will be reduced.

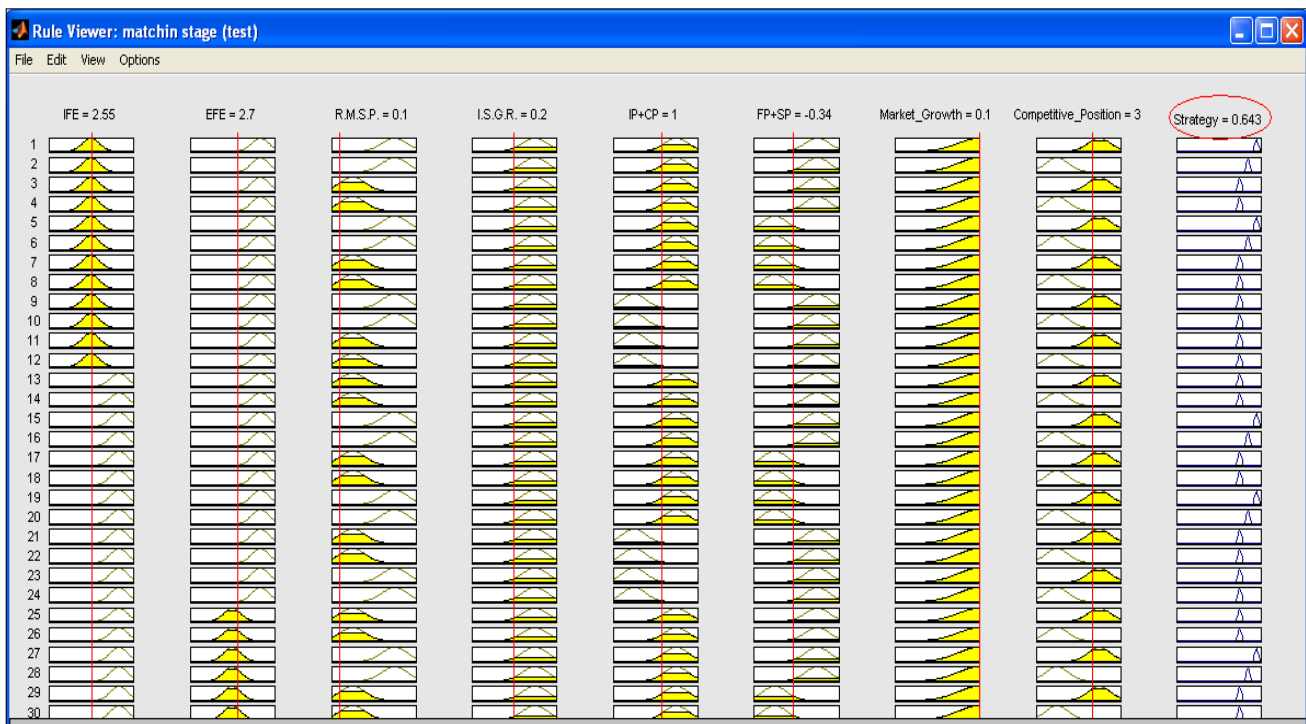


Figure 10. Defuzzified alternative output strategy in the Rule viewer screen

5. Conclusion and Recommendation

5.1. Conclusion

A Fuzzy Logic Decision Support System Model had been developed for generating and evaluating the alternative output strategies which are the most important step in the formulation stage of a strategic planning process. In this model, the four matching stage matrices (SPACE, IE, BCG, and Grand Strategy Matrix) have been combined by relations to select the most frequent alternative output strategy or set of strategies among these

matrices. There are 134 relations or conditions that resulted from the combination. One relation may be occurred when evaluating the eight variables of the four matrices from the internal and external audit of an organization's environment.

The relations among the four matrices had been entered as rules in this model using Fuzzy Logic Toolbox™ software which is a collection of functions built on the MATLAB® technical computing environment, and relies heavily on graphical user interface (GUI) tools. This model is simple and unsophisticated and no previous experience with computers or knowledge of

strategic planning is required for the users. The point of value of work is to map an input space (the inputs of the four matrices) to an output space (the alternative output strategies), and the primary mechanism that is used for doing this is a list of if-then rule statements. These rules are useful because they refer to variables and the adjectives that describe those variables.

A fuzzy logic based approach to model inconsistencies during the model development process is proposed. Fuzzy logic offers a unique opportunity to model methodological rules and handle inconsistencies within the same framework. Linguistic variables allows capturing, as much as possible, the software engineer's perception in a natural way, as well as helping an organization to reach its destination with incomplete information and under uncertain circumstances. It is much easier to express the state of the variables of each matrix of the four matching stage matrices in linguistic terms (High, Low, Strong, Weak ...) rather than using numbers. However, linguistic variables contain ambiguity and multiplicity of meanings and therefore, the information obtained can be expressed as a range of fuzzy sets, instead of a single value in the traditional methods. Also the developed model solved the problem of having values or inputs for the four matrices that fall in the middle or on the centreline between the strategic positions.

The main purpose for combining the four matrices in this model is for strategies who depend on more than two fuzzy factors of the external and internal organization's environment (as required by each matrix alone) to have an accurate strategies selection. On the other hand, an effective strategic plan for an organization that has limited resources can be obtained.

5.2. Limitations and Recommendations

This model is applicable only in profit making companies because of the nature of the eleven alternative output strategies in the four matching stage matrices. The aim of this work is to generate and evaluate these strategies to select the most frequent using Fuzzy Logic Decision Support System. Grand strategy matrix and IE matrix usually take into account the different strategic planning units (SBU) in an organization while SPACE Matrix and Grand strategy matrix don't take it into account. So this model is valid only for organizations that have no SBU's.

More alternative output strategies such as Michael Porter's Five Generic Strategies that are not found in the four matching stage matrices (SPACE, IE, BCG, and Grand Strategy Matrix) can be

inserted in a future improvement of the developed model. One alternative output strategy is recommended to be selected in further researches to help an organization that has limited resources by having a mechanism to select the best alternative output strategy to achieve its long term objectives.

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Power System Contingency Ranking using Fuzzy Logic Based Approach

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Abstract: Voltage stability is a major concern in planning and operations of power systems. It is well known that voltage instability and collapse have led to major system failures. Modern transmission networks are more heavily loaded than ever before to meet the growing demand. One of the major consequences resulted from such a stressed system is voltage collapse or instability. This paper presents fuzzy approach for ranking the contingencies using composite-index based on parallel operated fuzzy inference engine. The Line Flow index (L.F) and bus Voltage Magnitude (VM) of the load buses are expressed in fuzzy set notation. Further, they are evaluated using Fuzzy rules to obtain overall Criticality Index. Contingencies are ranked based on decreasing order of Criticality Index and then provide the comparison of ranking obtained with Fast Voltage Stability Index (FVSI) method.

Keywords: Contingency Ranking, Fuzzy Sets, Line Flow Index, Fuzzy Inference System.

1. Introduction

Voltage stability has been identified as a crucial issue in power system study and one of the causes that lead to cascading power system blackout in many parts of the world. This phenomenon has made this subject a very relevant issue in power system planning and operation. There are many incidents of power system blackouts, due to voltage collapse, as reported in [1-3]. Thus, it is very important to know the maximum permissible loading of a system so that it can be operated with an adequate voltage stability margin to prevent voltage collapse. Due to the fact that many systems have not expanded their transmission and generation capacity in recent years, many utilities are operating closer to their maximum capacity. For a system with smaller margin, more contingencies are considered as severe contingencies, and the system is exposed to more frequent voltage collapses [4]. Many power systems are now experiencing voltage problems more frequently and voltage studies have gained increasing attention from operating and planning points of views. It is vital, then, for the electric utility planners and operators to know the impact of every contingency on the voltage profile. Ranking all possible contingencies based on their impact on the system voltage profile will help the operators in choosing the most suitable remedial actions before the system moves toward voltage collapse. To maintain the system reliability, it is desirable to study the impact of the contingency on the power system, and to categorize them based on their severities.

The change in loading margin to voltage collapse when line outages occur is estimated, a nose curve is computed by continuation to obtain a nominal loading margin. Then linear and quadratic sensitivities of the loading margin to each contingency are computed and used to estimate the resulting change in the

loading margin [5]. A Fuzzy Set theory based algorithm is used to identify the weak buses in a power system. Bus voltage and reactive power loss at that bus are represented by membership functions for voltage stability study [6].

Newton optimal power flow is used to identify the weakest bus / area, which is likely to cause voltage collapse. The complex power – voltage curve is examined through Newton optimal power flow. The indicator, which identifies the weakest bus, was obtained by integrating all the marginal costs via Kuhn-Tucker theorem [7]. A Fast Voltage Stability Index is used to estimate the maximum loadability for identification of weak bus. The indicator is derived from the voltage quadratic equation at the receiving bus in a two bus system. The load of a bus, which is to be ranked is increased till maximum value of FVSI is reached and this load value is used as an indicator for ranking the bus [8].

A weak bus-oriented criterion is used to determine the candidate buses for installing new VAR sources in VAR planning problem. Two indices are used to identify weak buses based on power flow jacobian matrix calculated at the current operating point of the system [9]. A neural network method for the identification of voltage weak buses/areas uses singular value decomposition method. Kohonen neural network is trained to cluster/rank buses in terms of voltage stability [10].

Also the energy function is used for voltage stability assessment of multi-machine power system [11]. The formulated energy function provides an excellent indicator of the system vulnerability to voltage collapse. It is, also, used to rank the system buses according to their contributions to voltage collapse. Also, a multi-layer feed-forward ANN with error back-propagation learning algorithm is proposed for calculation of voltage stability margins (VSM).

This paper is organized as follows; section 2 explains the static voltage stability indicators which provide reliable information about proximity of voltage instability in a power system. Usually, their values change between 0 (no load) and 1 (voltage collapse).

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Section 3 illustrates the Fuzzy Inference System (FIS) which formulating the mapping from selected inputs to outputs through fuzzy decision rules; Section 4 shows the numerical results of applying the algorithm to IEEE-14 and IEEE-30 bus test systems and provides performance comparison by comparing results obtained from fuzzy based algorithm to FVSI method. Finally the conclusion is given in Section 5.

2. Static Voltage Stability Indicators

2.1. Fast Voltage Stability Index

Voltage stability index proposed by [12] can be conducted on a system by evaluating the voltage stability referred to a line. The voltage stability index referred to a line is formulated from the 2-bus representation of a system. The voltage stability index developed is derived by first obtaining the current equation through a line in a 2-bus system. Representation of the system illustrated in Fig. 1.

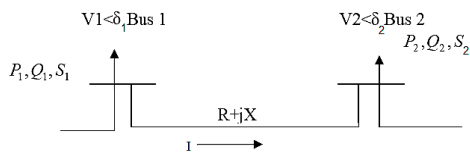


Figure 1. 2- Bus system model

$$FVSI_{ij} = \frac{4 \cdot Z^2 \cdot Q_j}{V_i^2 \cdot X} \quad (1)$$

where,

Z: line impedance

X: line reactance

Qj: reactive power at the receiving end

Vi: sending end voltage

2.2. Line Flow Index

The Line Flow (L.F) index proposed by [13] investigates the stability of each line of the system and they are based on the concept of maximum power transferred through a line as shown in Fig. 2.

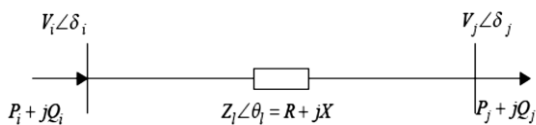


Figure 2. A transmission line of a power system network.

$$L.Findex = \frac{P_R}{P_{R(MAX)}} \quad (2)$$

Where the value of PR is obtained from conventional power flow calculations, and PR(max) is the maximum active that can be transferred through a line see (Equation 3). The Line Flow index varies from 0 (no load condition) to 1 (voltage collapse).

$$P_{R(MAX)} = \left(\frac{V_i^2}{2Z_l} \right) \left(\frac{\cos \phi}{1 + \cos(\theta_l - \phi)} \right) \quad (3)$$

Where Vi is the voltage magnitude of sending bus of branch i-j, Zl and θ_l are the magnitude and angle of branch impedance respectively, $\Phi = \arctan(Q_j / P_j)$

3. Fuzzy Inference System

In this formulation, L.F index values, which are linearly normalized into a [0, 1] range with the largest (L.F) having a value of 1 and the smallest having a value of 0, along with load bus Voltage magnitudes are the inputs to the fuzzy system that determines the severity indices of line flow and voltage profile by fuzzy inferencing. In fuzzy logic based approaches, the decisions are made by forming a series of rules that relate the input variables to the output variables using if-then statements. A set of multiple-antecedent fuzzy rules are established for determining the severity index of voltage profile (SIVP) and severity index for line flow (SIL.F), the input to the rules (L.F) and (VM) and the output consequent is (SIL.F) and (SIVP) respectively. The rules are summarized in the fuzzy decision matrix in table 8. Having related the input variables to the output variable, the fuzzy results are defuzzified through what is called a defuzzification process, to achieve a crisp numerical value. The most commonly used centroid or centre of gravity defuzzification strategy [14,15] is adopted. The fuzzy inference structure is tested in MATLAB R2008a fuzzy toolbox. The ranking obtained using fuzzy approach is verified with (FVSI).

3.1. Bus Voltage Profile (Selected Fuzzy Input)

The voltage profile at load buses is described using the linguistic variables as Low Voltage (LV), Normal Voltage (NV) and Over Voltage (OV) as shown in Fig. 3.

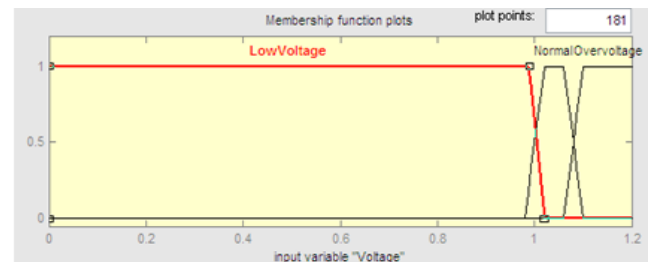


Figure 3. Voltage profiles membership function

3.2. Line Flow Index (Selected Fuzzy Input)

The Line Flow index is divided into five categories using fuzzy set notations: Very Small (VS), Small (S), Medium (M), High (H) and Very High (VH) as shown in Fig. 4. Fig. 5 and Fig. 6 show membership function chosen for linguistic output variables.

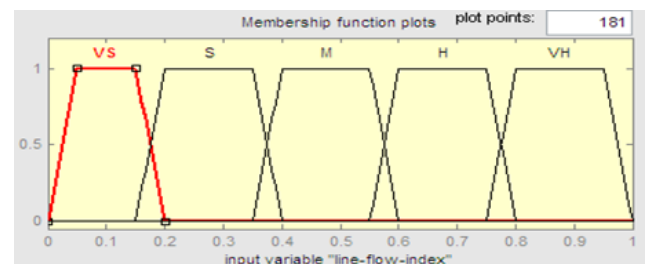


Figure 4. Line flow index membership function

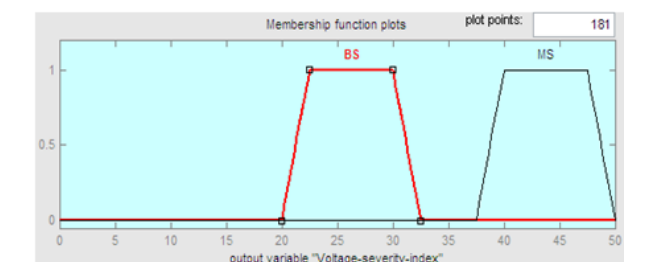


Figure 5. Severity index for Voltage Profile

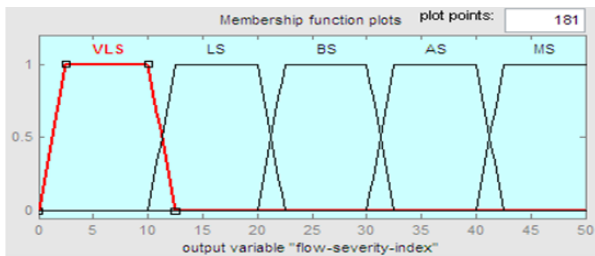


Figure 6. Severity index for line flow

3.3. Fuzzy Rules

The fuzzy rules, which are used for evaluation of severity indices of bus voltage profiles and line flow indices, are given in Table 1.

Table 1. Fuzzy rules

Input Variable			Output Variable						
Voltage			SI _{VP}						
LV	NV	OV	MS	BS	AS	MS			
L.F index			SI _{LF}						
VS	S	M	H	VH	VLS	LS	BS	AS	MS

VLS: Very Low Severe, LS: Low Severe, BS: Below Severe, AS: Above Severe, MS: More Severe.

3.4. Fuzzy Output (Composite Index)

The overall severity index (Composite index) for a particular line outage is given by $CI = \sum SI_{LF} + \sum SI_{VP}$ [16] as shown in Fig. 7; where, $\sum SI_{LF}$ is the severity index of all line flow index and $\sum SI_{VP}$ is severity index of all load bus voltage profiles for selected contingencies. Thus, the overall severity index indicates the actual severity of the system for a contingency.

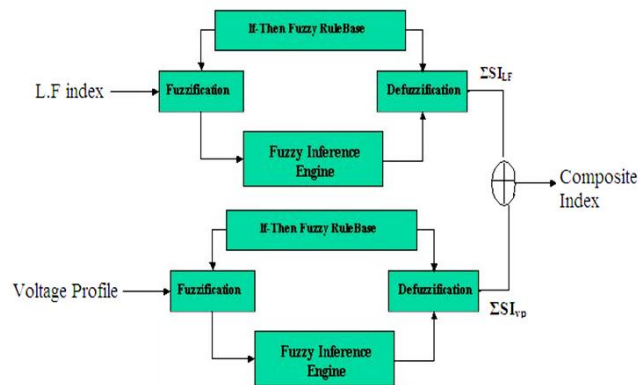


Figure 6. Severity index for line flow

4. Results and Discussion

4.1. IEEE 14 Bus Test System

The fuzzy logic approach is tested on IEEE-14 bus system. The line outages considered for ranking are listed in Table 2.

Table 2. List of selected contingencies

Contingency No.	Type of Contingency	From	to
1	Single Line Outage	10	11
2	Single Line Outage	4	9
3	Single Line Outage	5	6
4	Single Line Outage	12	13
5	Double Line Outage	9	10
		13	14

4.1.1. Contingency No.1 Analysis

Tables 3 and Table 4 show severity index for voltage profiles and line flow index calculated using fuzzy rules

Table 3. Severity indices for voltage profiles

Bus No.	Voltage (p.u)	SI _{VP}
Bus 4	1.0169	28.3
Bus 5	1.0193	27.6
Bus 7	1.0596	26.3
Bus 9	1.0524	26.3
Bus 10	1.0449	26.3
Bus 11	1.0635	28
Bus 12	1.055	26.3
Bus 13	1.0497	26.3
Bus 14	1.0332	26.3

$$\sum SI_{VP} = 241.7$$

Table 4. Severity indices for L.F index

Line	From	to	L.F index	SI _{LF}
1	1	2	0.195553	15.3
2	1	5	0.358289	18
3	2	3	0.320033	16.3
4	2	4	0.235641	16.3
5	2	5	0.185067	13.2
6	4	3	0.097943	6.25
7	5	4	0.059059	6.25
8	4	7	0.083447	6.25
9	4	9	0.168847	10.1
10	5	6	0.246067	16.3
11	6	11	0.027215	6.25
12	6	12	0.068677	6.25
13	6	13	0.08762	6.25
15	7	9	0.072067	6.25
16	9	10	0.031477	6.25
17	9	14	0.08226	6.25
19	12	13	0.020885	6.25
20	13	14	0.081216	6.25

$$\sum SI_{LF} = 174.25$$

$$CI = \sum SI_{LF} + \sum SI_{VP} = 415.95$$

4.1.2. Contingency No.2 Analysis

$$\sum SI_{VP} = 239.5$$

$$\sum SI_{LF} = 171.6$$

$$CI = \sum SI_{LF} + \sum SI_{VP} = 411.1$$

4.1.3. Contingency No.3 Analysis

$$\sum SI_{VP} = 275.6$$

$$\sum SI_{LF} = 183.98$$

$$CI = \sum SI_{LF} + \sum SI_{VP} = 459.58$$

4.1.4. Contingency No.4 Analysis

$$\sum SI_{VP} = 239.3$$

$$\sum SI_{LF} = 171.66$$

$$CI = \sum SI_{LF} + \sum SI_{VP} = 410.96$$

4.1.5. Contingency No.5 analysis

$$\sum SI_{VP} = 239.5$$

$$\sum SI_{LF} = 164.36$$

$$CI = \sum SI_{LF} + \sum SI_{VP} = 403.86$$

4.2. IEEE 30 Bus Test System

The fuzzy logic approach is tested on IEEE-30 bus system. The system consists of 6 generators, 2 shunt capacitors and 41 transmission lines. The line outages considered for ranking are listed in Table 5.

Table 5. List of selected contingencies

Contingency No.	Type of Contingency	From	to
1	Single Line Outage	2	5
2	Single Line Outage	16	17
3	Single Line Outage	5	7
4	Double Line outage	8	28
		6	28
5	Double Line outage	14	15
		18	19

4.2.1. Contingency No.1 Analysis

Table 6 and Table 7 shows severity index for voltage profiles and line flow index calculated using fuzzy rules

Table 6. Severity indices for voltage profiles

Bus No.	Voltage (p.u)	SI _{vp}	Bus No.	Voltage (p.u)	SI _{vp}
Bus3	0.99505	37.9	Bus19	1.0008	35.8
Bus4	0.98118	43.2	Bus20	1.004	34.5
Bus6	0.97038	43.8	Bus21	1.0043	34.4
Bus7	0.92842	43.8	Bus22	1.0049	34.2
Bus9	1.0238	26.3	Bus23	1.0034	34.8
Bus10	1.0171	28.2	Bus24	0.99286	38.7
Bus12	1.0411	26.3	Bus25	0.98311	42.3
Bus14	1.0244	26.3	Bus26	0.96479	43.8
Bus15	1.0178	27.7	Bus27	0.98594	41.1
Bus16	1.0228	26.3	Bus28	0.96758	43.8
Bus17	1.0138	30.1	Bus29	0.96527	43.8
Bus18	1.0051	34.1	Bus30	0.95331	43.8

$$\Sigma SI_{vp} = 865$$

Table 7. Severity indices for L.F index

From	To	L.F index	SI _{LF}	From	To	L.F index	SI _{LF}
1	2	0.212443	16.3	18	19	0.02022	6.25
1	3	0.42102	26.3	20	19	0.01551	6.25
2	4	0.338385	16.3	10	20	0.061961	6.25
3	4	0.109873	6.25	10	17	0.01404	6.25
2	6	0.433222	26.3	10	21	0.054117	6.25
4	6	0.124829	6.25	10	22	0.051897	6.25
7	5	0.323465	16.3	22	21	0.002316	6.25
6	7	0.296271	16.3	15	23	0.060376	6.25
6	8	0.027194	6.25	22	24	0.05018	6.25
6	9	0.062973	6.25	23	24	0.044053	6.25
6	10	0.137157	6.25	25	24	0.003431	6.25
9	10	0.068137	6.25	25	26	0.076142	6.25
4	12	0.269247	16.3	27	25	0.023568	6.25
12	14	0.077079	6.25	28	27	0.169291	10.2
12	15	0.100466	6.25	27	29	0.10137	6.25
12	16	0.07644	6.25	27	30	0.161603	8.68
14	15	0.026142	6.25	29	30	0.06504	6.25
16	17	0.041595	6.25	28	8	0.004531	6.25
15	18	0.059468	6.25	6	28	0.029831	6.25

$$\Sigma SI_{LF} = 334.23$$

$$CI = \Sigma SI_{LF} + \Sigma SI_{vp} = 1199.23$$

4.2.2. Contingency No.2 Analysis

$$\Sigma SI_{vp} = 691.7$$

$$\Sigma SILF = 307.15$$

$$CI = \Sigma SILF + \Sigma SI_{vp} = 998.85$$

4.2.3. Contingency No.3 Analysis

$$\Sigma SI_{vp} = 701.6$$

$$\Sigma SILF = 310.55$$

$$CI = \Sigma SILF + \Sigma SI_{vp} = 1012.15$$

4.2.4. Contingency No.4 Analysis

$$\Sigma SI_{vp} = 793$$

$$\Sigma SILF = 314.15$$

$$CI = \Sigma SILF + \Sigma SI_{vp} = 1107.15$$

4.2.5. Contingency No.5 analysis

$$\Sigma SI_{vp} = 691.6$$

$$\Sigma SILF = 299.75$$

$$CI = \Sigma SILF + \Sigma SI_{vp} = 991.35$$

In order to evaluate the fuzzy logic based algorithm, so results obtained will be compared with FVSI results by calculation of FVSI value for every line in the system using equation (1). Firstly the corresponding line which gives the highest FVSI must be identified. During these contingencies No. (1, 2, 3, 4, 5) at IEEE-14 bus case study, line connected between bus 7 to bus 8 demonstrates the highest FVSI with values 0.1084, 0.1074, 0.1298, 0.1022 and 0.0955 respectively. At IEEE-30 bus case study, line connected between bus 9 to bus 11 demonstrates the highest FVSI with values 0.167, 0.1162, 0.1168, 0.1316 and 0.1139 respectively. Table 8 and Table 9 provide the comparison of ranking obtained from Fuzzy logic based algorithm and FVSI method. The rankings obtained from fuzzy logic method are matched to the results obtained using FVSI method.

Table 8. Comparison of contingency ranking using fuzzy logic and FVSI method at IEEE 14 bus

Contingency No.	CI = $\Sigma SI_{LF} + \Sigma SI_{vp}$	Rank	FVSI	Rank
1	415.95	2	0.1084	2
2	411.1	3	0.1074	3
3	459.58	1	0.1298	1
4	410.96	4	0.1022	4
5	403.86	5	0.0955	5

Table 9. Comparison of contingency ranking using fuzzy logic and FVSI method at IEEE 30 bus

Contingency No.	CI = $\Sigma SI_{LF} + \Sigma SI_{vp}$	Rank	FVSI	Rank
1	1199.23	1	0.167	1
2	998.85	4	0.1162	4
3	1012.15	3	0.1168	3
4	1107.15	2	0.1316	2
5	991.35	5	0.1139	5

5. Conclusions

The contingencies ranked using composite index provides very useful information about the impact of the contingency on the system as a whole and helps in taking necessary control measures to reduce the severity of the contingency. The fuzzy logic based algorithm is efficient, simple and effectively ranks the contingencies. Based on composite index, suitable location for installing FACTS or any other corrective actions such as load shedding can be identified to avoid voltage collapse.

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Particle Swarm Optimization Design of Optical Directional Coupler Based on Power Loss Analysis

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Abstract: In this work, possible design is presented as an optimization problem for an optical directional coupler by using particle swarm optimization (PSO). In this PSO design, identical, parallel, lossless and slab optical waveguides are supposed to be coupled to each other weakly. The power loss and the propagation constant change of TE and TM modes in these coupled optical waveguides are analysed by the modal analysis and PSO. PSO design of an optical directional coupler is an optimization problem consisting of input variables and design parameters within a fitness function (FF). FF is the power loss of TE and TM modes. PSO should minimize the FF and obtain design criteria. The analysis shows that the PSO results are compatible with modal analysis results. The availability of the optical coupler design by PSO has been presented and tested successfully.

Keywords: Optical fiber, Optical communication, Optical directional coupler, Particle swarm optimization

1. Introduction

Optical fiber is the basic element of optical networks. The elements such as optical directional couplers, optical fiber sensors, optical amplifiers, optical filters, optical reflectors and optical detectors are commonly used in optical communication networks. In this work, an optical directional coupler is analysed and optimized considering the optical communication principles.

An optical directional coupler consists of two parallel optical waveguides or two bent optical waveguides or one straight and one bent optical waveguides. Evanescent fields affect each other because the distance between the axis of the optical waveguides is much smaller than the working wavelength. Coupling can occur between different modes propagating in distinct guides and also between different modes propagating in the same guide. In this study, coupling between two distinct parallel optical guides is investigated and optimized with PSO.

The PSO algorithm was introduced in 1995 as an efficient intelligent evolutionary optimization, which can be applied to solve most optimization problems [1]. In engineering, PSO has been successfully applied to various areas [2]. Optic studies with PSO are also common in engineering [3, 4].

The remainder of this paper is organized as follows: In Section 2, the coupling mechanism of an optical directional coupler is analysed by using the Coupled Mode Theory and Perturbation Theory. The interaction within an optical directional coupler, which consists of identical, slab, parallel, weakly guiding and lossless optical waveguides, is analysed for a time dependent term of $\exp(j\omega t)$ [5, 6]. TE and TM modes are determined by using the Maxwell Equations, Helmholtz Equations and dielectric-dielectric boundary conditions [7-10]. The power loss is analysed by

Poynting Theorem in order to use in PSO as FF. In Section 3, applicability of PSO in optical directional coupler is investigated by taking relevant criteria, parameters and constraints. The propagation constant change for each mode is calculated and drawn with the result values of PSO. Satisfactory numerical and graphical results are presented. The study is concluded in section 4 and determined that the PSO results are compatible with the analytical results.

2. Coupling Analysis in Optical Directional Coupler

An optical directional coupler consists of two parallel optical waveguides or two bent optical waveguides or one straight and one bent optical waveguides. Evanescent fields affect each other because the distance between the axis of the optical waveguides is much smaller than the working wavelength. Mutual coupling between optical waveguides is expressed with the help of Coupled Mode Theory and Perturbation Theory.

The propagation constant change β is analysed for a time dependent term of $\exp(j\omega t)$ with coupling coefficients which are independent of the propagation direction as follows:

$$\Delta\beta = j(c_{12}c_{21})^{1/2} \quad (1)$$

where c_{12} and c_{21} are the coupling coefficients of the optical guides affecting each other.

The change occurs in the form of an increase or decrease between the propagation constants of coupled modes. It takes complex or real values depending on whether the coupled waveguides are lossy or lossless. The value is a real value if lossless, a complex value if lossy [5, 6].

2.1. Propagation Constant Change in the Couple Uncladded Optical Waveguides

In Figure 1, coupled, parallel, identical, slab and uncladded optical waveguides are present. The radius of the core is d , and the

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distance between the core axis is U . The refractive index of the core is n_1 and the refractive index of the region surrounding the core is n_2 . The propagation is available with the z-direction in the waveguides extending to infinity with the y-direction.

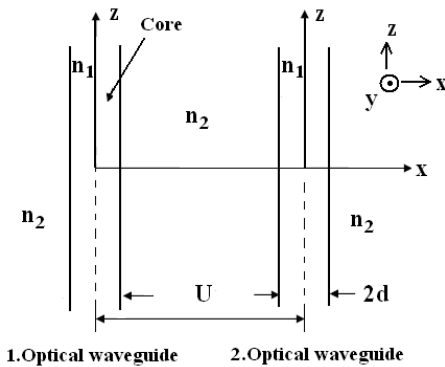


Figure 1. The coupling between two parallel, identical, slab and uncladded optical waveguides

TE and TM modes are examined as a result of solving the Maxwell Equations, Helmholtz Equations and boundary conditions through optical waveguides [5-14]. TE even and odd guided field definitions in the core and the surrounding region respectively are given by the following equation:

$$E_y = \begin{cases} A \begin{cases} \cos(\kappa x) \\ \sin(\kappa x) \end{cases} & , 0 \leq x \leq d \\ B \exp(-\gamma(|x| - d)) & , d \leq x \leq \infty \end{cases} \quad (2)$$

Where " κ " is the eigenvalue of core region and γ is the eigenvalue of the region surrounding the cores. The power loss by using Poynting theorem is given by:

$$2\alpha = \frac{2 \operatorname{Im}(\gamma)}{\beta(1 + \gamma d)} \begin{cases} \cos^2(\kappa d) \\ \sin^2(\kappa d) \end{cases} \exp(-2\gamma(|x| - d)) \quad (3)$$

The propagation constant change within two couple identical, parallel, slab and uncladded optical waveguides having the same β propagation constant is expressed as the following:

$$|\Delta\beta| = \begin{cases} \left\{ \frac{k_0^2}{4\gamma} (n_1^2 - n_2^2) \left[\frac{\gamma^2}{\beta^2 (1 + \gamma d)^2} \right]^{1/2} \right\} \\ \begin{cases} \cos^2(\kappa d) \\ \sin^2(\kappa d) \end{cases} \exp^2(\gamma d) \exp(-\gamma U) \end{cases} \quad (4)$$

TM even and odd guided field definitions in uncladded optical waveguides in the core and the surrounding region respectively are as follows:

$$H_y = \begin{cases} A \begin{cases} \cos(\kappa x) \\ \sin(\kappa x) \end{cases} & , 0 \leq x \leq d \\ B \exp(-\gamma(|x| - d)) & , d \leq x \leq \infty \end{cases} \quad (5)$$

The power loss by using Poynting theorem is given by:

$$2\alpha = \begin{cases} \left\{ 2 \operatorname{Im}(\gamma) \frac{1}{\beta \left[d + \frac{n_1^2 n_2^2}{\gamma} \frac{\kappa^2 + \gamma^2}{n_2^4 \kappa^2 + n_1^4 \gamma^2} \right]} \right\} \\ \begin{cases} \cos^2(\kappa d) \\ \sin^2(\kappa d) \end{cases} \exp(-2\gamma(|x| - d)) \end{cases} \quad (6)$$

The propagation constant change within two couple identical, parallel, slab and uncladded optical waveguides having the same β propagation constant is expressed as follows:

$$|\Delta\beta| = \begin{cases} \left\{ \frac{\omega^2 \epsilon_0^2 n_1^2 (n_1^2 - n_2^2)}{4\gamma} \frac{1}{\beta \left(d + \frac{n_1^2 n_2^2}{\gamma} \frac{\kappa^2 + \gamma^2}{n_2^4 \kappa^2 + n_1^4 \gamma^2} \right)} \right\} \\ \begin{cases} \cos^2(\kappa d) \\ \sin^2(\kappa d) \end{cases} \exp^2(\gamma d) \exp(-\gamma U) \end{cases} \quad (7)$$

In this PSO design, the modes corresponding to azimuthal mode number $v=l$ are investigated.

V_c is the normalized frequency and the relation is given by:

$$(\kappa d)_c \cong V_c = v \frac{\pi}{2} \quad (8)$$

$v = 0, 1, 2, 3 \dots$

2.2. Propagation Constant Change in the Couple Cladded Optical Waveguides

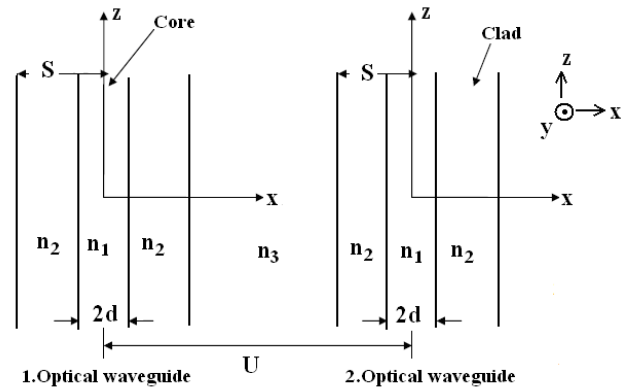


Figure 2. The coupling between two parallel, identical, slab and cladded optical waveguides

In Figure 2, coupled parallel, identical, slab and cladded optical waveguides are present. The radius of the core is d , and the radius with cladding is S . The distance between the core axis is U . The refractive index of the core is n_1 and the refractive index of the region surrounding the cladding is n_3 . The refractive index of the region surrounding the cladding is n_2 . The propagation is available with the z-direction in the waveguides extending to infinity with the y-direction [1-10].

The electric field in the core, cladding and surrounding region is expressed as the following:

$$E_y = \begin{cases} A \begin{cases} \cos(\kappa x) \\ \sin(\kappa x) \end{cases} & , 0 \leq x \leq d \\ B \exp(\gamma x) + C \exp(-\gamma x), & d \leq x \leq S \\ F \exp(-\rho x) & , S \leq x \leq \infty \end{cases} \quad (9)$$

where

$$\rho = (\beta^2 - n_3^2 k_0^2)^{1/2} \quad (10)$$

is the eigenvalue of the region surrounding the cladding.

The propagation constant change within two couple identical cladded optical waveguides having the same β propagation constant is expressed as:

$$|\Delta\beta| = \left\{ \frac{4\kappa^2 \gamma^3 \rho}{\beta(1 + \gamma d)(\gamma + \rho)^2 (\kappa^2 + \gamma^2)^{1/2}} \right\} \exp[-2\gamma(S - d)] \exp[-\rho(U - 2S)] \quad (11)$$

The power loss by using Poynting theorem is defined as the following equation:

$$2\alpha = \frac{8\kappa^2 \gamma^3 \text{Im}(\rho)}{\beta(1 + \gamma d)(\kappa^2 + \gamma^2) |\gamma + \rho|^2} \exp(-2\gamma(S - d)) \quad (12)$$

The magnetic field in the core, cladding and surrounding region as follows:

$$H_y = \begin{cases} A \begin{cases} \cos(\kappa x) \\ \sin(\kappa x) \end{cases}, & 0 \leq x \leq d \\ B \exp(\gamma x) + C \exp(-\gamma x), & d \leq x \leq S \\ F \exp(-\rho x) & , S \leq x \leq \infty \end{cases} \quad (13)$$

The propagation constant change within two couple identical cladded optical waveguides having the same β propagation constant is expressed by:

$$|\Delta\beta| = \frac{\left\{ \frac{n_1^2 n_2^2 \kappa^2 \gamma^2 [2 \exp((\gamma - \rho)(S - d)) - 1]}{\exp(-2\gamma(S - d)) \exp(-\rho(U - 2S))} \right\}}{\beta [(n_1^4 \kappa^2 + n_1^4 \gamma^2) \gamma d + n_1^2 n_2^2 (\kappa^2 + \gamma^2)]} \quad (14)$$

The power loss by using Poynting theorem is defined as the following:

$$2\alpha = \left\{ \frac{8n_1^2 n_2^4 |n_3|^4 \kappa^2 \gamma^3 \text{Im}(\frac{\rho}{n_3})}{\beta [(n_1^4 \kappa^2 + n_1^4 \gamma^2) \gamma d + n_1^2 n_2^2 (\kappa^2 + \gamma^2)] |n_2^2 \rho + n_3^2 \gamma|^2} \right\} \exp(-2\gamma(S - d)) \quad (15)$$

3. PSO Design of Optical Directional Coupler

3.1. PSO

In Particle Swarm Optimization (PSO), solutions are called particles in the search space. All particles have fitness values evaluated by the fitness function and velocity information that directs their movements. Particles follow the existent optimum particles in the problem space.

PSO is initiated with a random particle swarm and is updated to search for the best value. Each particle is updated according to the best value obtained in each of the iterations. One of the updates is related to the best fit value of the particle titled pbest (personal best). This value is kept in the memory to be used later. The second best value is the global best value (gbest) obtained by any particle in the swarm. It is the best global value in the swarm.

The swarm matrix where D is the swarm dimension and n is the particle number is given below:

$$x = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1D} \\ x_{21} & x_{22} & \dots & x_{2D} \\ x_{31} & x_{32} & \dots & x_{3D} \\ \dots & \dots & \dots & \dots \\ x_{n1} & x_{n2} & \dots & x_{nD} \end{bmatrix}_{n \times D}$$

According to swarm matrix; i^{th} particle;

$$x_i = [x_{i1}, x_{i2}, x_{i3}, \dots, x_{iD}]$$

And the personal best fit value obtained until that point pbest:

$$pbest_i = [p_{i1}, p_{i2}, p_{i3}, \dots, p_{iD}]$$

And the global best in the population is:

$$gbest = [p_1, p_2, p_3, \dots, p_D]$$

Velocity vector that presents the amount of transformation for i^{th} particle in each position is displayed as follows:

$$v_i = [v_{i1}, v_{i2}, v_{i3}, \dots, v_{iD}]$$

The velocity and position of the particle are updated according to the equations below respectively:

$$v_i^{k+1} = v_i^k + c_1 \cdot rand_1^k \cdot (pbest_i^k - x_i^k) + c_2 \cdot rand_2^k \cdot (gbest^k - x_i^k)$$

$$x_i^{k+1} = x_i^k + v_i^{k+1}$$

Where k is the number of iterations and i is the number for the particle. When the particle swarm matrix is composed of n lines, there is an i^{th} line as well. The learning factors c_1 and c_2 values pull the particle towards pbest and gbest values. c_1 and c_2 are generally selected as equals and in $[0,4]$ range. c_1 allows the particle to move according to its own experiences whereas c_2 allows it to move according to the experiences of the other particles in the swarm.

3.2. Optical Directional Coupler Design

In this study, the optical directional coupler is chosen to investigate the usage of PSO in optical communication systems.

Figure 3 shows a basic illustration for the PSO design of optical directional coupler with uncladded optical waveguides. The input variables are mode type, frequency of the mode, n_2 and the value range of n_1 and d . The output variable is the exact value of n_1 and d .

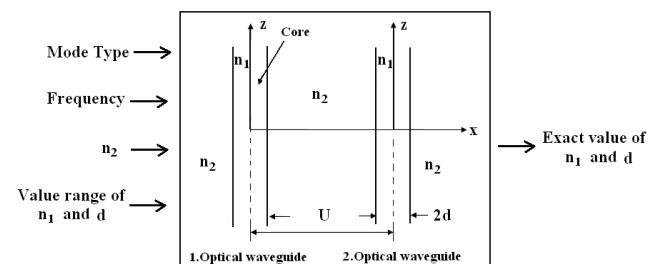


Figure 3. Optical Directional Coupler with uncladded fibers

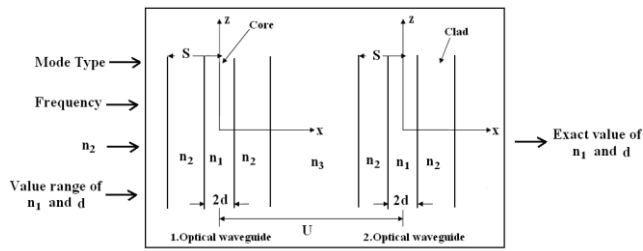


Figure 4. Optical Directional Coupler with cladded fibers

Figure 4 shows a basic illustration for the PSO design of optical directional coupler with cladded optical waveguides. The refractive index of the region surrounding the cladding is air and $n_3=1$. The input variables are mode type, frequency of the mode, n_2 and the value range of n_1 and d . The output variable is the exact value of n_1 and d .

The optimal design is aimed at taking relevant criteria, parameters and constraints. PSO should minimize the FF and obtain design criteria and constraints. In this study, FF is the power loss of TE and TM modes obtained by using the Poynting theorem.

In the beginning of the algorithm a certain constraint is adjusted because of the azimuthal mode number for both design criteria and design parameters by the designer. Initial population matrix size is 10×2 . The row number indicates the number of particles in the population and the column denotes n_1 and d parameters. The learning factors c_1, c_2 and w are 1.4, 1.4 and 0.9 respectively while the algorithm runs for 100 iterations. Propagation constant change is calculated and drawn with the result values of PSO.

3.3 Numerical and Graphical Results

A directional coupler is designed consisting of uncladded and cladded fibers guided with TE, TM even and odd modes. Table 1 shows that design criteria of directional coupler worked with uncladded waveguides.

Table 1. PSO design criteria of an optical directional coupler with uncladded waveguides.

Mode type	Frequency (THz)	n_1	n_2	The value range of d for single-mode ($\times 10^{-7}$)
TE even	200	1.5	1.49	1.77 - 3.54
TE odd	352	1.5	1.46	1.90 - 2.25
TM even	190	1.77	1.45	1.56 - 2.33
TM odd	230	1.48	1.33	1.57 - 1.96

As mentioned before, PSO starts with a random particle swarm. PSO algorithm runs for TE, TM even and odd modes considering design constraints given in Table 1. Optimal results which satisfy min loss power equality are given in Table 2.

Table 2. PSO results of n_1 and d

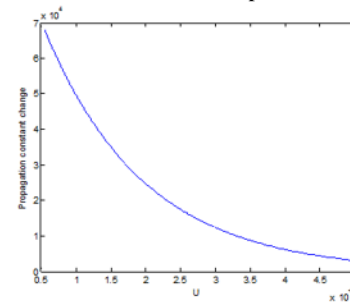
Mode type	$n_1 (>n_2)$	$d (\times 10^{-7})$
TE even	1.53 (>1.49)	2.57
TE odd	1.74 (>1.46)	2.18
TM even	1.69 (>1.45)	1.90
TM odd	1.66 (>1.33)	1.70

For comparison, using PSO results of output n_1 and d values, optical directional couplers with uncladded fibers are designed and drawn (Figure 5, Figure 6).

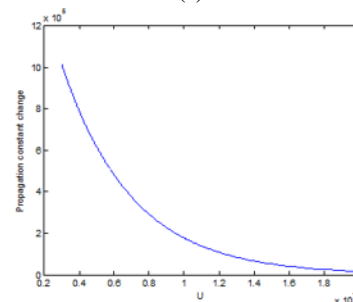
In conclusion, the coupling between TE modes is more efficient than the coupling between TM modes. In addition, propagation constant change decreases with increasing distance between two axis of the fiber core.

After a directional coupler consisting of uncladded fibers, a directional coupler with cladded fibers is taken. Table 3 shows the

PSO design criteria of a directional coupler with TE, TM modes.

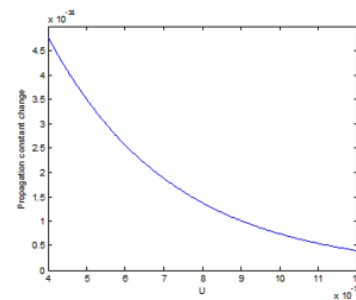


(a)

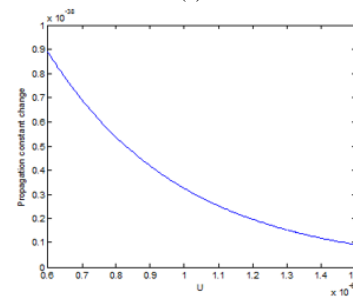


(b)

Figure 5. TE even and odd modes guided in uncladded coupled optical waveguides



(a)



(b)

Figure 6. TM even and odd modes guided in uncladded coupled optical waveguides

Table 3. PSO design criteria of an optical directional coupler with cladded waveguides.

Mode type	Frequency (THz)	n_1	n_2	The value range of d for single-mode ($\times 10^{-7}$)
TE even	220	1.49	1.33	1.58 - 2.71
TE odd	180	1.77	1.46	1.55 - 2.43
TM even	200	1.5	1.49	1.77 - 3.54
TM odd	195	2.42	1.48	1.22 - 2.78

Optimum results which satisfy min loss power equality are shown in Table 4.

For comparison, using PSO results of output n_1 and d values, optical directional couplers with cladded fibers are designed and drawn (Figure 7, Figure 8).

Table 4. PSO results of n_1 and d

Mode type	$n_1 (> n_2)$	$d (x10^{-7})$
TE even	1.44 (>1.33)	2.11
TE odd	1.90 (>1.46)	2.05
TM even	1.52 (>1.49)	2.66
TM odd	1.55 (>1.48)	1.58

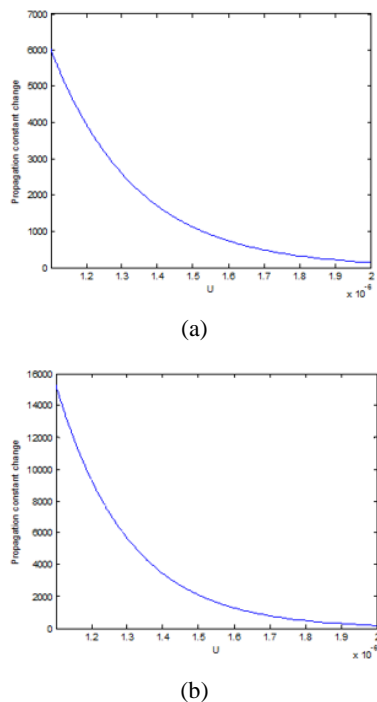


Figure 7. TE even and odd modes guided in cladded coupled optical waveguides

As seen in the graphics, the coupling between the uncladded optical fibers is more effective than the coupling between the cladded optical fibers. Also, the coupling between TE modes is more efficient than the coupling between TM modes. Moreover, propagation constant change decreases with increasing distance between two axis of the fiber core.

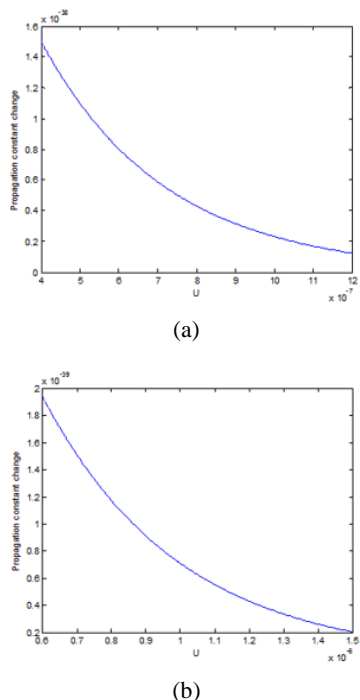


Figure 8. TM even and odd modes guided in cladded coupled optical waveguides

4. Conclusions

As a result of the modal analysis and PSO, the coupling between TE modes is more efficient than the coupling between TM modes. In addition, propagation constant change decreases with increasing distance between two axis of the fiber core. Moreover, the coupling between the uncladded optical fibers is more effective than the coupling between the cladded optical fibers. Analytical results are in agreement with the results of the PSO. PSO is a fast and good alternative method for designing a complex optical directional coupler.

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AIR: An Agent for Robust Image Matching and Retrieval

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Abstract: This paper presents a novel scheme coined AIR (Agent for Image Recognition), acting as an agent, to oversee the image matching and retrieval processes. Firstly, neighboring keypoints within close spatial proximity are examined and used to hypothesize true keypoint matches. While this approach is robust to noise (e.g. a tree) since spatial relation is considered, missing (undetected) keypoints in one image can also be recovered resulting in more keypoint matches. Secondly, the agent is able to recognize instability of projective transformations in certain cases (e.g. non-planar scenes). The geometric approach is substituted with LIS (Longest Increasing Subsequence) approach which does not require any complex geometric transformations. The effectiveness of AIR is substantiated by an image retrieval experiment which demonstrates that it achieves a twofold increase in true matches and higher matching accuracy when compared to RANSAC homography approach.

Keywords: Image recognition, Image matching, Image retrieval, Spatial relation approach, longest increasing subsequence

1. Introduction

The brute force method of comparing every pixel in two images is computationally prohibitive. Intuitively, one can relate the two images by matching only regions in the images that are in some way interesting. They are local as they are related to small regions on objects instead of the whole object itself. This property makes them distinctive as well as robust to occlusion and clutter. These regions are referred to as local features, and sometimes known as interest points or keypoints. Today, the use of keypoints to find correspondences across multiple images is a key step in many image processing and computer vision applications. Some of the most notable examples are panorama stitching [1-3], wide baseline matching [4-7], image retrieval [8, 9], object recognition [10-12], and object class recognition [13-15]. Differences between the images can be a substantial range of affine distortion, noise level, change in illumination, scaling, rotation, and viewpoint. The keypoints should be invariant to these differences in order to robustly match two images of the same object or scene. A good keypoint should be highly distinctive in the way that a single keypoint can be correctly matched with high probability against a large database of keypoints from many images. Nevertheless, the more invariant it is, the less distinctive it will be, which the trade-off between invariance and distinctiveness is. Typically, there are one or more follow-up verification steps to verify the keypoint matches. Without prior knowledge what types of images we are receiving, we often assume that the scenes are composed entirely of planes; and that all planes can be detected whereby planar homographies can be derived. When this assumption is invalid, the matching fails.

In this paper, we propose an intelligent scheme coined AIR (Agent for Image Recognition), acting as an agent, to hypothesize true keypoint matches, or in fact overseeing the keypoint

matching process. Neighboring keypoints within close spatial proximity are examined and used to hypothesize true keypoint matches. The fundamental idea behind this approach is that if two keypoints are true corresponding keypoints in the two images, at least some of their neighboring keypoints should be corresponded. By building a relationship between each keypoint and its neighboring keypoints, our approach can robustly deal with two common problems.

- 1) Asymmetric numbers of keypoints detected in the two images, since a keypoint detected in one image may not appear in the other image and therefore results in a lesser number of keypoint matches. These missing (undetected) keypoints can never be recovered.
- 2) False corresponding keypoints found in the two images after projectivity due to noise, e.g., a tree in one of the images will comprise a massive number of keypoints which can be easily mismatched after projectivity.

AIR is also able to recognize instability of the approach in some cases (e.g. non-planar scenes) after projectivity from the low number of keypoint matches. It substitutes with LIS (Longest Increasing Subsequence) approach which allows less rigid correspondence between the matched image pairs. This approach finds a subsequence (of keypoints in the first image) of a sorted sequence (of corresponding keypoints in the second image), in which the subsequence elements are in sorted order and is as long as possible. The subsequence is not necessarily contiguous, or unique. The concept is that an image pair is geometrically consistent if the geometric order of their corresponding keypoints is consistent. The rest of the paper is organized as follows. Section 2 discusses related work. In Section 3, our image recognition methodology is presented whereby AIR is described. Section 4 provides the experimental results. Section 5 concludes the paper.

2. Related Work

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Mikolajczyk and Schmid [16] evaluated a variety of object recognition algorithms and identified that the SIFT [12] (Scale-Invariant Feature Transform) and SIFT-based algorithms such as SURF [17] (Speeded Up Robust Features) are the most resistant to common image deformations and have achieved the best performance. SIFT-based features are invariant to image scale, translation, rotation, and partially invariant to illumination and viewpoint changes. Details on application of these features can be found in [3, 18, 19]. In its original matching scheme, a pair of keypoints is considered a match if the distance ratio between the closest match and the second closest match is below a certain threshold. While the distance ratio can eliminate some of the false keypoint matches, we often still need to identify correct subsets of keypoints containing less than 1% inliers.

To solve the outlier problem, the RANSAC [20] (Random Sample Consensus) algorithm and other similar hypothesize-and-verify methods have been proposed in the literature. The RANSAC algorithm is a robust method based on random sampling and rejects all keypoint matches not conforming to the found homography model [21] or epipolar geometry [21]. Although this method works fine in many applications, they perform poorly when the number of false keypoint matches outnumbers the number of true keypoint matches; or when the number of keypoint matches is modest (limited). The RANSAC idea was modified by Nister and Stewenius [22] to include competitive verification of models. The algorithm named "Preemptive RANSAC" was demonstrated to perform well in a real-time structure-from-motion system. The limitation is that only a fixed number of models are evaluated, which is equivalent to a priori assumption that a lower bound on the fraction of inliers is known. This limits the applicability of preemptive RANSAC in wide baseline stereo where the fraction of inliers varies widely.

Another popular method is the Hough Transform [23-25], which clusters keypoints in pose space. The Hough Transform identifies clusters of keypoints with a consistent interpretation by using each keypoint to vote for all object poses that are consistent with the keypoint. However, in [12], it was shown that follow-ups are required after Hough Transform is performed in order to eliminate more false keypoint matches, e.g., least-squares pose determination, followed by a probabilistic model given in [18]. Moreover, Hough Transform requires a huge computation load in pixel transformation and a large storage (or memory) is also required for the voted Hough space. Without proper parallelization, it will be very difficult for Hough Transform to achieve real-time performance.

3. Image Recognition Methodology

This section describes our image recognition technique used to identify objects in different images. Keypoints between two images of the same scene or object must be robustly detected, described, matched and verified. We exploit Fast-Hessian detector and SURF descriptor proposed by Bay et al. [17] due to its speed and accuracy. We find the best match between a query image and the database images by Euclidean distance, using the k-d data structure and search algorithm [25]. The algorithm generalizes classical binary trees to higher dimensional spaces so that one can locate nearest neighbors to a descriptor vector in $O(\log N)$ time instead of the brute-force $O(N)$ time, with N being the size of the images in the image database.

The Agent (AIR) as illustrated in Fig. 1 inspects the keypoint matches based on spatial relations. Once the keypoints passed the inspection, the agent examines the reliability of the matched image. If the match is not satisfactory (unreliable), it will automatically

switch to the LIS approach. The two tasks will be discussed in details in the following sections.

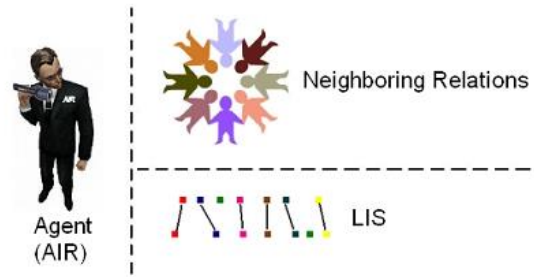


Figure 1. Agent (AIR) is responsible for two main tasks

3.1. Spatial Relations

For each image, we save each detected keypoint and its nearest 12 neighboring points. Let us consider a 3D coordinate frame and two planar surfaces of the same scene but with different camera angles. Let's call the two planar surfaces R_1 and R_2 as shown in Fig. 2. R_2 is defined by the point \vec{b}_0 and two linearly independent vectors \vec{b}_1 and \vec{b}_2 contained in the region. Let us consider a keypoint \vec{P}_2 in R_2 . Since the vectors \vec{b}_1 and \vec{b}_2 form a basis in R_2 , we can express \vec{P}_2 as

$$q_1\vec{b}_1 + q_2\vec{b}_2 + \vec{b}_0 = (\vec{b}_1, \vec{b}_2, \vec{b}_0) \begin{pmatrix} q_1 \\ q_2 \\ 1 \end{pmatrix} = B\vec{q}, \quad (1)$$

where $B = (\vec{b}_1, \vec{b}_2, \vec{b}_0) \in \mathbb{R}^{3 \times 3}$ defines the planar surface R_2 , and $\vec{q} = (q_1, q_2, 1)^T$ defines the 2D coordinates of \vec{P}_2 with respect to the basis (\vec{b}_1, \vec{b}_2) . We can compute a similar identity for planar surface R_1 as

$$\vec{P}_1 = A\vec{s}, \quad (2)$$

where $A = (\vec{a}_1, \vec{a}_2, \vec{a}_0) \in \mathbb{R}^{3 \times 3}$ defines R_1 , and $\vec{s} = (s_1, s_2, 1)^T$ defines the 2D coordinates of \vec{P}_1 with respect to the basis (\vec{a}_1, \vec{a}_2) . We impose the constraint that point \vec{P}_1 maps to point \vec{P}_2 under perspective projection centered at the origin:

$$\vec{P}_1 = \alpha(\vec{q})\vec{P}_2, \quad (3)$$

where $\alpha(\vec{q})$ is a scalar that depends on \vec{P}_2 , and consequently on \vec{q} . By combining the equation above with the constraint that \vec{P}_1 and \vec{P}_2 must be situated in its corresponding planar region, we obtain the relationship between the 2D coordinates of these points:

$$\vec{s} = \alpha(\vec{q})A^{-1}B\vec{q}, \quad (4)$$

where the role of $\alpha(\vec{q})$ is to simply scale the term $\alpha(\vec{q})A^{-1}B\vec{q}$ such that its third coordinate is 1. We can represent $\alpha(\vec{q})A^{-1}B$ as a homography matrix H_m and compute the above equation as

$$\vec{s} = H_m\vec{q}, \quad (5)$$

If R_1 and R_2 are true corresponding planars, the keypoint \vec{P}_2 and its 6 nearest neighboring points (shaded in different colors in Fig. 2) in R_2 should fit the homography matrix H_m to correctly locate the 7 corresponding keypoints in R_1 . There are more than 6 nearest neighbors (total of 12) stored in a descriptor vector although only 6 are used.

This is to solve the asymmetric problem where a point can be detected in one quadrilateral region but not in the other, and thus the nearest 6 neighbors may be slightly different in this case. E.g., the 6th nearest neighbor for \vec{P}_2 in R_2 may be the 7th nearest neighbor for \vec{P}_1 in R_1 . Therefore, we stored slightly more than 6 nearest neighbors to overcome this problem.

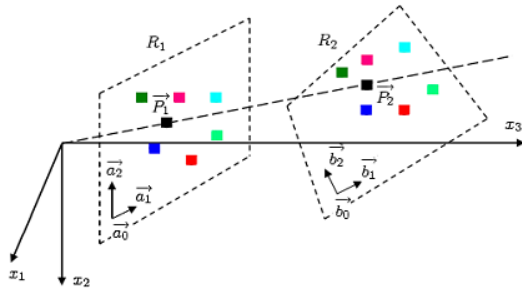


Figure 2. Point projectivity between two planar surfaces R_1 and R_2 of the same scene but different camera angles

Observations after various experiments summarized the point-to-point homography matching results into different cases. Given two planar surfaces (may be true or false corresponding pair) R_3 and R_4 , a keypoint (let's call it \vec{P}_3), its 6 nearest neighboring points $\{\vec{P}_{3a}, \vec{P}_{3b}, \dots, \vec{P}_{3f}\}$, and their corresponding points in R_4 (may be true or false corresponding points) denoted as \vec{P}_4 , $\{\vec{P}_{4a}, \vec{P}_{4b}, \dots, \vec{P}_{4f}\}$, the 7 frequent cases after projectivity of the 7 points by H_m from R_3 to R_4 can be categorized as:

- \vec{P}_4 , All 7 points $\vec{P}_{4a}, \vec{P}_{4a}$ to \vec{P}_{4f} found.
- \vec{P}_4 , \vec{P}_4 found, and most neighboring points (≥ 4) found.
- \vec{P}_4 , \vec{P}_4 not found, but most neighboring points (≥ 5) found.
- \vec{P}_4 , \vec{P}_4 found, and few neighboring points (< 4) found.
- \vec{P}_4 , \vec{P}_4 not found, and few neighboring points (< 5) found.
- \vec{P}_4 , \vec{P}_4 found, but no neighboring points (< 1) found.
- All 7 points \vec{P}_4, \vec{P}_{4a} to \vec{P}_{4f} not found.

We determine from experiments that candidate points falling within cases 1-3 give very high accuracy of correct matches. As a result, we only consider a candidate point as valid if it falls within these three cases. This approach is very robust since it does not evaluate the pair of candidate points in the two planar surfaces individually, but as a whole with the neighbors in order to adjudicate if the pair is a correct match. Consequently, keypoints coming from other objects or structures, e.g., the dense keypoints detected on a tree, will have very low chances of being matched. We are also the first to recover keypoints that are not detected in one image but in the other. This can be easily understood from case 3 where \vec{P}_4 is not found but most of the neighboring points (at least 5) are found. We can recover such missing (undetected) keypoint in an image if the point exists in the other image and most of their neighboring points are corresponded. This solves the common problem in keypoint detection where the numbers of detected keypoints in two images are asymmetric. Given that the numbers of keypoints are asymmetric in the pair of planar surfaces, in this case R_3 and R_4 , we apply an inverse homography matrix H_m^{-1} to the unmatched keypoints in R_4 to locate any corresponding keypoints in R_3 in the same manner as before. Eventually, the best pair of corresponding planar surfaces in the two images will obtain the largest list of corresponding keypoints.

However, homography based method only works well on images comprising planar rectangular structures or quadrilateral regions as described above. When it comes to non-planar scenes, the approach is unstable. AIR recognizes this failure by examining the number of keypoint matches, e.g. low number of matches indicates unreliability of matched image. Alternate approach (LIS) is substituted, which will be discussed next.

3.2. Longest Increasing Subsequence (LIS)

In this method, we find true keypoint matches by imposing a geometrical constraint on the matches. This constraint applies to a set of matched keypoint-pairs. Our assumption is that true keypoint matches are the elements of the largest subset of corresponding

keypoints which are consistent. The set of matched keypoint-pairs are consistent if the keypoints order is the same in both images. We can find the largest subset by calculating the longest increasing subsequence [26]. The pseudo-code for the algorithm is provided in Algorithm 1. First we sort the keypoints according to their x coordinate in the first image. Then we create a sequence from the x coordinates of the keypoints in the second image (keeping the order after sorting). The longest increasing subsequence of this sequence will give us the indexes of the keypoints we want. They will be geometrically consistent and they will be the largest subset of such. Fig. 3 illustrates this.

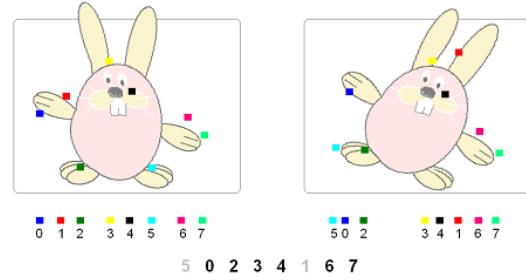


Figure 3. An illustration of Longest Increasing Subsequence (LIS) on two images with detected keypoints as shown. The LIS output in this case is 0, 2, 3, 4, 6, 7

This geometrical constraint is invariant to translation and scaling. Those transformations do not have effect on the geometrical orders of the keypoints. It also allows a little elasticity. However, it is not invariant to rotation, affine transformation, and homographic transformations. Those can change the relative order of the keypoints. We can address this problem by calculating the LIS on rotated images. We only have to apply the rotation on the keypoint coordinates, so we do not rotate the whole image and extract the keypoints again. Assuming we have found the “right” angles of rotation, we can prove that LIS is invariant to affine transformations.

Algorithm 1 Find LIS in Array A

```

n := A.length
m-idx := newArray(n)
m-val := newArray(n)
previous := newArray(n)
maxLength := 0
for i from 0 to n-1: do
    //Binary search in the m-val array
    idx := lowerJbound(m-val[0 : max-length], A[i])
    mJdx[idx] := i
    mval[idx] := A[i]
    if idx > 0 then
        previous[i] := mJdx[idx - 1] end if
    if idx = max-length then
        maxLength := max-length + 1
    end if
end for
LIS := newArray(max-length) idx := mJdx[max-length - 1]
for i from max-length-i to 0 do
    LIS[i] := idx
    idx := previous[idx]
end for
return LIS

```

The proof of affine invariance is as follows: Let Affine : $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ be an affine transformation. A fine $(\vec{p}) = A\vec{p} + \vec{t}$ where $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is the affine transformation matrix, and

$\vec{t} = \begin{pmatrix} e \\ f \end{pmatrix}$ is the translation vector. Let $\text{Rotate} : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a rotation. $\text{Rotate}(\vec{p}) = R\vec{p}$ where

$$R = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

and θ is the angle of the rotation. If we apply an affine transformation on a set of points $P = \{\vec{p}_i \in \mathbb{R}^2\}$, and also apply the rotation, we will get $P' = \{\vec{p}'_i \in \mathbb{R}^2\}$ where $\vec{p}'_i = \text{Rotate}(\text{Affine}(\vec{p}_i))$. The detailed equation is as follows:

$$\begin{pmatrix} x'_i \\ y'_i \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_i \\ y_i \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix} \quad (6)$$

where

$$\begin{pmatrix} x_i \\ y_i \end{pmatrix} = \vec{p}_i \text{ and } \begin{pmatrix} x'_i \\ y'_i \end{pmatrix} = \vec{p}'_i.$$

Our only concern is the difference between the x coordinates of two points $(P_{-0}) \rightarrow$ and $(P_{-1}) \rightarrow$ (the original points) and the difference between the x coordinates of $(P_{-1'}) \rightarrow$ and $(P_{-0'}) \rightarrow$ (the transformed points). From eq. 6 we can derive the following:

$$x'_1 - x'_0 = (a \cos \theta - c \sin \theta)(x_1 - x_0) + (b \sin \theta - d \cos \theta)(y_1 - y_0) \quad (7)$$

We can choose θ (and also r) from the equations $b = r \sin \theta$ and $d = r \cos \theta$, where $\theta, r \in \mathbb{R}$ and $r \det(A) > 0$ assuming $\det(A) \neq 0$. This is always true in practice. The θ will be the "right" angle we should choose so that $x'_1 - x'_0$ will not be dependent on the y coordinates y_1 and y_0 . After substitution, eq. 7 can be rewritten as:

$$x'_1 - x'_0 = \left(\frac{ad-bc}{r} \right) (x_1 - x_0). \quad (8)$$

As $\left(\frac{ad-bc}{r} \right) = \frac{\det(A)}{r} > 0$, we showed that we can preserve the relative order of points by applying a carefully chosen rotation.

In practice we do not know θ , because we do not know the parameters of the affine transformation. But computing LIS is very fast, so we can afford to do it multiple times. We randomly choose several angles for the first and for the second image. We compute the LIS for all possible angle-pairs ($K \times L$ times if the numbers of angles are K and L for the images respectively). We keep the true matches from the largest subset we obtained. We create the angles for the first image by the following steps. First we choose a random angle $\theta = \text{rand}(0, 2\pi)$. Then the set of angles becomes

$$\theta_k = \left\{ \theta + \frac{k \cdot 2\pi}{K} \right\},$$

where $k=0, 1, \dots, K-1$. We choose the θ_L angles for the second images similarly. $K = 3$ and $L=7$ are good choices based on our experiments.

4. Experiment

We validate the AIR approach in an image retrieval experiment. We compare it with three other schemes. Thus, the four schemes in our evaluations are as follows: (1) SURF + RANSAC Homography [21, 27], (2) SURF + LIS standalone, (3) SURF + Spatial Relations standalone, and (4) SURF + AIR. We include SURF + LIS standalone and SURF + Spatial Relations standalone in our evaluation although AIR is comprising both LIS and Spatial Relations. The reason is because we want to determine from the

results whether the agent (AIR) is intelligent enough to switch between the two approaches for different images. The results should be improved with AIR.

4.1. Data set and Evaluation measures

We use the Stanford Mobile Visual Search data set proposed in [28] for our evaluation. This data set has several key characteristics that are lacking in existing data sets: rigid objects, widely varying lighting conditions, perspective distortion, typical foreground and background clutter, realistic ground-truth reference data, and query data collected from heterogeneous low and high-end camera phones. The data are in several different categories: CDs, DVDs, books, business cards, text documents, video clips, and museum paintings. Some sample query and database images are shown in Fig. 4. The number of database and query images for different categories is shown in Table 1. There are a total of 2800 query images for 700 distinct classes across 7 image categories used in the evaluation. The original resolution of the images varies for all categories, and we deliberately reduce the size of the images to 320×240 to make them more compact for efficient transmission and storage, well-suited for mobile visual search applications. This also makes the evaluation more challenging (dealing with low resolution images).

The evaluation measures are straightforward. We report the percentage of correct images retrieved and the average number of matched keypoints for each category. These measurements are similar to the ones used in [28].

Table 1. Number of query and database images for different categories used in the evaluation.

Category	Database	Query
CDs	100	400
DVDs	100	400
Books	100	400
Video Clips	100	400
Business Cards	100	400
Text Documents	100	400
Paintings	100	400



Figure 4. Stanford Mobile Visual Search data set (Chandrasekhar et al., 2011) used for our evaluation. We used a total of 7 categories as shown. The images are captured with a variety of camera-phones, and under widely varying lighting conditions

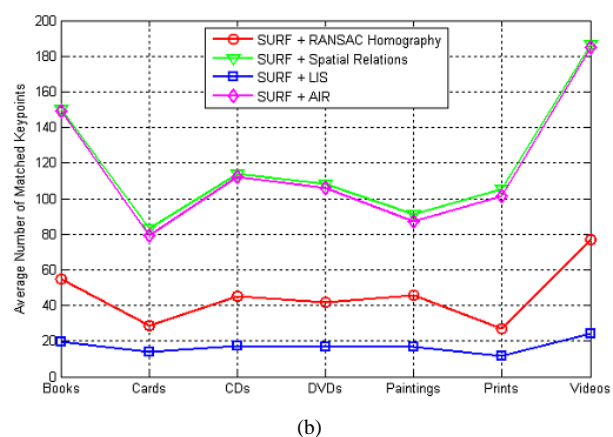
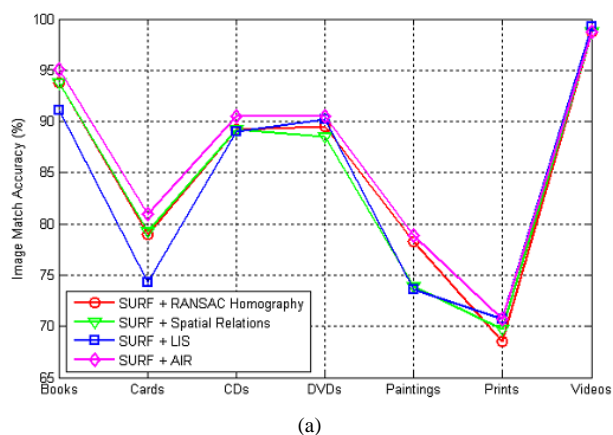


Figure 5. Results of the four approaches for each data set’s category (total of 7). (a) shows the image matching accuracy (correct retrieval) in percentage. (b) shows the average number of matched keypoints

4.2. Results

In Fig. 5, we report results for the four schemes as described above. Firstly, we observe that SURF + Spatial Relations standalone and SURF + LIS standalone do not give the best results. However, when they are combined into the AIR approach, it translates into better retrieval results. Out of the 7 categories in the data set, SURF + AIR dominates 6 categories as shown in Fig. 5(a), with 1 category (videos) having the same matching accuracy with SURF + RANSAC homography. LIS achieves highest matching accuracy among all in the video clips.

Secondly, we note that SURF + AIR and SURF + Spatial Relations both give very high average number of matched keypoints as shown in Fig. 5(b). The average number of matched keypoints in each category is about twofold and more of SURF + RANSAC homography. This is predictable as the spatial relations approach can recover missing (undetected) keypoints based on neighboring relations as explained in Section 3.1.

5. Conclusions

In this paper, we have proposed a novel design of an image recognition agent called AIR (Agent for Image Recognition) showing high potential in image matching and image retrieval applications. AIR is able to verify true keypoint matches while recovering missing (undetected) keypoints in one image by exploiting the spatial relations approach as described in Section 3.1. It is more robust to false keypoint matches or noise as it does

not only evaluate each pair of candidate keypoints in the two images, but also on each of their neighboring keypoints based on spatial proximity.

AIR is also able to recognize instability of the homography-based approach in certain images, and automatically switches to the LIS (Longest Increasing Subsequence) approach as proposed in Section 3.2. The LIS approach allows less rigid correspondence between the matched image pairs.

We have demonstrated AIR in an image retrieval experiment on the Stanford Mobile Visual Search data set, where the results favored AIR for its increased accuracy and larger number of matched keypoints. It achieved a twofold more matched keypoints when compared to the state-of-the-art approach (SURF + RANSAC homography).

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