TURKISH JOURNAL OF SCIENCE

e-ISSN 2587-0971

TURKISH JOURNAL OF SCIENCE

(An International Peer-Reviewed Journal)
ISSN: 2587-0971

Volume: V, Issue: I, 2020

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ISSN: 2587–0971

Caputo and Atangana-Baleanu-Caputo Fractional Derivative Applied to Garden Equation

Mustafa Ali Dokuyucu[a]

[a]*Department of Mathematics, Faculty of Science and Arts, Ağrı Ibrahim Çeçen University, Ağrı, Turkey*

Abstract. In this study, the garden equation which is a nonlinear partial differential equation is discussed. First, we will expand the garden equation to the Caputo derivative and Atangana-Baleanu fractional derivative in the sense of Caputo. Then, we will then demonstrate the existence of the new equation with the help of the fixed point theorem. Finally, we will examine uniqueness solution for the two fractional operators.

1. Introduction

Many nonlinear partial differential equations are used to describe real world problems. Such problems are used in many branches, especially in engineering, earth sciences and physics [5–7]. For example, one of these examples is the garden equation. The garden equation is a nonlinear differential equation used to describe some dynamics in hydrodynamics and plasma physics. For example, plasma physics is the study of the state of a substance that contains charged particles and liquids under the influence of electric and magnetic fields. It is possible to create plasma by heating a gas until it breaks chemical bonds that connect electrons to parent atoms or molecules. The subject of plasma is up to date and has many different application areas such as beam storage, accelerator physics, space and astrophysics.

To describe complex problems, the concept of a fractional-order derivative and a partial differential equation are used. One of the difficulties encountered in solving such equations is to predict the future behavior of the physical problem. Using fractional derivative operators to cope with this situation helps researchers [4]. Many fractional derivative definitions are used in the literature. The Caputo version [2] of the captive derivative is mostly used to model real world problems because it allows the use of initial conditions. The problem encountered in this version, however, is singularity due to the function used to induce the local derivative. Atangana-Baleanu fractional derivative [3] in the sense of Caputo is also quite assertive in this regard. Because the kernel used in this definition is both non-local and non-singular. This allows us to get rid of the singularity problem in the Caputo fractional derivative. Atangana, Akgul and Owolabi [8] present a detailed analysis including, numerical solution, stability analysis and error analysis. Atangana and Akgul [9] tried to construct new transfer functions that would lead the Sumudu transformation to new Bode, Nichols and Nyquist plots.

Corresponding author: MAD mail address: madokuyucu@agri.edu.tr ORCID: https://orcid.org/0000-0001-9331-8592
Received: 16 February 2020; Accepted: 17 March 2020; Published: 30 March 2020
Keywords. Atanagana-Baleanu-Caputo fractional derivative, Fixed-Point theorem, Garden equation, Existence and Uniqueness
2010 *Mathematics Subject Classification*. 26A33, 34K28, 39A14, 93A30
Cited this article as: Dokuyucu MA. Caputo and Atangana-Baleanu-Caputo Fractional Derivative Applied to Garden Equation. Turkish Journal of Science. 2020, 5(1), 1 – 7.

In this study, we devoted the first part to the general history and physical history of the garden equation. In the second part, the necessary definitions and theorems that will be used in the article are given. In the third section, we expanded the garden equation to the Caputo fractional derivative. Then we examined the existence and uniqueness solutions of the new equation. In the last chapter, we expanded the same equation to Atangana-Baleanu fractional derivative in the sense of Caputo. We have examined the existence and uniqueness solutions for this equation.

2. Preliminaries

We give in this section some fundamental definitions [1–3] on fractional derivative.

Definition 2.1. *The Caputo derivative of fractional derivative is defined as [2]:*

$$ {}^{C}_{a}D^{\nu}_{t}f(t)) = \frac{1}{\Gamma(n-\nu)}\int_{a}^{t}\frac{f^{(n)}(r)}{(t-r)^{\nu+1-n}}dr, \quad n-1<\nu<n\in\mathbb{N}. \tag{1}$$

Definition 2.2. *The Riemann-Liouville fractional integral is defined as [1]:*

$$ J^{\nu}f(t) = \frac{1}{\Gamma(\nu)}\int_{a}^{t}f(r)(t-r)^{\nu-1}dr. \tag{2}$$

Definition 2.3. *The Riemann-Liouville fractional derivative is defined as [1]:*

$$ {}^{R}_{a}D^{\nu}_{t}f(t) = \frac{1}{\Gamma(n-\nu)}\frac{d^{n}}{dt^{n}}\int_{a}^{t}\frac{f(r)}{(t-r)^{\nu+1-n}}dr, \quad n-1<\nu<n\in\mathbb{N}. \tag{3}$$

Definition 2.4. *The Sobolev space of order 1 in (a,b) is defined as [2]:*

$$ H^{1}(a,b) = \{u\in L^{2}(a,b) : u'\in L^{2}(a,b)\}. $$

Definition 2.5. *Let a function $u\in H^{1}(a,b)$ and $\nu\in(0,1)$. The AB fractional derivative in Caputo sense of order ν of u with a based point a is defined as [3]:*

$$ {}^{ABC}_{a}D^{\nu}_{t}u(t)) = \frac{B(\nu)}{1-\nu}\int_{a}^{t}u'(s)E_{\nu}\Big[-\frac{\nu}{1-\nu}(t-s)^{\nu}\Big]ds, \tag{4}$$

where $B(\nu)$ has the same properties as in Caputo and Fabrizio case, and is defined as

$$ B(\nu) = 1-\nu+\frac{\nu}{\Gamma(\nu)}, $$

$E_{\nu,\beta}(\lambda^{\nu})$ is the Mittag-Leffler function, defined in terms of a series as the following entire function

$$ E_{\nu,\beta}(z) = \sum_{k=0}^{\infty}\frac{(\lambda^{\nu})^{k}}{\Gamma(\nu k+\beta)}, \qquad \nu>0, \quad \lambda<\infty \quad \textit{and} \quad \beta>0, \quad \lambda=-\nu(1-\nu)^{-1}. \tag{5}$$

Definition 2.6. *Let a function $u\in H^{1}(a,b)$ and $\nu\in(0,1)$. The AB fractional derivative in Riemann-Liouville sense of order ν of u with a based point a is defined as [3]:*

$$ {}^{ABR}_{a}D^{\nu}_{t}u(t)) = \frac{B(\nu)}{1-\nu}\frac{d}{dt}\int_{a}^{t}u(s)E_{\nu}\Big[-\frac{\nu}{1-\nu}(t-s)^{\nu}\Big]ds, \tag{6}$$

when the function u is constant, we get zero.

Definition 2.7. *The Atangana-Baleanu fractional integral of order ν with base point a is defined as [3]:*

$$ {}^{AB}I^{\nu}_{t}u(t)) = \frac{1-\nu}{B(\nu)}u(t)+\frac{\nu}{B(\nu)\Gamma(\nu)}\int_{a}^{t}u(s)(t-s)^{\nu-1}ds, \tag{7}$$

when the function u is constant, we get zero.

3. Garden equation with Caputo derivative

Garden equation is given by,

$$u_t(x,t) = 6(u + \epsilon^2 u^2)u_x + u_{xxx} \tag{8}$$

The initial condition is $u(x,0) = f(x)$ for the equation (1). The equation (1) with Caputo derivative is given as below

$$ {}_0^C D_t^\nu u(x,t) = 6(u + \epsilon^2 u^2)u_x + u_{xxx}. \tag{9}$$

In this section we show existence and uniqueness solution of the equation (1). Let us present every continuous functions $G = C[a,b]$ in the Banach space defined in the closed set $[a,b]$ and consider $Z = \{\rho, a \in G, \rho(x,t) \geq 0 \quad and \quad a(x,t) \geq 0, a \leq t \leq b\}$

Definition 3.1. *[4] Let X be a Banach space with a cone H. H initiates a restricted order $\leq$ in E in the succeeding approach.*

$$y \geq x \Rightarrow y - x \in H$$

Now applying the fractional integral in equation (9), we obtain the following,

$$u(x,t) - u(x,0) = \frac{1}{\Gamma(\nu)} \int_0^t (t-r)^{\nu-1} \Big[6(u + \epsilon^2 u^2)u_x + u_{xxx} \Big] dr. \tag{10}$$

Now we can use system (10) to show the existence of equation (8). Necessary lemma for the existence of the solutions are given as Lemma 3.2. We now need to define an operator which $X : G \to G$.

$$Xu(x,t) = \frac{1}{\Gamma(\nu)} \int_0^t (t-r)^{\nu-1} \phi(x,r,\rho(x,r)) dr \tag{11}$$

To be dealt with more easily, let us consider below

$$\phi(x,r,u) = 6(u + \epsilon^2 u^2)u_x + u_{xxx} \tag{12}$$

Lemma 3.2. *The mapping $X : G \to G$ is completely continuous.*

Proof. Let $N \subset G$ be bounded. There exists a constants $l > 0$ such that $\|u\| < l$. Let,

$$T = max_{\substack{0 \leq t \leq 1 \\ 0 \leq u \leq l}} \phi(x,t,u(x,t))$$

$\forall u \in N$, we have,

$$\begin{aligned} \|Xu(x,t)\| &\leq \frac{1}{\Gamma(\nu)} \int_0^t (t-r)^{\nu-1} \|\phi(x,r,u(x,r))\| dr \\ &\leq \frac{T}{\Gamma(\nu)} \int_0^t (t-r)^{\nu-1} dr \\ &= \frac{T}{\Gamma(\nu+1)} t^\nu \end{aligned} \tag{13}$$

Hence $X(N)$ is bounded.
Now in the following part, we will consider $t_1 < t_2$ and $u(x,t), \in N$ and ; then for a given $\epsilon > 0$ if $|t_2 - t_1| < \delta$. We have,

$$\begin{aligned}
\|Tu(x,t_2)-Tu(x,t_1)\| =&\|\frac{1}{\Gamma(\nu)}\int_0^{t_2}(t_2-r)^{\nu-1}\|\phi(x,r,u(x,r))\|dr \\
&-\frac{1}{\Gamma(\nu)}\int_0^{t_1}(t_1-r)^{\nu-1}\|\phi(x,r,u(x,r))dr\| \\
=&\|\frac{1}{\Gamma(\nu)}\int_0^{t_2}(t_2-r)^{\nu-1}\|\phi(x,r,u(x,r))\|dr \\
&-\frac{1}{\Gamma(\nu)}\int_0^{t_2}(t_1-r)^{\nu-1}\|\phi(x,r,u(x,r))\|dr \\
&-\frac{1}{\Gamma(\nu)}\int_{t_1}^{t_2}(t_1-r)^{\nu-1}\|\phi(x,r,u(x,r))\|dr \\
\leq&\frac{1}{\Gamma(\nu)}\int_0^{t_2}\|(t_2-r)^{\nu-1}-(t_1-r)^{\nu-1}\|\,\|\rho(x,r,u(x,r))\|dr
\end{aligned} \tag{14}$$

$$\begin{aligned}
&+\frac{1}{\Gamma(\nu)}\int_{t_1}^{t_2}\|(t_1-r)^{\nu-1}\|\,\|\phi(x,r,u(x,r))\|dr \\
\leq&\frac{T}{\Gamma(\nu)}\int_0^{t_2}\Big((t_2-r)^{\nu-1}-(t_1-r)^{\nu-1}\Big)dr+\frac{T}{\Gamma(\nu)}\int_{t_1}^{t_2}(t_1-r)^{\nu-1}dr \\
=&\frac{T}{\Gamma(\nu)}\Big(\int_0^{t_2}(t_2-r)^{\nu-1}dr-\int_0^{t_2}(t_1-r)^{\nu-1}dr+\int_{t_1}^{t_2}(t_1-r)^{\nu-1}dr\Big) \\
=&\frac{T}{\Gamma(1+\nu)}\Big(t_2^{\nu}+(t_1-t_2)^{\nu}-t_1^{\nu}+(t_1-t_2)^{\nu}\Big) \\
\leq&\frac{2T}{\Gamma(1+\nu)}(t_1-t_2)^{\nu}+\frac{T}{\Gamma(1+\nu)}(t_1-t_1)^{\nu} \\
=&\frac{2T}{\Gamma(1+\nu)}(t_1-t_2)^{\nu} \\
<&\frac{2T}{\Gamma(1+\nu)}\delta^{\nu} \\
=&\epsilon
\end{aligned} \tag{15}$$

It is clear seen that, when the same steps are applied to the $a(x,t)$ function, we get same situation. Finally,

$$|Xu(x,t_2)-Xu(x,t_1)|\leq\epsilon$$

are satisfied. Where $\delta=\Big(\epsilon\Gamma(1+\nu/2T)\Big)^{1/\nu}$.Therefore $X(N)$ is equicontinuous. So that $\overline{X(N)}$ is compact via The Arzela-Ascoli theorem. □

Theorem 3.3. *Let $\chi:[u_1,u_2]\times[0,\infty)\to[0,\infty)$, then $\chi(x,t,.)$ is non-decreasing for each t in $[u_1,u_2]$. there exists a positive constants z_1 and z_2 such that $C(n)z_1\leq\chi(x,t,z_1)$, $C(n)z_2\geq\chi(x,t,z_2)$, $0\leq z_1(x,t)\leq z_2(x,t)$, $u_1\leq t\leq u_2$. This means that the new equation has a positive solution.*

Proof. We only need to consider the fixed point for operator of X. With framework of Lemma 3.2, the

considered operator $X : K \to K$ is completely continuous. Let us take two arbitrary u_1 and u_2,

$$\begin{aligned} Xu_1(x,t) =& \frac{1}{\Gamma(\nu)} \int_0^t (t-r)^{\nu-1} \phi(x,r,u_1(x,r))dr \\ &\leq \frac{1}{\Gamma(\nu)} \int_0^t (t-r)^{\nu-1} \phi(x,r,u_2(x,r))dr \\ &= Xu_2(x,t) \end{aligned} \tag{16}$$

Hence X is a non-decreasing operator. So that the operator $X : \langle z_1, z_2 \rangle \to \langle z_1, z_2 \rangle$ is compact and continuous via Lemma 3.2. In that case, K is a normal cone of X. □

3.1. Uniqueness of Solution

The aim of this chapter is to prove the uniqueness of solutions to the equation (10). So the uniqueness of the solution is presented as below,

$$\begin{aligned} \|Xu_1(x,t) - Xu_2(x,t)\| &= \|\frac{1}{\Gamma(\nu)} \int_0^t (t-r)^{\nu-1} \Big(\phi(x,r,u_1(x,r)) - \phi(x,r,u_2(x,r))\Big)dr\| \\ &\leq \frac{1}{\Gamma(\nu)} B_1 \int_0^t (t-r)^{\nu-1} \|u_1(x,r) - u_2(x,r)\|dr \end{aligned} \tag{17}$$

So that,

$$\|Xu_1(x,t) - Xu_2(x,t)\| \leq \left\{\frac{B_1 t^\nu}{\Gamma(\nu+1)}\right\} \|u_1(x,r) - u_2(x,r)\|$$

Therefore, if the following conditions hold,

$$\left\{\frac{B_1 t^\nu}{\Gamma(\nu+1)}\right\} < 1$$

Then mapping X is a contraction, which implies fixed point, and thus the model has a unique positive solution.

4. Garden equation with AB derivative in Caputo sense

We present in this chapter the existence and uniqueness of solutions of the garden equation using the Atangana-Baleanu derivative. Let $\Omega = (a,b)$ be an open and bounded subset of R^n. For a given $\nu \in (0,1)$ and functions $u(x,t) \in H^1(\Omega) \times [0,T]$. We apply the equation (8) to the Atangana-Baleanu fractional derivative,

$$ {}_0^{ABC}D_t^\nu u(x,t) = \xi(x,t,u) \tag{18}$$

where

$$\xi(x,t,u) = 6(u + \epsilon^2 u^2)u_x + u_{xxx} \tag{19}$$

Using the Atangana-Baleanu integral to (18) it yields

$$u(x,t) = u(x,0) + \frac{1-\nu}{B(\nu)} \xi(x,t,u(x,t)) + \frac{\nu}{B(\nu)\Gamma(\nu)} \int_0^t \xi(x,r,u(x,r))(t-r)^{\nu-1} dr \tag{20}$$

for all $t \in [0,T]$.

Theorem 4.1. *If the inequality (21) hold, ξ satisfies Lipshitz condition and contraction.*

$$0 < 3\varphi_1 d_1 + 2\epsilon^2 \varphi_1 d_2 + \varphi_1^3 \leq 1 \tag{21}$$

Proof. We would like to start with the kernel ξ. Let η and κ are two functions, the following equation is written:

$$\begin{aligned}\xi(x,t,\eta)-\xi(x,t,\kappa) =&\Big(6(\eta+\epsilon^2\eta^2)\eta_x+\eta_{xxx}\Big)-\Big(6(\kappa+\epsilon^2\kappa^2)\kappa_x+\kappa_{xxx}\Big)\\ &6(\eta\eta_x-\kappa\kappa_x)+6\epsilon^2(\kappa^2\kappa_x-\eta^2\eta_x)+(\eta_{xxx}-\kappa_{xxx})\end{aligned} \tag{22}$$

Then, applying the norm on both sides gives

$$\begin{aligned}\|\xi(x,t,\eta)-\xi(x,t,\kappa)\| =&\|6(\eta\eta_x-\kappa\kappa_x)+6\epsilon^2(\kappa^2\kappa_x-\eta^2\eta_x)+(\eta_{xxx}-\kappa_{xxx})\|\\ &\leq 6\|\eta\eta_x-\kappa\kappa_x\|+6\epsilon^2\|\kappa^2\kappa_x-\eta^2\eta_x\|+\|\eta_{xxx}-\kappa_{xxx}\|\\ &\leq 3\|\partial_x(\eta^2-\kappa^2)\|+2\epsilon^2\|\partial_x(\eta^3-\kappa^3)\|+\|\partial_{xxx}(\eta-\kappa)\|\end{aligned} \tag{23}$$

Using the Lipschitz condition for the first order derivative function ∂_x; we can find φ_1 such that

$$\begin{aligned}\|\xi(x,t,\eta)-\xi(x,t,\kappa)\| \leq&3\varphi_1\|\eta^2-\kappa^2\|+2\epsilon^2\varphi_1\|\eta^3-\kappa^3\|+\varphi_1^3\|\eta-\kappa\|\\ &\leq 3\varphi_1\|\eta+\kappa\|\cdot\|\eta-\kappa\|+2\epsilon^2\varphi_1\Big(\|\eta\|^2+\|\eta\|\cdot\|\kappa\|+\|\kappa\|^2\Big)\|\eta-\kappa\|+\varphi_1^3\|\eta-\kappa\|\\ &\leq \Big[3\varphi_1 d_1+2\epsilon^2\varphi_1 d_2+\varphi_1^3\Big]\|\eta-\kappa\|\end{aligned} \tag{24}$$

So the following inequality can be written.

$$\|\xi(x,t,\eta)-\xi(x,t,\kappa)\|\leq K\|(\eta(x,t)-\kappa(x,t)\|. \tag{25}$$

where

$$K=\Big(3\varphi_1 d_1+2\epsilon^2\varphi_1 d_2+\varphi_1^3\Big)$$

Therefore ξ satisfies the Lipschitz condition. Then we can say that it is a contraction.
In the another case, the following inequality can be written because our kernel is linear,

$$\xi_2(x,t,v_1)-\xi_2(x,t,v_2)\leq(c\vartheta_1^2+d)\|v_1(x,t)-v_2(x,t)\|$$

Hence, the proof is complete. We can now show that the uniqueness of the solution. □

4.1. Uniqueness of solution

The uniqueness solution for equation (20) is presented as below. Let $u_1,u_2\in H^1$ be two solutions of (20). Let $u=u_1-u_2$. the following equation can be written,

$$u=\frac{1-\nu}{B(\nu)}\Big(\xi(x,t,u_1(x,t))-\xi(x,t,u_2(x,t))\Big)+\frac{\nu}{B(\nu)\Gamma(\nu)}\int_0^t\Big(\xi(x,r,u_1(x,r))-\xi(x,r,u_2(x,r))\Big)dr,$$

If the norms of both sides are taken, by the Gronwall inequality [20],

$$\|u\|\leq\frac{1-\nu}{B(\nu)}\|\xi(x,t,u_1(x,t))-\xi(x,t,u_2(x,t))\|+\frac{\nu}{B(\nu)\Gamma(\nu)}\int_0^t\|\xi(x,r,u_1(x,r))-\xi(x,r,u_2(x,r))\|dr\leq K_1\int_0^t\|\xi(x,t,u_1(x,t))\|_{H^1}dr.$$

Finally, the equation (20) has a unique solution for the equation u.

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TURKISH JOURNAL OF SCIENCE
VOLUME 5, ISSUE 1, 8–15
ISSN: 2587–0971

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Some Estimates for the Spin-Submanifold Twisted Dirac Operators

Mehmet Ergen[a]

[a]*Eskişehir Technical University, Eskişehir, TURKEY*

Abstract. In this paper, we generalize lower bound estimates for the eigenvalue estimates of the submanifold twisted Dirac operator on a compact Riemannian Spin−submanifold proved by N. Ginoux and B. Morel in 2002.

1. Introduction

Defining some structure on the compact Riemannian manifolds as Spin and Spin^c−structure to obtain information about the topology and geometry of the manifold is the main way for mathematicians. Due to this feature, many authors have been systematically worked on these structures [2, 3, 7, 16]. One of the way to obtain these subtle information is the investigation the spectrum of the Dirac operator [4, 5, 7, 10, 13, 14]. The study of Dirac operators on the submanifolds was firstly started by E. Witten using the hypersurface Dirac operator to prove the positive energy-theorem [20]. Later on, this operator is investigated by the mathematicians and physicists to obtain subtle information about the topology and geometry of the manifolds. One of the ways to obtain this subtle information is done by investigating the spectrum of the Dirac operator [4–7, 9, 10, 14].

Obtaining lower bounds to the eigenvalues of the submanifold Dirac operator firstly was given by X. Zhang and O. Hijazi in [13] by generalized the results obtained on the hypersurfaces [12, 19]. The fundemantal tools used to estimate the lower bound are appropriately modified spinorial Levi−Civita connection and Schrödinger−Lichnerowicz formula.

In this paper, we will consider the generalization of the results for the the submanifolds obtained by N. Ginoux and B. Morel in [8] coming from the result for hypersurfaces given in [17]. In doing so, as in the papers of O. Hijazi and X. Zhang [12, 19], they started by restricted the spinor bundle of the Riemannian Spin−manifold to a submanifold equipped with an induced Riemannian metric. Then, they lifted the Levi−Civita connections defined on both the Riemannian Spin manifold and its submanifold onto the spinor bundle built on these manifolds, respectively. Finally, they defined the submanifold Dirac operator with the help of the spinorial Gauss formula. Some authors called this operator as a twisted Dirac operator [18]. In this paper we use this naming.

Later on, N. Ginoux and B. Morel in [8] obtain an estimates for the eigenvalues of the twisted Dirac operator on a compact Riemannian submanifold in terms of the scalar curvature, mean curvature and Energy−Momentum tensor.

Corresponding author: ME mail address: mergen@eskisehir.edu.tr ORCID: https://orcid.org/0000-0001-5255-3510
Received: 20 February 2020; Accepted: 25 March 2020; Publised: 30 March 2020
Keywords. (Spin and Spin^c geometry, Dirac operator, Estimation of eigenvalues)
2010 *Mathematics Subject Classification*. 53C27, 34L40, 35PL5
Cited this article as: Ergen M. Some Estimates for the Spin-Submanifold Twisted Dirac Operators. Turkish Journal of Science. 2020, 5(1), 8 - 15.

In this paper, by defining modified spinorial Levi−Civita connections we give estimates containing all inequalities obtained in [8] as special cases.

We would like to thank Nicolas Ginoux for his support during the prepation of this paper.

2. Twisted Dirac Operator on The Submanifolds

Let $\widetilde{M}$ be an $(m+n)-$dimensional compact Riemannian Spin−manifold and M be an $m-$dimensional immersed oriented Riemannian Spin−submanifold in $\widetilde{M}$ equipped with the induced Riemannian metric. Let NM be the normal bundle of M. As we know, the manifolds M and $\widetilde{M}$ defines an unique Spin−structure on the normal bundle NM [8]. Through out the whole paper $\mathbb{S}_M$, $\mathbb{S}_{MN}$ and $\mathbb{S}_{\widetilde{M}}$ denotes the spinor bundles over the manifolds M, NM and $\widetilde{M}$, respectively. The restricted spinor bundle $\mathbb{S} := \mathbb{S}_{\widetilde{M}}\big|_M$ is identified as follows:

$$\mathbb{S} := \begin{cases} \mathbb{S}_M \otimes \mathbb{S}_{MN}, & \text{if } n \text{ or } m \text{ is even,} \\ \mathbb{S}_M \otimes \mathbb{S}_{MN} \oplus \mathbb{S}_M \otimes \mathbb{S}_{MN}, & \text{otherwise.} \end{cases} \tag{1}$$

On this restricted spinor bundle exist a Hermitian inner product, denoted by $\big(\ ,\ \big)$, such that Clifford multiplication by a vector of $T\widetilde{M}\big|_M$ is skew−symmetric [15, 16].

As in [8] we denote the induced spinorial Levi−Civita connection on $\Gamma(\mathbb{S})$ by $\widetilde{\nabla}$ and ∇

$$\widetilde{\nabla} = \begin{cases} \left(\nabla^{\mathbb{S}_M} \otimes Id + Id \otimes \nabla^{\mathbb{S}_{MN}}\right) \oplus \left(\nabla^{\mathbb{S}_M} \otimes Id + Id \otimes \nabla^{\mathbb{S}_{MN}}\right), & \text{if } n \text{ and } m \text{ are odd,} \\ \left(\nabla^{\mathbb{S}_M} \otimes Id + Id \otimes \nabla^{\mathbb{S}_{MN}}\right), & \text{otherwise.} \end{cases}$$

For a fixed point $x \in M$, let $(e_1, ..., e_m, v_1, ..., v_n)$ be a positively oriented local orthonormal basis of $T\widetilde{M}\big|_M$ such that $(e_1, ..., e_m)$ $(resp(v_1, ..., v_n))$ is a positively oriented local orthonormal basis of TM (*resp* NM). Also we have the following identification between the Clifford multiplication on $\Gamma(\mathbb{S}_M)$ and $\Gamma(\mathbb{S}_{\widetilde{M}})\big|_M$

$$X \cdot_M \Phi \quad = \quad \left(X \cdot \omega_\perp \cdot \Psi\right)_M, \tag{2}$$

where $\Phi = \Psi\big|_M$, $\Psi \in \Gamma(\mathbb{S}_{\widetilde{M}})$, and

$$\omega_\perp = \begin{cases} \omega_n, & \text{for } n \text{ even,} \\ -i\omega_n, & \text{for } n \text{ odd,} \end{cases} \tag{3}$$

here ω_n denoting the complex volume form:

$$\omega_n = i^{[\frac{n+1}{2}]} v_1 \cdot ... \cdot v_n. \tag{4}$$

The spinorial Gauss formula [1]:

$$\widetilde{\nabla}_i \Psi = \nabla_i \Psi + \frac{1}{2} \sum_{j=1}^{m} e_j \cdot h_{ij} \cdot \Psi, \tag{5}$$

where $\Psi \in \Gamma(\mathbb{S})$ and h_{ij} is the component of the second fundemantal form at x. Accordingly, Dirac operators are defined as follows:

$$\widetilde{D} = \sum_{i=1}^{m} e_i \cdot \widetilde{\nabla}_i,\ D = \sum_{i=1}^{m} e_i \cdot \nabla_i, \tag{6}$$

and the twisted Dirac operator which also called submanifold Dirac operator and denoted by D_H is defined as

$$D_H := (-1)^n \omega_\perp \cdot \widetilde{D} = (-1)^n \omega_\perp \cdot D + \frac{1}{2} H \cdot \omega_\perp \cdot \Psi, \tag{7}$$

where $H = \sum_{i=1}^{m} h(e_i, e_i)$ denotes the mean curvature vector field. Moreover, by using the fact that $H \cdot \omega_\perp \cdot = (-1)^{n-1}\omega_\perp \cdot H$ one gets $\widetilde{D} = D - \frac{1}{2}H$ and

$$\begin{aligned} D &= \omega_\perp \cdot D_H + \frac{1}{2}H \cdot \omega_\perp \cdot \\ &= \lambda_H \cdot \omega_\perp \cdot + \frac{1}{2}H\cdot, \end{aligned} \tag{8}$$

where λ_H denotes the eigenvalue of the twisted Dirac operator D_H.

Finally, for any $\Psi \in \Gamma(\mathbb{S})$, we give the well−known formula called twisted Lichnerowicz formula as follows:

$$\Big(D^2\Psi, \Psi\Big) = \Big(\nabla^*\nabla\Psi, \Psi\Big) + \frac{1}{4}\Big(R + R^N_\Psi\Big)|\Psi|^2, \tag{9}$$

where R is the scalar curvature of M and $R^N_\Psi := 2\sum_{i,j=1}^{m}\Big(e_i \cdot e_j \cdot I_d \otimes R^N_{e_i,e_j}\Psi, \frac{\Psi}{|\Psi|^2}\Big)$ on $M_\Psi := \{x \in M : \Psi(x) \neq 0\}$, and $R^N_{e_i,e_j}$ stands for spinorial normal curvature tensor. Combining (8) and (9), we obtain

$$\int_M |\nabla\Psi|^2 = \int_M \Big(\lambda_H^2|\Psi|^2 + \frac{1}{4}\|H\|^2|\Psi|^2 + \lambda_H Re\Big(\omega_\perp \cdot \Psi, H \cdot \Psi\Big) - \frac{1}{4}(R + R^N_\Psi)|\Psi|^2\Big). \tag{10}$$

3. Lower bounds of Eigenvalues

In this section, two estimates are given. One of them is obtained in terms of the mean curvature and modified scalar curvature defined in [7, 12, 13] as follows:

$$R_{p,u,\Psi} = R + R^N_\Psi - 4p\nabla u + 4\nabla p\nabla u - 4\Big(1 - \frac{1}{m}\Big)p^2|du|^2, \tag{11}$$

where p and u are real valued functions defined on $\widetilde{M}$. If $p, u = 0$, then $R_{p,u,\Psi} = R + R^N_\Psi$. In this case all estimates coincides with the result obtained in [8]. The other is obtained in terms of above modified scalar curvature and Energy−Momentum tensor.

In the following theorem, we give an optimal lower bound to the eigenvalues of D_H by using an appropriate modified spinorial Levi−Civita connection.

Theorem 3.1. *Let $M \subset \widetilde{M}$ be a compact Riemannian Spin−submanifold of a Riemannian Spin−manifold $(\widetilde{M}, g)$. Consider a non−trivial eigenspinor field $\Psi \in \Gamma(\mathbb{S})$ such that $D_H\Psi = \lambda_H\Psi$. Assume that $m \geq 2$ and*

$$\Omega_{p,u,\Psi} = \{(p, u, \Psi)|mR_{p,u,\Psi} > (m-1)\|H\|^2 > 0\}, \tag{12}$$

on M_Ψ where p, u are real−valued functions. Then the following inequality is satisfied

$$\lambda_H^2 \geq \frac{1}{4}\sup_{\Omega_{p,u,\Psi}} Inf_{M_\Psi}\Big(\sqrt{\frac{m}{m-1}R_{p,u,\Psi}} - \|H\|\Big)^2. \tag{13}$$

Proof. Define a modified spinorial Levi−Civita connection on $\Gamma(\mathbb{S})$ by

$$\nabla^u_i\Psi = \nabla_i\Psi + \alpha e^i \cdot H \cdot \Psi + \beta\lambda_H e_i \cdot \omega_\perp \cdot \Psi + p\nabla_i u\Psi + q\nabla_j u e_i \cdot e_j \cdot \Psi \tag{14}$$

for any real−valued functions α, β, p and q. Then, for any $\Psi \in \Gamma(\mathbb{S})$ and for any i, $1 \leq i \leq n$, we have

$$\begin{aligned} |\nabla^u_i\Psi|^2 = {} & |\nabla_i\Psi|^2 + 2\alpha Re\Big(\nabla_i\Psi, e_i \cdot H \cdot \Psi\Big) + 2\lambda_H\beta Re\Big(\nabla_i\Psi, e_i \cdot \omega_\perp \cdot \Psi\Big) \\ & + 2pRe\Big(\nabla_i\Psi, \nabla_i u\Psi\Big) + 2qRe\Big(\nabla_i\Psi, \nabla_j u e_i \cdot e_j \cdot \Psi\Big) + \alpha^2\|H\|^2|\Psi|^2 \\ & + 2\alpha\beta\lambda_H Re\Big(e_i \cdot H \cdot \Psi, e_i \cdot \omega_\perp \cdot \Psi\Big) + 2\alpha pRe\Big(e_i \cdot H \cdot \Psi, \nabla_i u\Psi\Big) \\ & + 2\alpha qRe\Big(e_i \cdot H \cdot \Psi, \nabla_j u e_i \cdot e_j \cdot \Psi\Big) + \beta^2\lambda_H^2|\Psi|^2 \\ & + 2\beta p\lambda_H Re\Big(e_i \cdot \omega_\perp \cdot \Psi, \nabla_i u\Psi\Big) + 2\beta q\lambda_H Re\Big(e_i \cdot \omega_\perp \cdot \Psi, \nabla_j u e_i \cdot e_j \cdot \Psi\Big) \\ & + p^2|\nabla_i u|^2|\Psi|^2 + 2pqRe\Big(\nabla_i u\Psi, \nabla_j u e_i \cdot e_j \cdot \Psi\Big) \\ & + q^2|du|^2|\Psi|^2. \end{aligned} \tag{15}$$

Summing over i and using the fact that $\big(\omega_\perp \cdot \Psi, \Psi\big) = (-1)^n\big(\Psi, \omega_\perp \cdot \Psi\big)$, we have

$$\begin{aligned}|\nabla^u\Psi|^2 &= |\nabla\Psi|^2 - 2\alpha Re\big(D\Psi, H\cdot\Psi\big) - 2\lambda_H\beta Re\big(D\Psi, \omega_\perp\cdot\Psi\big)\\ &+2p\sum_{i=1}^{m}\nabla_i u Re\big(\nabla_i\Psi,\Psi\big) + m\alpha^2\|H\|^2|\Psi|^2 + 2m\alpha\beta\lambda_H Re\big(H\cdot\Psi, \omega_\perp\cdot\Psi\big)\\ &-2\alpha p Re\big(H\cdot\Psi, du\cdot\Psi\big) + 2m\alpha q Re\big(H\cdot\Psi, du\cdot\Psi\big) + m\beta^2\lambda_H^2|\Psi|^2\\ &-2\beta p\lambda_H Re\big(\omega_\perp\cdot\Psi, du\cdot\Psi\big) + 2m\beta q\lambda_H Re\big(\omega_\perp\cdot\Psi, du\cdot\Psi\big) + p^2|du|^2|\Psi|^2\\ &-2pq|du|^2|\Psi|^2 + mq^2|du|^2|\Psi|^2. \end{aligned} \tag{16}$$

Taking $q = \frac{p}{m}$, $\alpha = \frac{\beta-1}{2(m\beta-1)}$, for β nowhere equal to $\frac{1}{m}$ and using the equality obtained in (10), we get

$$\begin{aligned}\int_M\Big(1+m\beta^2-2\beta\Big)\lambda_H^2|\Psi|^2 &\geq \int_M\Big(\frac{(R+R^N_\Psi)}{4} + \Big(1-\frac{1}{m}\Big)p^2|du|^2 - p\Delta u + \nabla p\nabla u\Big)\\ &-\Big(\frac{m^2\beta^2-2m\beta-m\beta^2+2\beta-1)}{4(m\beta-1)^2}\Big)\|H\|^2\Big)|\Psi|^2. \end{aligned} \tag{17}$$

Using modified scalar curvature given in (11), we have

$$\lambda_H^2 \geq \frac{1}{4}\underset{p,u,\beta}{supinf}_{M}\Big(\frac{R_{p,u,\Psi}}{1+m\beta^2-2\beta} - \frac{(m-1)}{(m\beta-1)^2}\|H\|^2\Big). \tag{18}$$

Then, assuming $mR_{p,u,\Psi} > (m-1)\|H\|^2 > 0$ on M_Ψ, we can choose β so that

$$(1-m\beta)^2 = \frac{(m-1)\|H\|}{\sqrt{\frac{m}{m-1}R_{p,u,\Psi}} - \|H\|}, \text{ on } M_\Psi. \tag{19}$$

Inserting (19) in (18) we get (13). □

As in [8], κ_1 be the lowest eigenvalue of the self−adjoint operator $\mathcal{R}^N$ defined by

$$\begin{aligned}\mathcal{R}^N : \Gamma(\mathbb{S}) &\longrightarrow \Gamma(\mathbb{S})\\ \Psi &\longmapsto 2\sum_{i,j=1}^{m} e_i\cdot e_j\cdot Id\otimes R^N_{e_i,e_j}\Psi. \end{aligned} \tag{20}$$

Considering (20), tranforms the $R_{p,u,\Psi}$ as follows

$$R_{p,\kappa_1,u,\Psi} = R + \kappa_1 - 4p\nabla u + 4\nabla p\nabla u - 4\Big(1-\frac{1}{m}\Big)p^2|du|^2, \tag{21}$$

By using (21), Theorem (3.1) can be stengthened as follows:

Corollary 3.2. *Under the same conditions as in Theorem* (3.1)*, if* $m \geq 2$ *and*

$$\widetilde{\Omega}_{p,\kappa_1,u,\Psi} = \{(p,\kappa_1,u)|mR_{p,\kappa_1,u,\Psi} > (m-1)\|H\|^2 > 0\}$$

on M, then

$$\lambda_H^2 \geq \frac{1}{4}\underset{\widetilde{\Omega}_{p,\kappa_1,u,\Psi}}{sup}\ \underset{M_\Psi}{Inf}\Big(\sqrt{\frac{m}{m-1}R_{p,\kappa_1,u,\Psi}} - \|H\|\Big)^2. \tag{22}$$

In this part of the paper, concerning conformal change of the Riemannian metric and using the classic arguments given in [11–13], the optimal lower bounds is given for the square of the eigenvalue λ_H of the twisted submanifold Dirac operator D_H.

Consider the conformal change of the metric $\overline{g} = e^{2u}g$ given with any real−valued function u on $\widetilde{M}$. Let

$$\begin{array}{ccc} \mathbb{S} & \longrightarrow & \overline{\mathbb{S}} \\ \Psi & \longmapsto & \overline{\Psi} \end{array} \tag{23}$$

be the induced isometry between the two corresponding spinor bunbles. The Hermitian metrics defined on the corresponding two spinor bundles $\mathbb{S}$ and $\overline{\mathbb{S}}$, respectively satisfies:

$$\Big(\Psi, \Phi\Big) = \Big(\overline{\Psi}, \overline{\Phi}\Big)_{\overline{g}}. \tag{24}$$

Also, the Clifford multiplication on $\overline{\mathbb{S}}$ is defined as

$$\overline{e^i} \cdot \overline{\Psi} = \overline{e_i \cdot \Psi}, \tag{25}$$

where $\overline{e_i} = e^{-u}e_i$. Note that, $\overline{g} = e^{2u}g\Big|_M$ is denoted the restriction of $\overline{g}$ to M. Under this restriction, the following identity are satisfied:

$$\overline{D}\Big(e^{-\frac{(m-1)}{2}u}\overline{\Psi}\Big) = e^{-\frac{(m+1)}{2}u}\overline{D\Psi}, \tag{26}$$

where $\Psi \in \Gamma(\mathbb{S})$ and $\overline{D}$ is the Dirac operator with respect to $\overline{g}$. Also, the corresponding mean curvature vector field is given by

$$\widetilde{H} = e^{-2u}\Big(H - mgrad^N u\Big). \tag{27}$$

Let $grad^N u = 0$, then with respect to $\overline{g}$, the corresponding twisted Dirac operator $\overline{D}_H$ satisfies:

$$\overline{D}_H\Big(e^{-\frac{(m-1)}{2}u}\overline{\Psi}\Big) = e^{-\frac{(m+1)}{2}u}\overline{D_H\Psi}. \tag{28}$$

Finally, under the conformal change of the metric $\overline{g} = e^{2u}g$, $\overline{R}_{p,\kappa_1,u,\Psi}$ is written as

$$\begin{aligned} \overline{R}_{p,\kappa_1,u,\Psi} &= R + \kappa_1 + 4\Big(\frac{m-1}{2} - p\Big)\Delta u + 4\nabla p \nabla u - \Big(\big(m-1\big)(m-2) + 4(2-m)p \\ &\quad +4\Big(1 - \frac{1}{m}\Big)p^2\Big)|du|^2. \end{aligned} \tag{29}$$

In the next theorem, we will consider the regular conformal change of the metric $\overline{g}$ with $grad^N u = 0$, on M.

Theorem 3.3. *Let $M \subset \widetilde{M}$ be a compact Riemannian Spin−submanifold of a Riemannian Spin−manifold $(\widetilde{M}, g)$. Consider a non−trivial eigenspinor field $\Psi \in \Gamma(\mathbb{S})$ such that $D_H\Psi = \lambda_H\Psi$. For any regular conformal change of the metric $\overline{g} = e^{2u}g$ on $\widetilde{M}$, assume that*

$$\overline{\Omega}_{p,\kappa_1,u,\Psi} = \{(p, \kappa_1, u) | m\overline{R}_{p,\kappa_1,u,\Psi} > (m-1)\|H\|^2 > 0\}$$

on M_Ψ. Then the following inequality is satisfied

$$\lambda_H^2 \geq \frac{1}{4}\sup_{\overline{\Omega}_{p,\kappa_1,u,\Psi}} Inf_{M_\Psi}\Big(\sqrt{\frac{m}{m-1}\overline{R}_{p,\kappa_1,u,\Psi}} - \|H\|\Big)^2. \tag{30}$$

Proof. Let $\Psi \in \Gamma(\mathbb{S})$ be an eigenspinor of D_H with eigenvalue λ_H and let $\overline{\phi} := e^{-\frac{n-1}{2}u}\overline{\Psi}$. Then, by condidering $\overline{D}_H\overline{\phi} = \lambda_H e^{-u}\overline{\Phi}$, $\widetilde{H} = e^{-u}\overline{H}$, $\overline{R}^N_\Phi = e^{-2u}R^N_\Psi$ and applying $\overline{\Phi}$ to (17), we get

$$\begin{aligned} \int_M \Big(1 + m\beta^2 - 2\beta\Big)e^{-2u}\lambda_H^2|\overline{\Phi}|^2 &\geq \int_M \frac{1}{4}\Big(\overline{R}_{p,\kappa_1,u,\Psi} \\ &\quad -\Big(\frac{m^2\beta^2 - 2m\beta - m\beta^2 + 2\beta - 1)}{(mb-1)^2}\Big)\|H\|^2\Big)e^{-2u}|\overline{\Phi}|^2. \end{aligned} \tag{31}$$

As in the proof of Theorem 3.1, by considering

$$(1 - m\beta)^2 = \frac{(m-1)\|H\|}{\sqrt{\frac{m}{m-1}\overline{R}_{p,\kappa_1,u,\Psi}} - \|H\|}, \text{ on } M_\Psi, \tag{32}$$

we get the desired estimates given in (30). □

In the following theorem we improve our estimation in terms of the Energy−Momentum tensor Q^{Ψ} defined on M_{Ψ} as follows:

$$Q_{ij}^{\Psi} = \frac{1}{2}\Big(e_i\cdot\omega_{\perp}\cdot\nabla_j\Psi + e_j\cdot\omega_{\perp}\cdot\nabla_i\Psi, \frac{\Psi}{|\Psi|^2}\Big). \tag{33}$$

Theorem 3.4. *Let $M\subset\widetilde{M}$ be a compact Riemannian Spin−submanifold of a Riemannian Spin−manifold $(\widetilde{M},g)$. Consider a non−trivial eigenspinor field $\Psi\in\Gamma(\mathbb{S})$ such that $D_H\Psi=\lambda_H\Psi$. Assume that $m\geq 2$ and*

$$\Omega^{Q^{\Psi}}_{p,\kappa_1,u,\Psi} = \{(p,\kappa_1,u)|R_{p,\kappa_1,u,\Psi}+4|Q^{\Psi}|^2>\|H\|^2>0\}, \tag{34}$$

on M_{Ψ}. Then the following inequality is satisfied

$$\lambda_H^2 \geq \frac{1}{4}\sup_{\Omega^{Q^{\Psi}}_{p,\kappa_1,u,\Psi}}\inf_{M_{\Psi}}\Big(\sqrt{R_{p,\kappa_1,u,\Psi}+4|Q^{\Psi}|^2}-\|H\|\Big)^2. \tag{35}$$

Proof. Define a modified spinorial Levi−Civita connection on $\Gamma(\mathbb{S})$ by

$$\begin{aligned}\nabla_i^{Q^{\Psi}}\Psi &= \nabla_i\Psi+\alpha e^j\cdot H\cdot\Psi+\beta\lambda_H e_i\cdot\omega_{\perp}\cdot\Psi+p\nabla_i u\Psi+q\nabla_j u e_i\cdot e_j\cdot\Psi\\ &\quad +Q_{ij}^{\Psi}e_j\cdot\omega_{\perp}\cdot\Psi,\end{aligned} \tag{36}$$

where α,β,p and q are real−valued functions. Then, for any $\Psi\in\Gamma(\mathbb{S})$ and for any i, $1\leq i\leq n$, we have

$$\begin{aligned}|\nabla_i^{Q^{\Psi}}\Psi|^2 &= |\nabla_i\Psi|^2+2\alpha Re\Big(\nabla_i\Psi,e_i\cdot H\cdot\Psi\Big)+2\lambda_H\beta Re\Big(\nabla_i\Psi,e_i\cdot\omega_{\perp}\cdot\Psi\Big)\\ &\quad +2pRe\Big(\nabla_i\Psi,\nabla_i u\Psi\Big)+2qRe\Big(\nabla_i\Psi,\nabla_j u e_i\cdot e_j\cdot\Psi\Big)\\ &\quad +2Re\Big(\nabla_i\Psi,Q_{ij}^{\Psi}e_j\cdot\omega_{\perp}\cdot\Psi\Big)+\alpha^2\|H\|^2|\Psi|^2\\ &\quad +2\alpha\beta\lambda_H Re\Big(e_i\cdot H\cdot\Psi,e_i\cdot\omega_{\perp}\cdot\Psi\Big)+2\alpha pRe\Big(e_i\cdot H\cdot\Psi,\nabla_i u\Psi\Big)\\ &\quad +2\alpha qRe\Big(e_i\cdot H\cdot\Psi,\nabla_j u e_i\cdot e_j\cdot\Psi\Big)+2\alpha Re\Big(e_i\cdot H\cdot\Psi,Q_{ij}^{\Psi}e_j\cdot\omega_{\perp}\cdot\Psi\Big)\\ &\quad +\beta^2\lambda_H^2|\Psi|^2+2\beta p\lambda_H Re\Big(e_i\cdot\omega_{\perp}\cdot\Psi,\nabla_i u\Psi\Big)\\ &\quad +2\beta q\lambda_H Re\Big(e_i\cdot\omega_{\perp}\cdot\Psi,\nabla_j u e_i\cdot e_j\cdot\Psi\Big)\\ &\quad +2\beta\lambda_H Re\Big(e_i\cdot\omega_{\perp}\cdot\Psi,Q_{ij}^{\Psi}e_j\cdot\omega_{\perp}\cdot\Psi\Big)+p^2|\nabla_i u|^2|\Psi|^2\\ &\quad +2pqRe\Big(\nabla_i u\Psi,\nabla_j u e_i\cdot e_j\cdot\Psi\Big)+2pRe\Big(\nabla_i u\Psi,Q_{ij}^{\Psi}e_j\cdot\omega_{\perp}\cdot\Psi\Big)\\ &\quad +q^2|du|^2|\Psi|^2+2qRe\Big(\nabla_u e_i\cdot e_j\cdot\Psi,Q_{ij}^{\Psi}e_j\cdot\omega_{\perp}\cdot\Psi\Big)+|Q_{ij}^{\Psi}|^2|\Psi|^2.\end{aligned} \tag{37}$$

Summing over i and using the fact that $trQ^{\Psi}=\lambda_H+\frac{1}{2}Re\Big(H\cdot\Psi,\frac{\omega_{\perp}\cdot\Psi}{|\Psi|^2}\Big)$, we have

$$\begin{aligned}|\nabla^{Q^{\Psi}}\Psi|^2 &= |\nabla\Psi|^2-2\lambda_H\alpha Re\Big(\omega_{\perp}\cdot\Psi,H\cdot\Psi\Big)-\alpha\|H\|^2|\Psi|^2-2\lambda_H^2\beta|\Psi|^2\\ &\quad -\lambda_H\beta Re\Big(H\cdot\Psi,\omega_{\perp}\cdot\Psi\Big)+2p\sum_{i=1}^{m}Re\Big(\nabla_i\Psi,\nabla_i u\Psi\Big)-2|Q^{\Psi}|^2|\Psi|^2\\ &\quad +m\alpha^2\|H\|^2|\Psi|^2+2m\alpha\beta\lambda_H Re\Big(H\cdot\Psi,\omega_{\perp}\cdot\Psi\Big)-2\alpha pRe\Big(H\cdot\Psi,du\cdot\Psi\Big)\\ &\quad +2m\alpha qRe\Big(H\cdot\Psi,du\cdot\Psi\Big)+2\alpha\lambda_H Re\Big(H\cdot\Psi,\omega_{\perp}\cdot\Psi\Big)\\ &\quad +\frac{\alpha Re\Big(H\cdot\Psi,\omega_{\perp}\cdot\Psi\Big)^2}{|\Psi|^4}|\Psi|^2+m\lambda_H^2\beta^2|\Psi|^2-2\lambda_H\beta pRe\Big(\omega_{\perp}\cdot\Psi,du\cdot\Psi\Big)\\ &\quad +2\lambda_H\beta qmRe\Big(\omega_{\perp}\cdot\Psi,du\cdot\Psi\Big)+2\lambda_H^2\beta|\Psi|^2+\lambda_H\beta Re\Big(H\cdot\Psi,\omega_{\perp}\cdot\Psi\Big)\\ &\quad +p^2|du|^2|\Psi|^2-2pq|du|^2|\Psi|^2+mq^2|du|^2|\Psi|^2+|Q^{\Psi}|^2|\Psi|^2.\end{aligned} \tag{38}$$

Taking $q = \frac{p}{m}$, and using the equality obtained in (10), we get

$$\begin{aligned}\int_M |\nabla^{Q^\Psi}\Psi|^2 &= \int_M \Big(\big(1+m\beta^2\big)\lambda_H^2|\Psi|^2 - \frac{1}{4}\big(R+R^N\big)|\Psi|^2 + \big(1-\frac{1}{m}\big)p^2|du|^2|\Psi|^2 \\ &+\big(p\Delta_u - \nabla p\nabla u\big)|\Psi|^2 + \big(1+2m\alpha\beta\big)Re\big(H\cdot\Psi,\omega_\perp\cdot\Psi\big) - |Q^\Psi|^2|\Psi|^2 \\ &+\big(\frac{1}{4}+m\alpha^2-\alpha\big)||H||^2|\Psi|^2 + \frac{\alpha Re\big(H\cdot\Psi,\omega_\perp\cdot\Psi\big)^2}{|\Psi|^4}|\Psi|^2\Big). \end{aligned} \tag{39}$$

Using the definition given in (21) and taking $\alpha = -\frac{1}{2m\beta}$ we get

$$\begin{aligned}\int_M \Big[\big(1+m\beta^2\big)\lambda_H^2|\Psi|^2 &\geq \int_M \Big(\frac{R_{p,\kappa_1,u,\Psi}}{4}|\Psi|^2 - \big(\frac{1+m\beta^2}{4m\beta^2}\big)||H||^2|\Psi|^2 - \frac{1}{2m\beta}\big(||H||^2 \\ &-\frac{Re\big(H\cdot\Psi,\omega_\perp\cdot\Psi\big)^2}{|\Psi|^4}|\Psi|^2\big) + |Q^\Psi|^2|\Psi|^2\Big)\end{aligned} \tag{40}$$

Since $||H||^2 - \frac{Re\big(H\cdot\Psi,\omega_\perp\cdot\Psi\big)^2}{|\Psi|^4}|\Psi|^2 \geq 0$, we have
we have

$$\lambda_H^2 \geq \frac{1}{4}\inf_M\Big(\frac{R_{p,\kappa_1,u,\Psi}+4|Q|^2}{1+m\beta^2} - \frac{||H||^2}{m\beta^2}\Big). \tag{41}$$

If $R_{p,\kappa_1,u,\Psi} + 4|Q|^2 > ||H||^2 > 0$ on M_Ψ, we can choose β as

$$\beta = \sqrt{\frac{||H||}{\Big(m\sqrt{R_{p,\kappa_1,u,\Psi}+4|Q|^2} - ||H||\Big)}} \text{ on } M_\Psi. \tag{42}$$

□

In the next theorem, we will consider regular conformal change of the metric $\overline{g}$ with $grad^N u = 0$, on M as in Theorem (3.3).

Theorem 3.5. *Let $M \subset \widetilde{M}$ be a compact Riemannian Spin−submanifold of a Riemannian Spin−manifold $(\widetilde{M}, g)$. Consider a non−trivial eigenspinor field $\Psi \in \Gamma(\mathbb{S})$ such that $D_H\Psi = \lambda_H\Psi$. For any regular conformal change of the metric $\overline{g} = e^{2u}g$ on M, assume that*

$$\overline{\Omega}^{Q^\Psi}_{p,\kappa_1,u,\Psi} = \{(p,\kappa_1,u)|\overline{R}_{p,\kappa_1,u,\Psi} + 4|Q^\Psi|^2 > ||H||^2 > 0\}$$

on M_Ψ. Then the following inequality is satisfied

$$\lambda_H^2 \geq \frac{1}{4}\sup_{\overline{\Omega}^{Q^\Psi}_{p,\kappa_1,u,\Psi}} \inf_{M_\Psi}\Big(\sqrt{\overline{R}_{p,\kappa_1,u,\Psi}+4|Q|^2} - ||H||\Big)^2. \tag{43}$$

where p, u are real−valued functions.

Proof. As in Theorem (3.3, applying $\overline{\Phi}$ to (40), we get

$$\int_M \big(1+m\beta^2\big)e^{-2u}\lambda_H^2|\overline{\Phi}|^2 \geq \int_M \frac{1}{4}\Big(\overline{R}_{p,\kappa_1,u,\Psi}+4|Q^\Psi|^2 - \big(\frac{1+m\beta^2)}{(m\beta^2}\big)||H||^2\Big)e^{-2u}|\overline{\Phi}|^2. \tag{44}$$

As in the proof of Theorem 3.3, we finally by taking

$$\beta = \sqrt{\frac{||H||}{m\Big(\sqrt{\overline{R}_{p,\kappa_1,u,\Psi}+4|Q^\Psi|^2} - ||H||\Big)}} \tag{45}$$

we obtained the desired result given in (43). □

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TURKISH JOURNAL OF SCIENCE http:/dergipark.gov.tr/tjos
VOLUME 5, ISSUE 1, 16–22
ISSN: 2587–0971

Radii Problems for Normalized Hyper-Bessel Function

Evrim Toklu[a], Osman Kara[b]

[a]*Department of Mathematics, Faculty of Education, Ağrı İbrahim Çeçen University, Ağrı, Turkey*
[b]*Graduate School of Natural and Applied Sciences, Ağrı İbrahim Çeçen University, Ağrı, Turkey*

Abstract. The main purpose of the present paper is to ascertain the radii of starlikeness and convexity associated with lemniscate of Bernoulli and the Janowski function, $(1 + Az)/(1 + Bz)$ for $-1 \leq B < A \leq 1$, of normalized hyper-Bessel function.

1. Introduction and the main results

Let $\mathbb{D}_r$ be the open disk $\{z \in \mathbb{C} : |z| < r\}$ with the radius $r > 0$ and let $\mathbb{D} = \mathbb{D}_1$. By $\mathcal{A}$ we mean the class of analytic functions $f : \mathbb{D}_r \to \mathbb{C}$, which satisfy the usual normalization conditions $f(0) = f'(0) - 1 = 0$. Denote by $\mathcal{S}$ the class of functions belonging to $\mathcal{A}$ which are of univalent in $\mathbb{D}_r$. A function $f \in \mathcal{A}$ is said to be starlike function if $f(\mathbb{D})$ is starlike domain with the respect to the origin. It is well known fact that various subclasses of starlike function can be unified by making use of the concept of subordination. A function $f \in \mathcal{A}$ is said to be subordinate to a function $g \in \mathcal{A}$, written as $f(z) \prec g(z)$, if there exist a Schwarz function w with $w(0) = 0$ such that $f(z) = g(w(z))$. In addition, we know that if g is a univalent function, then $f(z) \prec g(z)$ if and only if $f(0) = g(0)$ and $f(\mathbb{D}) \subset g(\mathbb{D})$. For an analytic function φ, let $\mathcal{S}^\star(\varphi)$ denote the class of all analytic functions satisfying $1 + zf'(z)/f(z) \prec \varphi(z)$. By $\mathcal{K}(\varphi)$ we mean the class of all analytic functions satisfying $1 + zf''(z)/f'(z) \prec \varphi(z)$. It is worth mentioning that these classes include respectively several famous subclasses of starlike and convex functions. For instance, the class $\mathcal{S}^\star_{\mathcal{L}} := \mathcal{S}^\star(\sqrt{1+z})$ denotes the class of lemniscate starlike functions introduced and investigated by Sokól and Stankiewich [17] and the class $\mathcal{K}_{\mathcal{L}} := \mathcal{K}(\sqrt{1+z})$ represents the class of lemniscate convex functions. Moreover, for $-1 \leq B < A \leq 1$, the class $\mathcal{S}^\star[A, B] := \mathcal{S}^\star((1 + Az)/(1 + Bz))$ is the class of Janowski starlike functions and $\mathcal{K}[A, B] := \mathcal{K}((1 + Az)/(1 + Bz))$ is the class of Janowski convex functions [14].

Given a class of functions $\mathcal{M} \subset \mathcal{A}$ and a function $f \in \mathcal{A}$, the $\mathcal{M}-$radius of the function f is the largest number r with $0 \leq r \leq 1$ such that $f_r \in \mathcal{M}$, where $f_r(z) := f(rz)/r$. If we choose $\mathcal{M} = \mathcal{S}^\star_{\mathcal{L}}$, the $\mathcal{M}-$radius of the function f, which is represented by $r^\star_{\mathcal{L}}(f)$, is called the radius of lemniscate starlikeness. It is indeed the largest r with $0 \leq r \leq 1$ such that

$$\left| \left(\frac{zf'(z)}{f(z)} \right)^2 - 1 \right| < 1 \quad (|z| < r).$$

Corresponding author: ET mail address: evrimtoklu@gmail.com ORCID: https://orcid.org/0000-0002-2332-0336, OK ORCID: https://orcid.org/0000-0003-0650-4253
Received: 24 January 2020; Accepted: 21 February 2020; Published: 30 March 2020
Keywords. (Hyper-Bessel function; Lemniscate of Bernoulli; Janowski function.)
2010 *Mathematics Subject Classification*. 30C45, 30C15, 33C10
Cited this article as: Toklu E, Kara O. Radii Problems for Normalized Hyper-Bessel Function. Turkish Journal of Science. 2020, 5(1), 16–22.

If we choose $\mathcal{M} = \mathcal{K}_{\mathcal{L}}$, the $\mathcal{M}$−radius of the function f, which is represented by $r^c_{\mathcal{L}}(f)$, is called as the radius of lemniscate convexity. It is indeed the largest r with $0 \leq r \leq 1$ such that

$$\left|\left(1+\frac{zf''(z)}{f'(z)}\right)^2 - 1\right| < 1 \quad (|z| < r).$$

If we take $\mathcal{M} = \mathcal{S}^\star[A,B]$ or $\mathcal{M} = \mathcal{K}[A,B]$, the respective $\mathcal{M}$−radii, which are represented by $r^\star_{A,B}(f)$ and $r^c_{A,B}(f)$, are called as the radii of Janowski starlikeness and Janowski convexity. These are respectively the largest r with $0 \leq r \leq 1$ such that

$$\left|\frac{(zf'(z)/f(z))-1}{A-B\left[zf'(z)/f(z)\right]}\right| < 1 \text{ and } \left|\frac{zf''(z)/f'(z)}{A-B\left[1+zf''(z)/f'(z)\right]}\right| < 1 \quad (|z| < r).$$

Recently, there has been a vivid interest on some geometric properties such as univalency, starlikeness, convexity and uniform convexity of various special functions such as hyper-Bessel, Wright, q−Bessel and Mittag-Leffler functions (see [1], [2], [3], [4],[5], [6], [7], [9], [10], [12], [18]). Fore more details on the radius problems, one may consult on [8], [13], [15], [19]. Moreover, in [16] the authors determined the radii of starlikeness and convexity associated with lemniscate of Bernoulli and the Janowski function $(1+Az)/(1+Bz)$. Motivated by the above series of papers on geometric properties of special functions, in this paper our aim is to determine the radii of lemniscate starlikeness, lemniscate convexity, Janowski starlikeness and Janowski convexity of certain normalized hyper-Bessel function.

Let us consider the hyper-Bessel function defined by

$$J_{\alpha_d}(z) = \frac{\left(\frac{z}{d+1}\right)^{\alpha_1+\alpha_2+\cdots+\alpha_d}}{\Gamma(\alpha_1+1)\ldots\Gamma(\alpha_d+1)}\,{}_0F_d\left(\begin{matrix}-\\(\alpha_d+1)\end{matrix}\;;-\left(\frac{z}{d+1}\right)^{d+1}\right), \tag{1}$$

where the notation

$${}_pF_q\left(\begin{matrix}(\beta_p)\\(\gamma_q)\end{matrix}\;;x\right) = \sum_{n\geq 0}\frac{(\beta_1)_n(\beta_2)_n\ldots(\beta_p)_n}{(\gamma_1)_n(\gamma_2)_n\ldots(\gamma_q)_n}\frac{x^n}{n!} \tag{2}$$

stands for the generalized hypergeometric function, $(\beta)_n$ is the shifted factorial (or Pochhammer's symbol) defined by $(\beta)_0 = 1$, $(\beta)_n = \beta(\beta+1)\ldots(\beta+n-1), n \geq 1$ and the contracted notation α_d is used to abbreviate the array of d parameters $\alpha_1,\ldots\alpha_d$.

By using Eqs. (1) and (2) it is easy to deduce that the function $z \mapsto J_{\alpha_d}(z)$ has the following infinite sum representation:

$$J_{\alpha_d}(z) = \sum_{n\geq 0}\frac{(-1)^n}{n!\Gamma(\alpha_1+1+n)\ldots\Gamma(\alpha_d+1+n)}\left(\frac{z}{d+1}\right)^{n(d+1)+\alpha_1+\cdots+\alpha_d}. \tag{3}$$

It is obvious that by choosing $d = 1$ and puttinq $\alpha_1 = \nu$ in (3) we obtain the classical Bessel function. The normalized hyper-Bessel function $\mathcal{J}_{\alpha_d}(z)$ defined by

$$J_{\alpha_d}(z) = \frac{\left(\frac{z}{d+1}\right)^{\alpha_1+\cdots+\alpha_d}}{\Gamma(\alpha_1+1)\ldots\Gamma(\alpha_d+1)}\mathcal{J}_{\alpha_d}(z). \tag{4}$$

By combining Eqs. (3) and (4) we obtain the following infinite sum representation:

$$\mathcal{J}_{\alpha_d}(z) = \sum_{n\geq 0}\frac{(-1)^n}{n!(\alpha_1+1)_n\ldots(\alpha_d+1)_n}\left(\frac{z}{d+1}\right)^{n(d+1)}. \tag{5}$$

Since the function $\mathcal{J}_{\alpha_d}$ does not belong to the class $\mathcal{A}$ we focus on the following normalized form

$$f_{\alpha_d}(z) = z\mathcal{J}_{\alpha_d}(z) = \sum_{n\geq 0}\frac{(-1)^n}{n!(d+1)^{n(d+1)}(\alpha_1+1)_n\ldots(\alpha_d+1)_n}z^{n(d+1)+1} \tag{6}$$

so that the function $f_{\alpha_d} \in \mathcal{A}$.

The Weierstrassian canonical product expansion of the function J_{α_d} reads as (see [11, Eq.(5.5)])

$$J_{\alpha_d}(z) = \frac{(\frac{z}{d+1})^{\alpha_1+\cdots+\alpha_d}}{\Gamma(\alpha_1+1)\ldots\Gamma(\alpha_d+1)} \prod_{n\geq 1}\left(1-\frac{z^{d+1}}{j_{\alpha_d,n}^{d+1}}\right), \tag{7}$$

where $j_{\alpha_d,n}$ is the nth positive zero of the function $\mathcal{J}_{\alpha_d}$.

In light of Eqs. (4) and (7), we get

$$\mathcal{J}_{\alpha_d}(z) = \prod_{n\geq 1}\left(1-\frac{z^{d+1}}{j_{\alpha_d,n}^{d+1}}\right) \tag{8}$$

and consequently

$$f_{\alpha_d}(z) = z\prod_{n\geq 1}\left(1-\frac{z^{d+1}}{j_{\alpha_d,n}^{d+1}}\right). \tag{9}$$

1.1. Lemniscate starlikeness and lemniscate convexity of normalized hyper-Bessel function

This section is devoted to determine the radii of lemniscate starlikeness and lemniscate convexity of the normalized hyper-Bessel function.

Theorem 1.1. *Let $\alpha_i > -1$ for $i \in \{1,2,\ldots,d\}$. Then the radius of lemniscate starlikeness $r_{\mathcal{L}}^{\star}(f_{\alpha_d})$ of the normalized hyper-Bessel function $z \mapsto f_{\alpha_d}(z) = z\mathcal{J}_{\alpha_d}(z)$ is the smallest positive root of the equation*

$$r^2\left(\mathcal{J}'_{\alpha_d}(r)\right)^2 - 2r\mathcal{J}'_{\alpha_d}(r)\mathcal{J}_{\alpha_d}(r) + 3\mathcal{J}^2_{\alpha_d}(r) = 0.$$

Proof. By means of Eq. (8) we have

$$\frac{z\mathcal{J}'_{\alpha_d}(z)}{\mathcal{J}_{\alpha_d}(z)} = -(d+1)\sum_{n\geq 1}\frac{z^{d+1}}{j_{\alpha_d,n}^{d+1}-z^{d+1}}. \tag{10}$$

Taking into consideration the normalization (6), it follows from the equation (10) that

$$\frac{zf'_{\alpha_d}(z)}{f_{\alpha_d}(z)} = 1+\frac{z\mathcal{J}'_{\alpha_d}(z)}{\mathcal{J}_{\alpha_d}(z)} = 1-(d+1)\sum_{n\geq 1}\frac{z^{d+1}}{j_{\alpha_d,n}^{d+1}-z^{d+1}}. \tag{11}$$

In light of Eq. (11), we get

$$\begin{aligned}\left|\left(\frac{zf'_{\alpha_d}(z)}{f_{\alpha_d}(z)}\right)^2-1\right| &\leq \left((d+1)\sum_{n\geq 1}\frac{|z|^{d+1}}{j_{\alpha_d,n}^{d+1}-|z|^{d+1}}\right)\left(2+(d+1)\sum_{n\geq 1}\frac{|z|^{d+1}}{j_{\alpha_d,n}^{d+1}-|z|^{d+1}}\right)\\ &= \left(\frac{|z|\,f'_{\alpha_d}(|z|)}{f_{\alpha_d}(|z|)}\right)^2-4\left(\frac{|z|\,f'_{\alpha_d}(|z|)}{f_{\alpha_d}(|z|)}\right)+3.\end{aligned} \tag{12}$$

Suppose that $r^{\star}$ is the smallest positive root of the equation

$$\left(\frac{rf'_{\alpha_d}(r)}{f_{\alpha_d}(r)}\right)^2-4\left(\frac{rf'_{\alpha_d}(r)}{f_{\alpha_d}(r)}\right)+2=0,$$

then the inequality

$$\left|\left(\frac{rf'_{\alpha_d}(r)}{f_{\alpha_d}(r)}\right)^2-1\right|<1$$

holds true for $|z| < r^{\star}$. By virtue of Eq. (11) we deduce that the zeros of the above equation coincide with those of equation

$$r^2\left(\mathcal{J}'_{\alpha_d}(r)\right)^2 - 2r\mathcal{J}'_{\alpha_d}(r)\mathcal{J}_{\alpha_d}(r) + 3\mathcal{J}^2_{\alpha_d}(r) = 0. \tag{13}$$

In order to end the proof, we need to show that equation (13) has a unique root in $(0, j_{\alpha_d,1})$. The function $u_{\alpha_d}(z) : (0, j_{\alpha_d,1}) \to \mathbb{R}$, defined by

$$u_{\alpha_d}(z) = \left(\frac{rf'_{\alpha_d}(r)}{f_{\alpha_d}(r)}\right)^2 - 4\left(\frac{rf'_{\alpha_d}(r)}{f_{\alpha_d}(r)}\right) + 2,$$

is continuous and strictly increasing since

$$u'_{\alpha_d}(z) = 2\sum_{n\geq 1}\frac{(d+1)^2 r^d j^{d+1}_{\alpha_d,n}}{\left(j^{d+1}_{\alpha_d,n} - r^{d+1}\right)^2}\left(1 + (d+1)\sum_{n\geq 1}\frac{r^{d+1}}{j^{d+1}_{\alpha_d,n} - r^{d+1}}\right) > 0.$$

Observe also that

$$\lim_{r\searrow 0} u_{\alpha_d}(z) = -1 < 0 \quad \text{and} \quad \lim_{r\nearrow j_{\alpha_d,1}} u_{\alpha_d}(z) = \infty.$$

Due to the Intermediate Value Theorem, we conclude that the equation $u_{\alpha_d}(z) = 0$ has a unique root in $(0, j_{\alpha_d,1})$. This means that the lemniscate starlike radius of the function $f_{\alpha_d}(z)$, say $r^{\star}_{\mathcal{L}}(f_{\alpha_d})$, is the unique zero of $u_{\alpha_d}(z)$ in $(0, j_{\alpha_d,1})$ or of the equation (13). □

Theorem 1.2. *Let $\alpha_i > -1$ for $i \in \{1, 2, \ldots, d\}$. Then the radius of lemniscate convexity $r^{c}_{\mathcal{L}}(f_{\alpha_d})$ of the normalized hyper-Bessel function $z \mapsto f_{\alpha_d}(z) = z\mathcal{J}_{\alpha_d}(z)$ is the smallest positive root of the equation*

$$\left(\frac{r^2\mathcal{J}''_{\alpha_d}(r) + 2r\mathcal{J}'_{\alpha_d}(r)}{\mathcal{J}_{\alpha_d}(r) + r\mathcal{J}'_{\alpha_d}(r)}\right)^2 - 2\left(\frac{r^2\mathcal{J}''_{\alpha_d}(r) + 2r\mathcal{J}'_{\alpha_d}(r)}{\mathcal{J}_{\alpha_d}(r) + r\mathcal{J}'_{\alpha_d}(r)}\right) - 1 = 0.$$

Proof. From [6, Theorem 1] we have

$$f'_{\alpha_d}(z) = \prod_{n\geq 1}\left(1 - \frac{z^{d+1}}{\gamma^{d+1}_{\alpha_d,n}}\right), \tag{14}$$

where $\gamma_{\alpha_d,n}$ is the nth positive real zero of the function $f'_{\alpha_d}(z)$. With the aid of Eq. (14) it is easily seen that

$$1 + \frac{zf''_{\alpha_d}(z)}{f'_{\alpha_d}(z)} = 1 - (d+1)\sum_{n\geq 1}\frac{z^{d+1}}{\gamma^{d+1}_{\alpha_d,n} - z^{d+1}}. \tag{15}$$

By making use of (15) and triangle inequality for $|z| < \gamma_{\alpha_d,1}$, we get

$$\left|\left(1 + \frac{zf''_{\alpha_d}(z)}{f'_{\alpha_d}(z)}\right)^2 - 1\right| \leq \left(\frac{|z|\,f''_{\alpha_d}(|z|)}{f'_{\alpha_d}(|z|)}\right)^2 - 2\left(\frac{|z|\,f''_{\alpha_d}(|z|)}{f'_{\alpha_d}(|z|)}\right).$$

Hence, we deduce that the lemniscate convex radius of f_{α_d}, $r^{c}_{\mathcal{L}}(f_{\alpha_d})$, is the unique positive root of the equation

$$\left(\frac{rf''_{\alpha_d}(r)}{f'_{\alpha_d}(r)}\right)^2 - 2\left(\frac{rf''_{\alpha_d}(r)}{f'_{\alpha_d}(r)}\right) - 1 = 0 \tag{16}$$

which implies

$$\left(\frac{r^2\mathcal{J}''_{\alpha_d}(r) + 2r\mathcal{J}'_{\alpha_d}(r)}{\mathcal{J}_{\alpha_d}(r) + r\mathcal{J}'_{\alpha_d}(r)}\right)^2 - 2\left(\frac{r^2\mathcal{J}''_{\alpha_d}(r) + 2r\mathcal{J}'_{\alpha_d}(r)}{\mathcal{J}_{\alpha_d}(r) + r\mathcal{J}'_{\alpha_d}(r)}\right) - 1 = 0.$$

Now, we need to show that the above equation has a unique root in $(0, \gamma_{\alpha_d,1})$. To do this, let us consider the function $v_{\alpha_d} : (0, \gamma_{\alpha_d,1}) \to \mathbb{R}$ defined by

$$v_{\alpha_d}(z) = \left(\frac{rf''_{\alpha_d}(r)}{f'_{\alpha_d}(r)}\right)^2 - 2\left(\frac{rf''_{\alpha_d}(r)}{f'_{\alpha_d}(r)}\right) - 1.$$

It is obvious that the function is continous and strictly increasing, since

$$v'_{\alpha_d}(r) > (d+1)^3 \sum_{n\geq 1} \frac{r^d \gamma_{\alpha_d,n}^{d+1}}{\left(\gamma_{\alpha_d,n}^{d+1} - r^{d+1}\right)^2} \sum_{n\geq 1} \frac{r^{d+1}}{\gamma_{\alpha_d,n}^{d+1} - r^{d+1}} > 0.$$

Observe also that

$$\lim_{r\searrow 0} v_{\alpha_d}(r) = -1 < 0 \text{ and } \lim_{r\nearrow \gamma_{\alpha_d,1}} v_{\alpha_d}(r) = \infty.$$

Thus, we deduce that the root is unique in $(0, \gamma_{\alpha_d,1})$. This means that the lemniscate convex radius of f_{α_d} is the unique root of (16) in $(0, \gamma_{\alpha_d,1})$. □

1.2. Janowski starlikeness and Janowski convexity of normalized hyper-Bessel function

In this section, we turn our attention to determining the radii of Janowski starlikeness and Janowski convexity of the normalized hyper-Bessel function $f_{\alpha_d}(z)$.

Theorem 1.3. *Let $\alpha_i > -1$ for $i \in \{1, 2, \ldots, d\}$. Then the Janowski starlikeness radius $r^\star_{A,B}(f_{\alpha_d})$ is the smallest positive root of the equation*

$$\frac{r\mathcal{J}'_{\alpha_d}(r)}{\mathcal{J}_{\alpha_d}(r)} + \frac{A-B}{1+|B|} = 0.$$

Proof. In order to determine the radius of Janowski starlikeness of the normalization $f_{\alpha_d}(z)$ of $\mathcal{J}_{\alpha_d}(z)$, we need to find a real number $r^\star$ such that

$$\left|\frac{\frac{zf'_{\alpha_d}(z)}{f_{\alpha_d}(z)} - 1}{A - B\frac{zf'_{\alpha_d}(z)}{f_{\alpha_d}(z)}}\right| < 1 \quad (|z| < r^\star).$$

In light of Eq. (11) and by using triangle inequality it follows that the inequality

$$\left|\frac{\frac{zf'_{\alpha_d}(z)}{f_{\alpha_d}(z)} - 1}{A - B\frac{zf'_{\alpha_d}(z)}{f_{\alpha_d}(z)}}\right| \leq \frac{(d+1)\sum_{n\geq 1} \frac{|z|^{d+1}}{j_{\alpha_d,n}^{d+1} - |z|^{d+1}}}{A - B - |B|(d+1)\sum_{n\geq 1} \frac{|z|^{d+1}}{j_{\alpha_d,n}^{d+1} - |z|^{d+1}}} \quad (|z| < j_{\alpha_d,1})$$

holds for $\alpha_i > -1$, $i \in \{1, 2, \ldots, d\}$ with the equality at $z = |z| = r$. With the aid of Eq. (11) the above inequality yields

$$\left|\frac{\frac{zf'_{\alpha_d}(z)}{f_{\alpha_d}(z)} - 1}{A - B\frac{zf'_{\alpha_d}(z)}{f_{\alpha_d}(z)}}\right| \leq \frac{1 - \frac{|z|f'_{\alpha_d}(|z|)}{f_{\alpha_d}(|z|)}}{A - B + |B|\left(\frac{|z|f'_{\alpha_d}(|z|)}{f_{\alpha_d}(|z|)} - 1\right)}. \tag{17}$$

Hence we deduce that for $|z| < r^\star$, Janowski starlike radius $r^\star_{A,B}(f_{\alpha_d})$ is the unique positive root of the equation

$$\frac{1 - \frac{rf'_{\alpha_d}(r)}{f_{\alpha_d}(r)}}{A - B + |B|\left(\frac{rf'_{\alpha_d}(r)}{f_{\alpha_d}(r)} - 1\right)} - 1 = 0,$$

which implies

$$\frac{rf'_{\alpha_d}(r)}{f_{\alpha_d}(r)} = 1 - \frac{A-B}{1+|B|}. \tag{18}$$

We need to show that the above equation (that is Eq. (18)) has a unique root in $(0, j_{\alpha_d,1})$. Let us consider the function $u_{\alpha_d} : (0, j_{\alpha_d,1}) \to \mathbb{R}$ defined by

$$u_{\alpha_d}(r) = \frac{rf'_{\alpha_d}(r)}{f_{\alpha_d}(r)} - 1 + \frac{A-B}{1+|B|}.$$

It is clear that the above mentioned function is continuous and strictly decreasing, since

$$u'_{\alpha_d}(r) = -(d+1)^2 \sum_{n\geq 1} \frac{r^d j_{\alpha_d,n}^{d+1}}{\left(j_{\alpha_d,n}^{d+1} - r^{d+1}\right)^2} < 0.$$

Observe also that

$$\lim_{r\searrow 0} u_{\alpha_d}(r) = \frac{A-B}{1+|B|} > 0 \text{ and } \lim_{r\nearrow j_{\alpha_d,1}} u_{\alpha_d}(r) = -\infty.$$

Therefore, the Intermediate Value Theorem ensures the existence of the unique root of $u_{\alpha_d}(r) = 0$ in $(0, j_{\alpha_d,1})$. That is, the Janowski starlikeness radius $r^{\star}_{A,B}(f_{\alpha_d})$ is the unique root of equation (18) in $(0, j_{\alpha_d,1})$. □

Theorem 1.4. *Let $\alpha_i > -1$ for $i \in \{1, 2, \ldots, d\}$. Then the Janowski convexity radius $r^{c}_{A,B}(f_{\alpha_d})$ is the smallest positive root of the equation*

$$\frac{r^2\mathcal{J}''_{\alpha_d}(r) + 2r\mathcal{J}'_{\alpha_d}(r)}{\mathcal{J}_{\alpha_d}(r) + r\mathcal{J}'_{\alpha_d}(r)} + \frac{A-B}{1+|B|} = 0.$$

Proof. In order that the function f_{α_d} is Janowski convex in the disk $\{z : |z| < r\}$, the inequality

$$\left|\frac{\frac{zf''_{\alpha_d}(z)}{f'_{\alpha_d}(z)}}{A - B\left(1 + \frac{zf''_{\alpha_d}(z)}{f'_{\alpha_d}(z)}\right)}\right| < 1$$

must be valid for $|z| < r$. It is easily seen that the function f_{α_d} satisfies the inequality

$$\left|\frac{\frac{zf''_{\alpha_d}(z)}{f'_{\alpha_d}(z)}}{A - B\left(1 + \frac{zf''_{\alpha_d}(z)}{f'_{\alpha_d}(z)}\right)}\right| \leq \frac{(d+1)\sum_{n\geq 1} \frac{z^{d+1}}{\gamma_{\alpha_d,n}^{d+1} - z^{d+1}}}{A - B - |B|\left((d+1)\sum_{n\geq 1} \frac{z^{d+1}}{\gamma_{\alpha_d,n}^{d+1} - z^{d+1}}\right)}$$

for $|z| < \gamma_{\alpha_d,1}$ with the equality at $z = |z| = r$. The above inequality implies that

$$\left|\frac{\frac{zf''_{\alpha_d}(z)}{f'_{\alpha_d}(z)}}{A - B\left(1 + \frac{zf''_{\alpha_d}(z)}{f'_{\alpha_d}(z)}\right)}\right| \leq \frac{-\frac{|z|f''_{\alpha_d}(|z|)}{f'_{\alpha_d}(|z|)}}{A - B + |B|\frac{|z|f''_{\alpha_d}(|z|)}{f'_{\alpha_d}(|z|)}}. \tag{19}$$

In this case, we say that the Janowski convexity radius $r^{c}_{A,B}(f_{\alpha_d})$ is the smallest positive root of the equation

$$\frac{rf''_{\alpha_d}(r)}{f'_{\alpha_d}(r)} + \frac{A-B}{1+|B|} = 0. \tag{20}$$

In order to finish the proof, we must show that the above mentioned equation (that is (20)) has a unique root in $(0, \gamma_{\alpha_d,1})$. To reach our aim, we consider the function $v_{\alpha_d} : (0, \gamma_{\alpha_d,1}) \to \mathbb{R}$ defined by

$$v_{\alpha_d}(r) = \frac{r f''_{\alpha_d}(r)}{f'_{\alpha_d}(r)} + \frac{A-B}{1+|B|}.$$

It is obvious that the function v_{α_d} is strictly decreasing as

$$v'_{\alpha_d}(r) = -(d+1)^2 \sum_{n\geq 1} \frac{r^d \gamma_{\alpha_d,n}^{d+1}}{\left(\gamma_{\alpha_d,n}^{d+1} - r^{d+1}\right)^2} < 0.$$

Observe also that

$$\lim_{r\searrow 0} v_{\alpha_d}(r) = \frac{A-B}{1+|B|} > 0 \text{ and } \lim_{r\nearrow \gamma_{\alpha_d,1}} v_{\alpha_d}(r) = -\infty.$$

Therefore, by monotonicity of the function v_{α_d}, it is obvious that the function f_{α_d} is Janowski convex for $|z| < r_1$, where r_1 is the unique positive root of equation (20) in $(0, \gamma_{\alpha_d,1})$. □

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TURKISH JOURNAL OF SCIENCE
VOLUME 5, ISSUE 1, 23–27
ISSN: 2587–0971

http:/dergipark.gov.tr/tjos

On Some Inequalities for Product of Different Kinds of Convex Functions

Mustafa Gürbüz[a], M. Emin Özdemir[b]

[a]*Department of Elementary Mathematics, Faculty of Education, Agri Ibrahim Cecen University, Agri, Turkey.*
[b]*Department of Mathematics Education, Faculty of Education, Uludağ University, Görükle Campus, Bursa, Turkey.*

Abstract. In this paper some new inequalities for product of different kinds of convex functions are obtained. To put forward new results, basic definitions of convex functions are considered in different ways and fairly elementary analysis is used.

1. Introduction

It is seen that, in development of mathematics and many other applied sicences, inequalities are very important. Inequalities offer so wide and effective field of study and application. To meet the application area needs of inequalities, there has been a constantly increasing interest in such an area of research. As it is known that convex functions and various forms of it are closely related with integral inequalities, many researchers have revealed many classes of convex functions in line with the growing interest in convexity theory. Interested researchers can see various generalizations and applications of convex functions in [1], [3], [4], [6]-[9] and references therein.

The function f defined from $[a,b]$ which is a subset of $\mathbb{R}$ and defined to $\mathbb{R}$, is namely convex function if it holds the inequality

$$f\left(\zeta r+(1-\zeta)s\right)\leq\zeta f\left(r\right)+(1-\zeta)f\left(s\right)$$

for all $\zeta\in[0,1]$ and $r,s\in[a,b]$. Also f is said to be concave if $(-f)$ is convex.

E. K. Godunova and V. I. Levin established the $Q(I)$ class of function in 1985 which is defined in the following (see [2]):

Definition 1.1. *Let f be a nonnegative function defined from I to $\mathbb{R}$ belongs to the class $Q(I)$ if it satisfies the inequality*

$$f(\zeta r+(1-\zeta)s)\leq\frac{f(r)}{\zeta}+\frac{f(s)}{1-\zeta}$$

for all $\zeta\in(0,1)$ and $r,s\in I$.

Corresponding author: MG, mail address: mgurbuz@agri.edu.tr ORCID: https://orcid.org/0000-0002-7092-4298, MEÖ ORCID: https://orcid.org/0000-0002-5992-094X

Received: 13 February 2020; Accepted: 20 March 2020; Published: 30 March 2020
Keywords. $h-$convex functions, $Q(I)$ class of functions, convex function, $(\alpha,m)-$convex functions, integral inequalities
2010 *Mathematics Subject Classification*. 26D07, 52A40.
Cited this article as: Gürbüz M, Özdemir ME. On Some Inequalities for Product of Different Kinds of Convex Functions. Turkish Journal of Science. 2020, 5(1), 23–27.

The following class of functions are introduced by S. Varošanec in [10].

Definition 1.2. *Let h be a non-negative function defined from J (a subset of $\mathbb{R}$ and a subset of $(0,1)$) to $\mathbb{R}$ and $h \not\equiv 0$. A non-negative function f defined from I (a subset of $\mathbb{R}$) to $\mathbb{R}$ is said to be an $h-$convex function, or that f belongs to the class $SX(h,I)$, if we have*

$$f(\zeta r + (1-\zeta)s) \leq h(\zeta) f(r) + h(1-\zeta) f(s) \tag{1}$$

for all $r,s \in I$, $\zeta \in (0,1]$.

f is said to be $h-$concave, i.e. $f \in SV(h,I)$ if the inequality (1) is reversed.

On the other hand, Miheşan, introduced the following class of convexity in [5] which generalizes many types of function types.

Definition 1.3. *Let f be a function defined from $[0,b]$ to $\mathbb{R}$. If for every $\zeta \in [0,1]$, $r,s \in [0,b]$ and $(\alpha,m) \in [0,1]^2$ we have*

$$f(\zeta r + m(1-\zeta)s) \leq \zeta^{\alpha} f(r) + m(1-\zeta^{\alpha}) f(s)$$

then f is said to be $(\alpha,m)-$convex function.

Throughout we will use $M(r,s)$ and $N(r,s)$ in the meaning of "$f(r)g(r) + f(s)g(s)$" and "$f(r)g(s) + f(s)g(r)$" respectively.

In this paper, we obtained some new inequalities using some convex function classes given above. Also we used fairly elementary analysis to obtain new results.

2. Main Results

Theorem 2.1. *Let $f,g : I \to \mathbb{R}^+$ be functions and $r,s \in I$. If $f \in SX(h,I)$, $g \in Q(I)$ with $h \in L_1([0,1])$, for all $t \in (0,1)$ we have*

$$f\left(\frac{r+s}{2}\right) g\left(\frac{r+s}{2}\right) \leq 2h\left(\frac{1}{2}\right)[M(r,s) + N(r,s)] \int_0^1 \left(\frac{h(t) + h(1-t)}{t(1-t)}\right) dt.$$

Proof. Since we can write $\frac{r+s}{2} = \frac{tr+(1-t)s}{2} + \frac{(1-t)r+ts}{2}$ for all $t \in (0,1)$ and $f \in SX(h,I)$, $g \in Q(I)$ we have

$$\begin{aligned}
& f\left(\frac{r+s}{2}\right) g\left(\frac{r+s}{2}\right) \\
= \; & f\left(\frac{tr+(1-t)s}{2} + \frac{(1-t)r+ts}{2}\right) g\left(\frac{tr+(1-t)s}{2} + \frac{(1-t)r+ts}{2}\right) \\
\leq \; & 2h\left(\frac{1}{2}\right)[f(tr+(1-t)s) + f((1-t)r+ts)] \\
& \times [g(tr+(1-t)s) + g((1-t)r+ts)] \\
\leq \; & 2h\left(\frac{1}{2}\right)[h(t) f(r) + h(1-t) f(s) + h(1-t) f(r) + h(t) f(s)] \\
& \times \left[\frac{g(r)}{t} + \frac{g(s)}{1-t} + \frac{g(r)}{1-t} + \frac{g(s)}{t}\right] \\
= \; & 2h\left(\frac{1}{2}\right)[h(t)(f(r)+f(s)) + h(1-t)(f(r)+f(s))] \\
& \times \left[\frac{(1-t)(g(r)+g(s)) + t(g(r)+g(s))}{t(1-t)}\right]
\end{aligned}$$

$$\begin{aligned} &= 2h\left(\frac{1}{2}\right)(f(r)+f(s))(h(t)+h(1-t))\left[\frac{g(r)+g(s)}{t(1-t)}\right] \\ &= 2h\left(\frac{1}{2}\right)(f(r)+f(s))(g(r)+g(s))\left[\frac{h(t)+h(1-t)}{t(1-t)}\right] \\ &= 2h\left(\frac{1}{2}\right)\left[\frac{h(t)+h(1-t)}{t(1-t)}\right][M(r,s)+N(r,s)]. \end{aligned}$$

By integrating both sides respect to t over $[0,1]$, the proof is completed. □

Theorem 2.2. *Let $f, g : [r,s] \to \mathbb{R}^+$ be convex functions on $[r,s]$. Then we get*

$$\begin{aligned} &f\left(\frac{r+s}{2}\right)g\left(\frac{r+s}{2}\right)+\left(\frac{f(r)+f(s)}{2}\right)g\left(\frac{r+s}{2}\right)+\left(\frac{g(r)+g(s)}{2}\right)f\left(\frac{r+s}{2}\right) \\ \leq\ &\frac{3}{4}[M(r,s)+N(r,s)]. \end{aligned}$$

Proof. Using Hermite-Hadamard inequality, for all $t \in [0,1)$ we have

$$\begin{aligned} t\frac{f(r)+f(s)}{2}+(1-t)f\left(\frac{r+s}{2}\right) &\leq \frac{f(r)+f(s)}{2} \\ t\frac{g(r)+g(s)}{2}+(1-t)g\left(\frac{r+s}{2}\right) &\leq \frac{g(r)+g(s)}{2}. \end{aligned}$$

Multiplying the above inequalities side by side we get

$$\begin{aligned} &t^2\frac{(f(r)+f(s))(g(r)+g(s))}{4}+(1-t)^2 f\left(\frac{r+s}{2}\right)g\left(\frac{r+s}{2}\right) \\ &+t(1-t)\left[\left(\frac{f(r)+f(s)}{2}\right)g\left(\frac{r+s}{2}\right)+\left(\frac{g(r)+g(s)}{2}\right)f\left(\frac{r+s}{2}\right)\right] \\ \leq\ &\frac{(f(r)+f(s))(g(r)+g(s))}{4}. \end{aligned}$$

Then we have

$$\begin{aligned} &(1-t)^2 f\left(\frac{r+s}{2}\right)g\left(\frac{r+s}{2}\right) \\ &+t(1-t)\left[\left(\frac{f(r)+f(s)}{2}\right)g\left(\frac{r+s}{2}\right)+\left(\frac{g(r)+g(s)}{2}\right)f\left(\frac{r+s}{2}\right)\right] \\ \leq\ &\left(1-t^2\right)\frac{(f(r)+f(s))(g(r)+g(s))}{4}. \end{aligned}$$

Thus we get

$$\begin{aligned} &(1-t)f\left(\frac{r+s}{2}\right)g\left(\frac{r+s}{2}\right) \\ &+t\left[\left(\frac{f(r)+f(s)}{2}\right)g\left(\frac{r+s}{2}\right)+\left(\frac{g(r)+g(s)}{2}\right)f\left(\frac{r+s}{2}\right)\right] \\ \leq\ &(1+t)\frac{(f(r)+f(s))(g(r)+g(s))}{4}. \end{aligned} \tag{2}$$

By integrating both sides of (2) respect to t from 0 to 1, the desired inequality is obtained. □

Corollary 2.3. *Since f is chosen as convex function in Teorem 2.2, by using the inequality*

$$f\left(\frac{r+s}{2}\right) \leq \frac{f(r)+f(s)}{2}$$

which is a part of Hadamard's inequality we have

$$f\left(\frac{r+s}{2}\right) g\left(\frac{r+s}{2}\right) \leq \frac{1}{4}\left[M(r,s)+N(r,s)\right].$$

Theorem 2.4. *Let $f, g : [r,s] \to \mathbb{R}^+$ $(0 \leq r < s)$ be (α_1, m) −convex and (α_2, m) −convex functions on $[r,s]$ respectively. If $f, g \in L_1[r,s]$, for all $x \in [r,s]$ and $(\alpha_1, m), (\alpha_2, m) \in (0,1]^2$ we have*

$$\begin{aligned} & \frac{1}{s-r}\int_r^s f(x)\, g(r+s-x)\, dx \\ \leq\ & \beta(\alpha_1+1, \alpha_2+1)\left((fg)(r) + m^2 (fg)\left(\frac{s}{m}\right)\right) \\ & + \frac{m}{\alpha_1+\alpha_2+1}\left(f(r)\, g\left(\frac{s}{m}\right) + f\left(\frac{s}{m}\right) g(r)\right). \end{aligned}$$

Proof. Since f and g are (α_1, m) −convex and (α_2, m) −convex on $[r,s]$ respectively, for all $t \in [0,1]$ we can write

$$\begin{aligned} f(tr+(1-t)s) &\leq t^{\alpha_1} f(r) + m(1-t^{\alpha_1}) f\left(\frac{s}{m}\right) \\ g((1-t)r+ts) &\leq (1-t^{\alpha_2}) g(r) + m t^{\alpha_2} g\left(\frac{s}{m}\right). \end{aligned}$$

By multiplying the above inequalities side by side we get

$$\begin{aligned} & f(tr+(1-t)s)\, g((1-t)r+ts) \\ \leq\ & t^{\alpha_1}(1-t^{\alpha_2})(fg)(r) + m^2(1-t^{\alpha_1}) t^{\alpha_2} (fg)\left(\frac{s}{m}\right) \\ & + m\left[t^{\alpha_1+\alpha_2} f(r)\, g\left(\frac{s}{m}\right) + (1-t^{\alpha_1})(1-t^{\alpha_2}) f\left(\frac{s}{m}\right) g(r)\right]. \end{aligned}$$

Moreover, for all $t_1, t_2 \in [0,1]$ and $\alpha \in (0,1]$ we have

$$\left|t_1^{\alpha} - t_2^{\alpha}\right| \leq |t_1 - t_2|^{\alpha}.$$

So it is clear that

$$1 - t^{\alpha_1} = 1^{\alpha_1} - t^{\alpha_1} \leq (1-t)^{\alpha_1}$$

and

$$1 - t^{\alpha_2} = 1^{\alpha_2} - t^{\alpha_2} \leq (1-t)^{\alpha_2}.$$

With the help of above inequalities we get

$$\begin{aligned} & f(tr+(1-t)s)\, g((1-t)r+ts) \\ \leq\ & t^{\alpha_1}(1-t)^{\alpha_2}(fg)(r) + m^2(1-t)^{\alpha_1} t^{\alpha_2} (fg)\left(\frac{s}{m}\right) \\ & + m\left[t^{\alpha_1+\alpha_2} f(r)\, g\left(\frac{s}{m}\right) + (1-t)^{\alpha_1}(1-t)^{\alpha_2} f\left(\frac{s}{m}\right) g(r)\right] \\ =\ & t^{\alpha_1}(1-t)^{\alpha_2}(fg)(r) + m^2(1-t)^{\alpha_1} t^{\alpha_2} (fg)\left(\frac{s}{m}\right) \\ & + m\left[t^{\alpha_1+\alpha_2} f(r)\, g\left(\frac{s}{m}\right) + (1-t)^{\alpha_1+\alpha_2} f\left(\frac{s}{m}\right) g(r)\right]. \end{aligned} \tag{3}$$

Integrating both sides of (3) respect to t over $[0,1]$ we have

$$\begin{aligned}
&\int_0^1 f(tr+(1-t)s)\, g((1-t)r+ts)\, dt \\
\leq\ & \beta(\alpha_1+1,\alpha_2+1)\left((fg)(r)+m^2(fg)\left(\frac{s}{m}\right)\right) \\
&+\frac{m}{\alpha_1+\alpha_2+1}\left(f(r)\, g\left(\frac{s}{m}\right)+f\left(\frac{s}{m}\right)g(r)\right).
\end{aligned}$$

where

$$\beta(r,s)=\int_0^1 t^{r-1}(1-t)^{s-1}dt, \qquad r,s>0$$

and

$$\beta(r,s)=\beta(s,r).$$

So the proof is completed. □

Acknowledgement

This paper is taken from first author's Ph.D. thesis (see [3]).

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TURKISH JOURNAL OF SCIENCE
VOLUME 5, ISSUE 1, 28–33
ISSN: 2587–0971

http:/dergipark.gov.tr/tjos

Barbaloin Attenuates Oxidative Testicular Injury Induced by Ischemia Reperfusion via Antioxidant Effects

Ömer Topdağı[a], Ayhan Tanyeli[b], Fazile Nur Ekinci Akdemir[c], Ersen Eraslan[d], Mustafa Can Güler[e]

[a]*Department of Internal Medicine, Faculty of Medicine, Atatürk University, Erzurum, Turkey*
[b]*Department of Physiology, Faculty of Medicine, Atatürk University, Erzurum, Turkey*
[c]*Department of Nutrition and Dietetics, High School of Health, Ağrı İbrahim Çeçen University, Ağrı, Turkey*
[d]*Department of Physiology, Faculty of Medicine, Bozok University, Yozgat, Turkey*
[e]*Department of Physiology, Faculty of Medicine, Atatürk University, Erzurum, Turkey*

Abstract. The purpose of this research is to examine the protective effects of barbaloin on testes injury induced by ischemia reperfusion. In the experimental stage of our research, the rats were assigned to 4 groups. Our groups are scheduled as follows; sham, ischemia reperfusion, ischemia and reperfusion + DMSO and ischemia reperfusion+barbaloin. Some oxidant and antioxidant parameters were evaluated in testes tissues obtained at the end of the experiment. In ischemia reperfusion group it was found out that the oxidant parameters increased and antioxidant parameters decreased but on the contrary, the antioxidant parameters increased and oxidant parameters decreased in the treatment group. These results demonstrated that barbaloin administration performed a protective effect against oxidative testes injury induced by ischemia reperfusion.

1. Introduction

Among adolescents and infants, testes torsion is a critical urological emergency and may cause testicular necrosis and complete damage. At the early stages, diagnosis and treatment are important points to restrain infertility because of its 1 in 158 incidence by 25 years old age in males [1, 2]. As an urological emergency health condition, testes torsion often takes place in adolescents and children. A seasonable and effective cure must be applied to able to maintain testicular function. Detorsion of twisted testes is a part of the treatment. Testicular torsion detorsion (T/D) pathophysiology is described as acute ischemia reperfusion (I/R) injury [3, 4]. When the reactive oxygen species (ROS) are produced excessively, this may be a major reason of inducing I/R injury during testicular T/D [5, 6]. Spermatic cord T/D, a testicular I/R state, leads to testicular injury [7]. If it is untreated, testicular torsion may reduce fertility [8]. In rat spermatic cord torsion model, permanent spermatogenesis cessation occured due to germ cell spesific oxidative stress, inflammation and apoptosis [9]. Recent studies about oxidative stress and free oxygen radicals arising from testicular torsion demonstrated that antioxidants prevents ROS- induced testes injury [8, 10–12]. As carrying high economic value, Aloe is a *A Liliaceae* family member plant [13, 14]. Aloe plays role in immunity

Corresponding author: FNEA mail address: fneakdemir@agri.edu.tr ORCID:https://orcid.org/0000-0001-9585-3169, ÖT ORCID:https://orcid.org/0000-0002-9690-4447, AT ORCID:https://orcid.org/0000-0002-0095-0917, EE ORCID:https://orcid.org/0000-0003-2424-2269, MCG ORCID:https://orcid.org/0000-0001-8588-1035
Received: 15 January 2020 ; Accepted: 25 March 2020; Published: 30 March 2020
Keywords. (Ischemia reperfusion, barbaloin, testes, oxidative stress, rat)
Cited this article as: Topdağı Ö, Tanyeli A, Ekinci Akdemir FN, Eraslan E, Güler MC. Barbaloin Attenuates Oxidative Testicular Injury Induced by Ischemia Reperfusion via Antioxidant Effects. Turkish Journal of Science. 2020, 5(1), 28-33.

and acts via anti-oxidation and anti-inflammatory effects [13], and widely preferred in clinic [15]. Barbaloin, main ingredient in aloe, catches on from day to day [16]. Barbaloin (10-b-D-glycopyranosyl-1,8-dihydroxy-3-hydroxymethyl-9 (10H) -anthracenor) is the main compound of Traditional Chinese medicine aloe vera from the plant liliaceous. According to previous studies, many pharmacological effects of barbaloin have been shown, including anti-oxidant, anti-inflammatory and anti-tumor effects [17]. Different agents with anti-inflammatory, antioxidant and radical scavenging features, and it has been reported that they have beneficial effects in alleviation or elimination of I/R injuries [18–20]. This study was planned to search the protective effect of barbaloin against testicular oxidative damage induced by I/R.

2. Methods

2.1. Animals and Ethical Approval

The present study was admitted by Atatürk University Experimental Animal Ethics Committee (2019-66). Experiments of this study were performed at Atatürk University Experimental Animals Research and Application Center (ATADEM). Wistar albino male rats weighing 270-300 g, acquired from Atatürk University Experimental Animal Research and Application Center. Rats were housed in cages in laboratory conditions such as temperature of 22 ± 2 $^{\circ}C$, humidity of 55 ± 5% , and 12 light/12 darkness. Rats were fed with standard rat feed, and supplied drinking water. All animals were food deprived for 12 hours before the experiment, but were allowed to drink water.

2.2. Groups and Experimental Design

32 male rats were allocated into 4 experimental groups (n=8): Sham group; only laparotomy was made under anaesthesia. Ischemia reperfusion (I/R) group; 2 hours ischemia and 2 hours reperfusion was performed. In the midline field of lower abdomen, a longitudinal and 2 cm length incision was applied. After a small peritoneal incision, spermatic cords and testes were located. In I/R group, bilateral testes ischemia was carried out via a vascular clamp by applying below the testes. Ischemia reperfusion+DMSO (I/R+DMSO) group; 2 hours ischemia was fullfilled and DMSO 0.3 ml administered as intraperitoneal (i.p.). Then, 2 hours reperfusion stage was done. Ischemia reperfusion+barbaloin (I/R+barbaloin) group; 2 hours ischemia was done. Following, 20 mg/kg i.p. application of barbaloin (Sigma-Aldrich Co, USA) and then 2 hours reperfusion was carried out. All experimental steps were performed under deep anaesthesia by administration of ketamine-xylazine 60/10 mg/kg i.p. (Ketalar 50 mg/ml Pfizer İlaçları Limited Şirketi, İstanbul and Xylazine Rompun BAYER İstanbul, Turkey). The testes were cleaned in cold saline and then stored in a freezer at $-80^{\circ}C$ for biochemical measurements.

2.3. Biochemical Assessments

Total antioxidant status (TAS) value was detected with the commercial kit (Rel Assay Diagnostics). Total oxidant status (TOS) measurement was applied with commercially available kit (Rel Assay Diagnostics). TOS to TAS ratio was admitted as the oxidative stress index (OSI). OSI value was measured as follows: OSI = [(TOS, μPmol H2O2 equivalent L)/(TAS, mmol Trolox equivalent/L) $\times$ 10]. We preferred OSI as another indicant of oxidative stress. Xanthine and xanthine oxidase system produce superoxide radicals. Superoxide radicals, react with nitroblue tetrazolium to form formazan dye, form a basis of superoxide dismutase (SOD) evaluation [21]. Amounts of lipid peroxidation in testes tissue were measured by assessing malondialdehyde (MDA) using the thiobarbituric acid test [22]. The activity of myeloperoxidase (MPO) in the testes tissue was estimated according to methods described by Bradley et al. [23].

2.4. Statistical Analysis

Results were analyzed using One-way ANOVA and then Tukey test for pairwise comparisons of groups. All the results were presented as Means ± SEM. The differences were accepted significant when $p < 0.05$.

3. Results

3.1. Biochemical Results

In this study, when SOD, MPO activities and MDA levels were analyzed, it was found that MPO activity and MDA levels were significantly increased in I/R group, whereas SOD enzyme activity decreased due to insufficient antioxidant capacity. In addition, it was determined that SOD activity was increased by supporting antioxidant capacity in the group treated with barbaloin compared to I/R group but MPO activity and MDA levels decreased (See Table 1). When the findings of OSI, TAS and TOS levels were evaluated, it was found that TAS levels significantly decreased while TOS and OSI levels significantly increased in I/R group compared to sham and IR+DMSO groups ($P < 0.05$). However, TAS levels increased but TOS and OSI levels decreased in group treated with 20 mg/kg barbaloin (See Table 2).

Table 1: The Minimum, Maximum, Mean and Standard Error of Mean values of Superoxide Dismutase (SOD), Myeloperoxidase (MPO) activities of all experimental groups were presented.

Groups		MDA (µmol/gr tissue)	SOD (U/mg protein)	MPO (U/mg protein)
Sham	Minimum	164,83	342,52	28354,84
	Maximum	228,42	478,85	41325,94
	Mean	205,7816*	406,4826*	34973,9064*
	Std. Error of Mean	8,30084	15,44626	1401,19425
I/R	Minimum	359,29	163,54	76431,64
	Maximum	492,67	216,90	93245,80
	Mean	419,0717*,**	192,5102*,**	85023,8419*,**
	Std. Error of Mean	16,39499	5,96061	2102,09375
I/R+DMSO	Minimum	351,44	171,43	79304,19
	Maximum	503,22	223,33	93278,67
	Mean	423,4385*,#	194,0845*,#	85452,2270*,#
	Std. Error of Mean	20,61832	6,17502	1622,62726
I/R+ Barbaloin	Minimum	184,76	362,68	30426,12
	Maximum	221,44	447,45	40326,99
	Mean	208,8152**,#	408,3420**,#	35969,6675**,#
	Std. Error of Mean	4,88605	11,08708	1122,94026

*, **,# represent the statistically significant relationship between the groups. p value is less than 0.001.

Table 2: The Minimum, Maximum, Mean and Standard Error of Mean values of Total Antioxidant Status (TAS), Total Oxidant Status (TOS) and Oxidative Stress Index (OSI) levels of all experimental groups were presented.

Groups		TAS (mmol/L)	TOS (µmol/L)	OSI
Sham	Minimum	1,13	5,90	,43
	Maximum	1,62	7,78	,66
	Mean	1,3666*	6,9316*	,5141*
	Std. Error of Mean	,05620	,20719	,02918
I/R	Minimum	,63	9,78	1,12
	Maximum	,97	12,79	1,91
	Mean	,7842*,**	11,5362*,**	1,4955*,**
	Std. Error of Mean	,03817	,36801	,08720
I/R+DMSO	Minimum	,74	10,69	1,23
	Maximum	,90	13,56	1,67
	Mean	,8256*,#	11,8911*,#	1,4475*,#
	Std. Error of Mean	,02275	,33126	,05533
I/R+ Barbaloin	Minimum	,94	6,33	,47
	Maximum	1,59	7,65	,72
	Mean	1,3198**,#	7,2657**,#	,5613**,#
	Std. Error of Mean	,06976	,16209	,03165

*, **,# represent the statistically significant relationship between the groups. p value is less than 0.001.

4. Discussion

Testicular torsion is an urological health condition and may occur among adolescents, newborn males and children. A fast diagnosis and surgical interfedence matters against permenant fertility loss and testicular injury [24]. In order to prevent testicular injury, quick diagnosis and right management must be performed [25]. Testicular T/D leads to I/R injury, a harmful pathological situation. I/R injury causes to excessive free radicals production like ROS and takes role in inflammatory signaling pathway activation [26]. Free oxygen radicals, produced during I/R injury, can lead to oxidative damage in many cellular biomolecules containing proteins, lipids, and DNA [6, 27] By virtue of ischemia, oxygen supply reduces, cellular energy depletes and toxic metabolites accumulate and hereat, oxidative stess, germ cell death occur. ROS and reactive nitrogen species are much more produced due to reperfusion and these species lead to membrane lipid peroxidation. In conclusion tissue injury occurs, cell structure and function disorganize [28]. Reperfusion stage damages the tissue much more than the ischemic phase [29]. At the reperfusion period, ROS production is the key for uncontrolled oxidative stress and increased amounts of ROS may promote the inflammatory cascade [30]. Several studies have been reported that testicular I/R incremented the oxidative stress and decreased the level of "antioxidant enzymes" [31, 32]. Antioxidants take role in breaking down of chain oxidative reactions and reducing oxidative stress [33, 34]. Glutathione (GSH) and SOD are examples for the natural free radical scavengers which conserve tissues and organs injuries induced by ROS. ROS related injury and lipid peroxidation increase in case of a depletion in the amount of renal antioxidants (catalase, SOD, GSH and other free radical scavengers) [35]. For the male reproductive organs, glutathione poeroxidase and SOD are the main enzymes which scavenge harmful ROS [36]. TAS and TOS reflect the redox balance between oxidation and antioxidation. TAS measurement is an indicator of the activity of all antioxidants while TOS is an indicator of ROS [37, 38]. Oxidative stress is an oxidant-antioxidant imbalance status, due to oxidants which exceed the antioxidant capacity. OSI is the ratio of TOS to TAS and is an indicator of OSI degree [37, 38]. TAS and TOS are the well known methods in the biochemical analysis of I/R injury studies [39]. I/R study was showed that the TOS and OSI values increased, in contrast to the total antioxidant status decreased in the I/R group [40]. In a study, it was shown that SOD levels increased effectively with barbaloin [41]. Barbaloin alleviated lipopolisaccaride-induced acute lung injury by reducing intracellular ROS [42]. As the most deleterious free radical effect, lipid peroxidation in cells results in decrease in membrane potential and right after, cell injury. MDA occurs due to lipid peroxidation. It induces polymerization and causes cross linking in membrane components that result in serious cell damage [43]. MDA is preferred to evaluate free radical formation in postischemic tissue [44]. MDA represents lipid peroxidation degree and indirectly shows I/R injury degree [45]. MPO is particularly within neutrophils and so an increase in MPO levels also is an indicant for neutrophil activation [46]. Barbaloin-related MDA and MPO levels were not found in the literature, and in this study, it was significant that it was the first to reduce these levels. The most important biochemical data of barbaloin treatment can be expressed as follows: The I/R injury in testes was related to dramatic increases of MDA level, TOS and OSI values and MPO activity, and a decrease in SOD activity and TAS value in the testicular tissue. The novel result of the present study is that barbaloin significantly derogated testicular tissue damage induced by I/R. Barbaloin treatment effected in the positive direction changes of the findings of MDA, TOS, OSI and MPO and stimulated an overproduction of enzymatic antioxidant SOD activity and increased TAS value.

5. Conclusion

These results recommend that barbaloin may protect the testes by diminishing oxidative injury caused by I/R. We have found that treatment with barbaloin at single dose (20 mg/kg) reduces testicular damage induced by I/R in testes in experimental animals exposed to a torsion for 2 h and detorsion for 2 h model. Part of the mechanisms of these protective effects of barbaloin may be stemming from supporting the antioxidant capacity by barbaloin. New studies may be helpful to be able to find different protective mechanism on I/R-induced testicular tissue injury.

6. Conflict of interest

None

7. Financial Disclosure

There is no financial support organization in the implementation of this study.

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TURKISH JOURNAL OF SCIENCE
VOLUME 5, ISSUE 1, 34–40
ISSN: 2587–0971

http:/dergipark.gov.tr/tjos

Demographic and Clinical Characteristics of Patients Diagnosed with Colorectal Cancer: Six-Year Experience

Gülşen Yurttaş[a], Ömer Topdağı[b]

[a]*Department of Internal Medicine, Medicine Faculty, Atatürk University, Erzurum, Turkey*
[b]*Department of Internal Medicine, Medicine Faculty, Atatürk University, Erzurum, Turkey*

Abstract. Background: Cancer is the second cause of death after cardiovascular disease all over the world. Colorectal cancer, as one of the leading cause of cancer deaths, is the third most common cancer in men and the second most common cancer in women. Determination of the demographic and clinical characteristics of colorectal cancer is critical for early diagnosis and treatment of the disease. Aim: To investigate the epidemiological and clinical characteristics of patients with colorectal cancer at medical oncology clinic of Atatürk University. Study Desing: Cross-sectional study. Methods: The study was conducted in hospitalised male, and female patients with colorectal cancer admitted to our clinic from 2010 to 2016. Information such as the demographic characteristics (age, sex, etc.) of the patients, the place they live in, the location of the tumor, the results of pathologic evaluation, the stage of the cancer, the presence of metastasis, chemotherapy and radiotherapy were obtained from the hospital automation system and the patient's inpatient and outpatient files. The study included only patients who were diagnosed and followed up in our clinic. Information on survival status and the date of death were reached from Death Notification System. Results: In our study, 269 patients with colorectal cancer were evaluated. 50.6% of the patients are male, 49.4% female. The mean age was 58.9±14 (29-91) in males and 58.2 ± 14.7 (19-91) in females. 27.9% of patients had a history of smoking whereas 2.2% had a history of alcohol use. The most common subtype was adenocarcinoma (49.4%). The most common site (39.4%) was rectum, and (16.7%) was sigmoid colon. Of the patients, 53.1% were metastatic, and liver metastasis was the most common (44.7%). The most common symptom in patients receiving colorectal cancer was abdominal pain (29%). 32.3% patients underwent palliative surgery. 28.3% patients had comorbid disorder and 4.5% were accompanied by another malignancy. 49.4% of the patients had stage 4, 26% stage 3, 10.8% stage 2, 2.2% stage 1 and 0.4% intramucosal carcinoma. CEA levels were mostly 0-5 ng/Ml (46.1%). CA 9-19 level was mostly 0-40 U/ml (56,5%). Of the patients, 85.5% received chemotherapy and 19% received radiotherapy. Conclusion: There is no effect of sex, smoking and alcohol use, pathologic typing, tumor location, symptoms of arrival, comorbid disorder at survival. Age, presence of distant metastasis, history of operation, CEA and CA 19-9 levels, chemotherapy and radiotherapy status were found to be effective on survival.

1. Introduction

Cancer is the second cause of death after cardiovascular disease all over the world [1]. Colorectal cancer, as one of the leading cause of cancer deaths, causes about 694 000 deaths per year [2]. Colorectal cancer is

Corresponding author: ÖT mail address: omertopdagi@hotmail.com ORCID: https://orcid.org/0000-0002-9690-4447, GY ORCID: https://orcid.org/0000-0001-8972-6809
Received: 02 January 2020; Accepted: 25 March 2020; Published: 30 March 2020
Keywords. (Colorectal cancer, demographic and clinical characteristics, survival)
Cited this article as: Yurttaş G, Topdağı Ö. Demographic and Clinical Characteristics of Patients Diagnosed with Colorectal Cancer: Six-Year Experience, Turkish Journal of Science, 2020, 5(1), 34-40

the third most common cancer in men (after lung and prostate cancer) and the second most common cancer in women (after breast cancer). There are about 1.360 000 new cases annually and constitutes 10% of total cancer in the world. In recent years, the most appropriate method for screening in the normal population (those aged 54-70 years, without colorectal cancer family history) is to have stool occult blood test every year or every two years, flexible sigmoidoscopy every 5 years and colonoscopy every 10 years [3]. The symptoms of colorectal cancer vary according to the localisation of a tumour. Cancer from the ascending colon and cecum can reach rather large sizes without producing any obstructive symptoms or the change in bowel habits. Patients with colorectal cancer in that localisation often have nonspecific symptoms such as fatigue, palpitation, and angina due to chronic and insidious blood loss. Patients with colorectal cancer during that period often have nonspecific symptoms such as fatigue, palpitation, and angina due to chronic and occult blood loss. Anemia of iron deficiency in blood sample and positivity in fecal occult blood test may be seen in laboratory findings. Patients with cancer from transverse colon and descending colon can grow inside the colon lumen and may cause abdominal cramps, obstruction or even perforation symptoms. Cancers that are located in the rectosigmoid region produce symptoms such as hematochezia, tenesmus and thinning in the faeces diameter [4]. Approximately half of the patients have metastases during follow-up. 25% of patients have liver metastasis, the most common metastatic site of colorectal cancer, at the time of diagnosis [5]. The prognosis of metastatic colorectal cancer is poor, and the 5-year survival rate is 5-13% [6]. Inhere, we aimed to investigate the epidemiological and clinical profiles of a sample of patients with colorectal cancer at our medical oncology clinic.

2. Material and Methods

The study was conducted in hospitalised male, and female patients with colorectal cancer admitted (diagnosed histopathologically) to the Medical Oncology Clinic, Atatürk University Medical Faculty Hospital from 1 January 2010 to 31 December 2015. Inpatient and outpatient records of all patients were reviewed retrospectively. The study included only patients who were diagnosed and followed up in our clinic. Information such as the demographic characteristics (age, sex, etc.) of the patients, the place they live in, the location of the tumor, the results of pathologic evaluation, the stage of the cancer, the presence of metastasis, chemotherapy and radiotherapy were obtained from the hospital automation system and the patient's inpatient and outpatient files. Information on whether the patients were alive or dead and the date of death were reached by the Death Notification System of the Ministry of Health. The study was carried out according to the principles of the Declaration of Helsinki and approved by the local ethical committee (24.10.2016-6-09).

2.1. Statistical Analysis

SPSS 20.0 (SPSS Inc. Chicago, IL, USA) program was used to analyse the data. Data are presented as the number, percent, median, mean, standard deviation and standard error. Mann-Whitney test, Kruskal Wallis test and Post Hoc test were used for statistical analysis. The statistical significance level was taken as $p < 0.05$.

3. Results

Of the 269 colorectal cancer patients who were taken into the study, 136 (50.6%) were male and 133 (49.4%) were female. The mean age was 58.5±14.3 (19-91) years. The mean age was 58.9± 14(29-91) in males and 58.2±14.7 (19-91) in females. The distribution of histological subtypes and stage of colorectal cancer, the location of tumours, surgical treatment status, co-morbid disease status, CEA and CA 19-9 levels (at diagnosis) are summarised in Table 1. In Table 1, the properties of these variables are given in numbers and percentages. In Table 2, admission symptoms of the patients with colorectal cancer were shown. Of the patients whose smoking status was known, 27,9% (n=75) had a history of smoking and 64,7% (n=174) had no history of smoking. Of the patients whose using alcohol status was known, 2.2% (n=6) had a history of using alcohol and 90,3% (n=243) had no history of using alcohol. The smoking and using alcohol status

of 20 patients was unknown. Symptoms of patients with colorectal cancer were abdominal pain (n=78, 29%), constipation (n=59, 21.9%), bleeding from the rectum (n=39, 14,5%), pain in back passage (n=11, 4,1%), weight loss (n=10, 3,7%), constipation+abdominal distension (n=9, 3,3%), abdominal swelling (n=8, 3%), diarrhoea (n=5, 1,9%), weakness (n=4, 1,5%), jaundice (n=3, 1,1%), lumbar pain (n=1, 0,4%), faecal incontinence (n=1, 0,4%), and various combination of these symptoms at the admission to the hospital. Twenty-five patients had no information about admission to the hospital. Comorbid diseases (malignancy) were breast cancer (n=3, 1.1%), prostate cancer (n=2, 0.7%), stomach cancer and chronic myeloid leukemia (n=1, 0.37, together), bladder cancer (n=1, 0.37%), endometrium cancer (n=1, 0.37%), over cancer (n=1, 0.37%), basal cell carcinoma (n=1, 0.37%), thyroid cancer and intraabdominal carcinoma. Of the 143 metastatic patients, 60 (44,7%) had liver metastases and 17 (12,6%) had abdomen metastases (omentum, peritoneum, the abdominal wall), whereas metastases were observed in various combinations (including liver, lung, abdomen, bladder, bone, adrenal gland, over, brain, endometrium) in other metastatic patients. 15 patients the site of metastases could not be determined. Mean survival was 40 months in non-distant metastases, whereas 17.3 months in patients with distant metastases. Mean survival was 40 months in patients with non-distant metastases, whereas 17.3 months in patients with distant metastases. Received cycles of chemotherapy and patient numbers were as follows; 1 cycle 82 (30.5%) patients, 2 cycles 63 (23.4%) patients, 3 cycles 26 (9.7%) patients, 4 cycles 28 (10.4%) patients, 5 cycles 15 (5.6%) patients, 6 cycles 9 (3.3%) patients, 7 cycles 2 (0.7%) patients, 8 cycles 4 (1.5%) patients, and 9 cycles 1 (0.4%) patients. Seven patients (2.6%) did not receive chemotherapy, and there were no data on chemotherapy for 32 (11.9%) patients. The mean survival time of patients with colorectal cancer was 26.4±24.1 (1-168) months, whereas it was found 26.4±21.9 months for males and 26.3±26.4 months for females. Demographic and clinical variables were compared according to survival status (survived or died). There were no statistical differences concerning age, gender, using alcohol status, smoking status, histopathologic subtypes of cancer, localisation of a tumour, comorbid status ($p > 0,05$). There was a significant difference in survival between the patients who underwent surgery and didn't undergo surgery ($p = 0,02$). The mean survival time was 39.1 months in the patients with curative surgery whereas it was 19.9 months in the patients with palliative surgery ($p < 0.001$). Initially, of the 269 patients with colorectal cancer, 218 (81%) received radiotherapy, while 51 (19%) didn't receive radiotherapy. In patients had radiotherapy, the mean survival was 41.3 months, whereas in patients had no radiotherapy, it was 22.7 months ($p < 0.001$). It was observed that patients who received 7-cycles chemotherapy live longer than patients who received other serial chemotherapy ($p < 0.001$). In patients receiving 7-cycles chemotherapy, mean survival was found 90 months, whereas was found 17 months in patients receiving 1-cycle. Mean survival was 22 months in 5 patients who did not receive chemotherapy. Patients who did not have distant metastasis were observed to live longer than patients who had distant metastasis ($p < 0.001$). Patients with CEA levels 0-5 were found to have significantly higher survival times in patients with CEA levels between 100-1000 ($p = 0.001$).

4. Discussion

Cancer is the second most common cause of death after cardiovascular disease, according to the World Health Organization data. Colorectal cancers, the 4th most frequent cancer of all cancers, have a rate of 9 %, preceded by lung cancer, breast cancer and prostate cancer. This rate differs according to countries and races. Colorectal cancers are more common in European countries and the United States, and less frequently in Asian and African countries [7]. Also, recent studies have shown that colorectal cancer is the third most common cancer in both women and men in Turkey [8]. Colorectal cancers are more common in men than in women. This gender difference of colorectal cancer is not very obvious. Boyle et al. reported that colorectal cancer is 1.1 times more common in male than in females [9]. The findings of our study support these results. Another risk factor for colorectal cancer is advanced age. In a systematic review, it has been reported that colorectal cancers reach a peak during the 7th decade of life [10]. In the present study, colorectal cancers increased with increasing age, and it was found mostly in the age range of 70-80 years. Smoking plays an important role in etiology of many cancers, including colorectal cancer. In two case-control studies conducted in the Turkish population, the rate of smoking in the patients with colorectal was found to be higher than the controls [11, 12]. The findings of our study were not consistent

with these results. The frequency of smoking (27%) in our study was found to be similar to the general population, according to the Turkish Statistical Institute data for 2016 [13]. Alcohol is also known to trigger cancer and have a carcinogenic effect. Colon and rectum cancer are among the cancers caused by alcohol. Studies emphasise the total amount of alcohol consumed daily is more important than the variety of alcohol consumed. Alcohol consumption in the Turkish population is one-third of the world average [14]. In our study, alcohol consumption rate was found low in colorectal cancer patients. In the country where this study is done, it is reported that adenocarcinoma is the most common colorectal cancer subtype in many studies [15–18]. We found that adenocarcinoma was the most common subtype. Approximately 30% of all colorectal cancers are located in the rectum, and 20% are located in the sigmoid colon [19–21]. In this study, it was observed that 106 (39,4%) of the cancer cases were located in the rectum and 45 (16,7%) were located in the sigmoid colon. The results of our study were consistent with the literature. It was also found that there was no significant survival effect of the cancer subtype and primary tumour site. Mehrkhani et al. reported that distant metastasis is the most important factor that worsens survival prognosis [22]. We also found that distant metastasis worsened the prognosis. The mean survival for patients without distant metastasis was 40 months, while it was 17.3 months for patients with distant metastases. Early diagnosis in patients with colorectal cancer is critical in terms of treatment and survival time. Therefore, it is important to evaluate the complaints of patients and to use appropriate diagnostic methods. Changes in bowel habit, weight loss, bloody mucus diarrhea and anaemia are the most frequent complaints of colorectal cancer [23, 24]. The most common complaints in this study were abdominal pain, constipation and rectal bleeding, respectively. Resection of primary tumour in colorectal cancers significantly improves survival. Ruo et al. suggested that primary tumour resection of even in patients with distant organ metastasis [25]. In addition, Ruo et al. said that patients with colorectal cancer who were initially treated without bowel resection, need subsequent operations for complications. In this study, mean survival times in patients with the operation were found to be approximately 5 times those of patients without operation history. In patients operated for curative purposes, the survival time was approximately 2 times longer than patients operated for palliative purposes. The pathologic stage has repeatedly been reported as an important prognostic factor by many investigators. Newland et al. said that survival worsened with increasing pathologic stage [26]. Survival time was 113.3 months in patients with stage 1, 36.7 months in patients with stage 2, 17.4 months in patients with stage 3 and 34.9 months in stage 4, according to the oncologist's the initial diagnosis. The survival of one patient evaluated as stage 0 intramucosal carcinoma was 61 months and comorbid diseases were considered to affect survival (i.e. chronic renal failure, diabetes mellitus and hypertension). The patient died due to an acute episode of chronic renal failure. In the study conducted by Shikhani, the risk of developing second cancer in primer-tumour cases was found 1.29 times higher than in healthy individuals [27]. These study findings showed that there was no significant difference between those who had the comorbid disorder as malignancy and those who did not have. These results suggest that colon cancer is the determinant of cancer deaths in patients with multiple malignancies. Approximately 60-70% of colorectal cancer patients have a high CEA level in serum. The sensitivity and specificity of serum CEA were reported 55%-89% and 75%-98%, respectively. Serum CEA has been shown to detect recurrence of colorectal cancer before periodic clinical examinations, other laboratory tests, radiographic imaging [28]. In our study, the CEA level at the time of admission was found to be 0-5 in 124 (46.1%) of the patients. There was a significant difference between CEA levels concerning survival. Mean survival was found 32.4 months in patients with CEA levels< 5 ng/mL (low) whereas was found 16 months in patients with CEA levels¿1000ng/mL (high). In the present study, the levels of CA 9-19 were also similar to these results. In the randomised study with advanced colorectal cancer patients, the median survival time was found 11 months in the chemotherapy group and 5 months in the supportive treatment group [5]. Patients receiving higher chemotherapy (7 cycles) had longer survival times than patients receiving lower chemotherapy (1 cycle). The mean survival time of 5 patients who had never received chemotherapy was approximately 2 years.

Table 1. Distribution of clinic characteristics related to colorectal cancer

Histologic subtype (n=269)		Localization (n=269)		Operation (n=269)		Stage (n=269)		Comorbid disease (n=269)	
n (%)		n (%)		n (%)		n (%)		n (%)	
Adenocarcinoma-NOS	133 (49.4%)	Rectum	106 (39.4%)	Palliative	87 (32.3%)	Intramucosal carcinoma	1 (0.4%)	Non-malignancy	64 (23.8%)
Moderately differentiated adenocarcinoma	53 (19.7%)	Sigmoid colon	45 (16.7%)	Curative	49 (18.2%)	Stage 1	6 (2.2%)	Malignancy	12 (4.5%)
Mucinous adenocarcinoma	29 (10.8%)	Common kolon	34 (12.6%)	Unknown purpose	41 (15.2%)	Stage 2	29 10.8%)	Unknown	24 (8.9%)
Low differentiated adenocarcinoma	10 (3.7%)	Cecum	29 (10.8%)	Unknown	20 (7.4%)	Stage 3	70 (26%)	Non-comorbid disease	169 (62.8%)
Malignant epithelial tumor	7 (2.6%)	Ascending colon	22 (8.2%)	Total	197 (73.2%)	Stage 4	133 (49.4%)		
Signet ring cell carcinoma	4 (1.5%)	Transverse colon	12 (4.5%)	Non-operation	72 (26.8%)	Unknown	30 (11.2%)		
Intramucosal carcinoma	4 (1.5%)	Rectosigmoid kolon	5 (1.9%)	CA levels (n=269)		CA 19-9 levels (n=269)			
Neuroendocrine	3 (1.1%)	Appendix colon	3 (1.1%)	0-5	124 (46.1%)	0-40	152 (56.2%)		
Well differentiated adenocarcinoma	2 (0.7%)	Unknown	13 (4.8%)	5-10	24 (8.9%)	40-100	32 (11.9%)		
Intraepithelial carcinoma	1 (0.4%)			10-20	12 (4.5%)	100-1000	33 (12.3%)		
Desmoid tumor	1 (0.4%)			20-100	32 (11.9%)	1000	16 (5.9%)		
GIST	1 (0.4%)			100-1000	33 (12.3%)	Unknown	36 (13.4%)		
Unknown	21 (7.8%)			1000 over	9 (3.3%)				
				Unknown	35 (13%)				

n/%=number/percent, NOS: Not specified otherwise GIST: Gastrointestinal Stromal Tumor

Table 2. Admission symptoms of the patients with colorectal cancer

Admission Symptoms (n=269)	
Abdominal Pain	78 (29%)
Constipation	59 (21.9%)
Rectal Bleeding	39 (14,5%)
Anal Pain	11 (4.1%)
Weight Loss	10 (3.7%)
Abdominal Distension	8 (3%)
Diarrhea	5 (1.9%)
Jaundice	2 (0.7%)
Back Ache	1 (0.4%)
Feces in Urine	1 (0.4)

5. Conclusion

As a result, the organisation of screening programs may be considered in future for the early diagnosis of colorectal cancers, which are common in our country and have high mortality. Future studies will provide more information about the demographic and clinical characteristics of patients with colorectal cancer.

6. Conflict of interest

No conflict of interest was declared by the authors.

7. Financial Disclosure

The authors declared that this study has received no financial support.

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TURKISH JOURNAL OF SCIENCE
VOLUME 5, ISSUE 1, 41–47
ISSN: 2587–0971

http:/dergipark.gov.tr/tjos

Glycated Hemoglobin (HbA1C): A Predictor of In-hospital Short Therm All Cause Mortality

Ömer Topdağı[a], Yasemin Kaya[b], Oktay Gülcü[c], Halil İbrahim Tanboğa[d], Nuri Bakan[e]

[a]*Department of Internal Medicine, Medicine Faculty, Atatürk University, Erzurum, Turkey*
[b]*Department of Internal Medicine, Medicine Faculty, Ordu University, Ordu, Turkey*
[c]*Department of Cardiology, Erzurum Training and Research Hospital, Erzurum, Turkey*
[d]*Department of Cardiology, Hisar Hospital, Istanbul, Turkey*
[e]*Department of Biochemistry, Faculty of Medicine, Ataturk University, Erzurum, Turkey*

Abstract. Background and objectives: HbA1c shows the mean glucose level in blood and is a biochemical parameter used in follow-up and diagnosis of diabetic patients. We aimed to investigate the association between HbA1c and in-hospital all cause mortality in diabetic patients who were admitted due to any diagnosis. Methods: 3207 diabetic patients included study who had been diagnosed with diabetes mellitus. Trauma patients, type 1 diabetes were excluded. Patients' age, gender, admission diagnosis, duration of hospitalization, whether they died in-hospital, laboratory parameters and HbA1c levels were recorded. Results: The mean age of patients was 50.53±17 years with 59.7 % (n:1913) being females. Patients who died in hospital had higher HbA1c, age, BUN (blood urea nitrogen), creatine and uric acid levels according to the Univariate analysis ($p = 0.000, p = 0.000; p = 0.004, p = 0.04, p = 0.03$; respectively). In the model 1 in multivariate analysis, there was a significant correlation between HbA1c level and in-hospital mortality (uncorrected OR: 1.216, 95 % CI 1.116-1.326,$p < 0.001$). In the model 2, the significant correlation between HbA1c level and in hospital mortality continued when corrected with age and gender (corrected OR: 1.150, 95 % CI 1.046-1.265, $p : 0.004$). In the model 3, which was created with covariates that were found significant in the univariate analysis, the correlation between HbA1c level and in hospital mortality still continued (corrected OR: 1.151, 95 % CI 1.041-1.271, $p : 0.006$). Interpretation and conclusions: There was a positive correlation between in hospital all cause mortality and HbA1c level in diabetic patients who had admitted any diagnosis. HbA1c level predict in hospital short term all cause mortality.

1. Introduction

Diabetes mellitus is a disease with increasing incidence worldwide and is a significant cause of morbidity and mortality [1]. An early and accurate assessment of mortality is critical in preventing the progression of diabetes mellitus and thereby in improving the lives of diabetic patients. Identifying high-risk patients is critical when applying additional pharmacological and mechanical treatment approaches, which provide prognostic benefits. It has been shown that various biochemical markers (insulin-like growth factor 1, B-type natriuretic peptide (BNP) and N-terminal prohormone BNP, etc.) are potentially effective in the early

Corresponding author: ÖT mail address: omertopdagi@hotmail.com ORCID: https://orcid.org/0000-0002-9690-4447, YK ORCID: https://orcid.org/0000-0001-7360-8090, OG ORCID: https://orcid.org/0000-0003-4479-2997, HİT ORCID: https://orcid.org/0000-0003-4546-9227, NB ORCID: https://orcid.org/0000-0002-2139-7268

Received: 12 January 2020; Accepted: 20 March 2020; Published: 30 March 2020

Keywords. (Tip 2 Diabetes Mellitus, Hb-A1c, In hospital mortality)

Cited this article as: Topdağı Ö, Kaya Y, Gülcü O, Tanboğa Hİ, Bakan N. Glycated Hemoglobin (HbA1C): A Predictor of In-hospital Short Therm All Cause Mortality. Turkish Journal of Science. 2020, 5(1), 41-47

detection of mortality in those with diabetes mellitus [2, 3]. HbA1c data provides retrospective information about blood glucose levels without being influenced by intraday changes, fasting, postprandial states, exercise or blood glucose transients [4]; HbA1c is created upon the non-enzymatic binding of haemoglobin A to glucose [5, 6]. Since the mean lifespan of erythrocytes is 2 to 3 months, HbA1c represents the mean level of glucose exposed and is a biochemical parameter used in the diagnosis and follow-up assessments of diabetic patients [7–9]. Many previous studies have shown that HbA1c levels are associated with all-cause long-term mortality in diabetic patients [10–13]. This study's goal was to discover whether HbA1c levels predict all-cause in-hospital mortality in patients diagnosed with type 2 diabetes mellitus (T2DM) who were hospitalized due to various diagnoses.

2. Material and Methods

A total of 40,947 patients aged 18 and over and who were hospitalized between 2010 and 2013 were screened retrospectively. Among these, 3,207 diabetics were included in the study; these patients were previously diagnosed with diabetes mellitus according to ICD diagnostic codes and were on oral antidiabetics and insulin therapy. Trauma patients and those diagnosed with type 1 diabetes were excluded from the study. Patient data regarding age, gender, diagnosis, duration of hospitalization, whether they died in hospital, hemograms, biochemical parameters and HbA1c levels were recorded upon admission. Admissions were grouped according to ICD diagnostic codes as follows: cardiac diseases, pulmonary diseases, acute/chronic renal failure, infectious diseases due to bacterial, viral, and other infectious agents, malignant diseases, endocrine disorders, septicaemia, general signs and symptoms, acute abdominal, cerebrovascular diseases, psychiatric diseases, intracranial haemorrhage, liver diseases and rheumatic diseases. Subgroupings were as follows: 1- Cardiac disease group: ischemic heart disease, cardiac failure, valve diseases and hypertension; 2- Pulmonary disease group: asthma, chronic obstructive pulmonary disease and pulmonary embolism; 3- Malignant disease group: thyroid cancer, oesophageal cancer, pancreas cancer, ovarian cancer, gastric cancer, bladder cancer, laryngeal cancer, pharyngeal cancer, hepatocellular cancer, endometrial cancer, ampulla of Vater cancer, lung cancer, renal-cell cancer, acute myeloblastic leukaemia, acute lymphoblastic leukaemia, chronic myeloid leukaemia, myelodysplastic syndromes, multiple myeloma; 4- Endocrine disorder group: type 1 diabetes mellitus, type 2 diabetes mellitus, thyroid diseases, diabetes insipidus, acromegaly, Addison's disease, hypophysis adenoma, surrenal adenoma, insipidus, pheochromocytoma, parathyroid adenoma; 5- General signs and symptoms group: carbon monoxide intoxication, ulcerative colitis, Chron's disease, burns, diffuse body pain, entrapment neuropathy, fibromyalgia, tachycardia, benign prostatic hyperplasia, anaemia; 6- Acute abdominal disease group: mesenteric artery ischemia, acute cholecystitis, volvulus, acute perforation, acute appendicitis, urolithiasis, choledocholithiasis, acute pancreatitis; 7- Psychiatric disease group: schizoaffective disorder, manic attack, depression and anxiety, bipolar disorder; 8- Liver disease group: cirrhosis, hepatitis; 9- Rheumatic disease group: rheumatoid arthritis, ankylosing spondylitis, systemic lupus erythematosus, Behcet's disease; 10- Infectious diseases due to bacterial, viral, and other infectious agents: pneumonia, Crimean Congo haemorrhagic fever, infective endocarditis, and 11- Intracranial haemorrhage disease group: subdural hematoma, subarachnoid haemorrhage, epidural haemorrhage, aneurysm. The blood samples were collected in gel tubes, which did not contain anticoagulants, to measure biochemical markers using the Beckman Coulter AU 5800 for all patients. HbA1c was measured using High Performance Liquid Chromatography (HPLC) with Bio-Rad's VARIANT^{TM} II. Venous blood was routinely collected in a tube containing EDTA for measuring haemoglobin, total WBC, neutrophils, lymphocytes; the values of these parameters were determined using the Sysmex XN 9000, an automated blood cell counter, for all patients. The local ethics committee approved of this study.

3. Statistical Analysis

All statistical studies were carried out with SPSS (version 20.0, SPSS, Chicago, Illinois, USA). Continuous variables are expressed as the mean ± SD. Categorical variables are expressed as percentages. T-tests were

used to compare parametric continuous variables. Multiple logistic regression analysis was used to identify the independent predictors of in-hospital mortality. All variables showing significance values < 0.10 in a univariate analysis were included in the model. Two-tailed p values < 0.05 were considered statistically significant.

4. Results

A total of 3,207 patients were included in the study. The mean age was 50.53 ± 17 years with 59.7 % (n = 1913) being females. Of all patients, 29.5% (n = 946) were hospitalized due to ICD diagnoses of cardiac diseases, 1.3% (n = 43) for pulmonary diseases, 1.9% (n = 62) for acute/chronic renal failure, 1.8% (n = 57) for infectious diseases due to bacterial, viral and other infectious agents, 11.9% (n = 381) for malignant diseases, 24% (n = 770) for endocrine disorders, 3.6% (n = 117) for septicaemia, 7.6% (n = 243) for general signs and symptoms, 6.6% (n = 213) for acute abdominal diseases, 5.3% (n = 169) for cerebrovascular diseases and multiple sclerosis, 0.5% (n = 16) for psychiatric diseases, 1% (n = 33) for intracranial haemorrhage, 2.9% (n = 93) for liver diseases, and 2% (n = 64) for rheumatic diseases (Table 1). The mean duration of hospitalization was 5.5 ± 4.4 days and ranged from 1 to 67. During the follow-up, it was discovered that in-hospital mortality occurred in 2.7% of patients (n = 88) (Table 1). Patients who died in-hospital were of greater age and had higher HbA1c levels, blood urea nitrogen (BUN), creatine and uric acid levels according to the univariate analysis (p = 0.000, p = 0.000; p = 0.004, p = 0.04 and p = 0.03, respectively) (Table 2). In the first model of multivariate analysis, there was a significant correlation between HbA1c levels and in-hospital mortality (uncorrected OR: 1.216, 95% CI 1.116-1.326, $p < 0.001$). In model 2, the significant correlation between HbA1c levels and in-hospital mortality persisted even when corrected with age and gender (corrected OR: 1.150, 95% CI 1.046-1.265, p = 0.004). In model 3, which was created with covariates that were found significant during univariate analysis, the correlation between HbA1c levels and in-hospital mortality also continued (corrected OR: 1.151, 95% CI 1.041-1.271, p = 0.006) (Table 3). When HbA1c values were analysed after being divided into quartiles, there was a significant difference between the groups in terms of in-hospital mortality (1.4% vs. 2.1% vs. 2.9% vs. 5%, chi-square p value < 0.001, p for trend < 0.001).

Table 1: Baseline clinical and biochemical characteristics of patients, hospitalized diagnostic groups.

	% n / mean±std deviation
Age	50,52±17,09
Gender male (n%)	%40,3 (n=1294)
female (n%)	%59,7 (n=1913)
Discharged type exitus (n%)	% 2,7 (n=88)
alive (n%)	%97,3 (n=3119)
Cardiac diseases	% 29.5 (n=946)
Pulmonary diseases	%1,3 (n= 43)
Acute / chronic renal failure	%1.9 (n=62)
Maling diseases	%11.9 (n=381)
Endocrine disorders	% 24 (n=770)
Septicemia	%3.6 (n=117)
General symptoms	% 7.6 (n= 243)
Acute abdomen	% 6.6 (n=213)
Cerebrovascular diseases and multiple sclerosis	%5.3 (n=169)
Psychiatric diseases	%0.5 (n=16)
Intracranial hemorrhage	%1 (n=33)
Liver diseases	%2.9 (n=93)
Rheumatoid diseases	%2 (n=64)
Infectious diseases caused by bacterial, viral and other infectious agents	%1.8 (n=57)
TSH	
Triglycerides	2,051±6,571
Total cholesterol	212,700±281,778
HDL	198,690±62,532
LDL	41,621±11,980
Albumin	131,83±39,83
Total bilirubin	3,62±0,74
Direct bilirubin	0,96±2,130
Glucose	0,34±1,213
Blood urea nitrogen	283,03±121,149
Creatinin	26,59±21,63
Na	1,23±1,23
K	135,78±5,16
Uric acid	4,36±0,66
Hemoglobin	5,64±2,65
White blood cell	13,83±2,41
Platelets	10,40±8,18
Sedimentation	233,46±94,68
C reactive protein	24,75±25,40
HbA1c	37,98±59,98
	6,30±1,77

TSH: Thyroid Stimulating Hormone, HDL: High Density Lipoprotein, LDL: Low Density Lipoprotein

Table 2: Comparison of Laboratory Parameter Between Groups.

	Alive	**In hospital death**	**P**
Hba1c	6,27± 1,74	7,15± 2,40	***0,000***
Age	50,17±16,994	63,18±15,71	***0,000***
TSH	2,057±6,62	1,75±2,03	0,80
Triglycerides	212,17±283,40	234,88±203,46	0,61
Total cholesterol	198,59±62,79	202,87±50,84	0,67
HDL	41,65±11,99	40,14±11,17	0,43
LDL	131,75±39,96	135,24±33,84	0,58
Total protein	6,67±0,98	6,54±1,10	0,28
Albumin	3,63±0,74	3,47±0,82	0,07
Total bilirubin	0,95±2,11	1,20±2,50	0,30
Direct bilirubin	0,34±1,20	0,48±1,41	0,30
Glucose	281,37±116,52	340,80±224,01	***0,000***
Blood urea nitrogen	26,40±21,49	33,15±25,31	***0,004***
Creatinin	1,23±1,23	1,51±1,28	***0,04***
Na	135,79±5,12	135,53±6,44	0,65
K	4,36±0,66	4,37±0,73	0,97
Uric acid	5,62±2,63	6,46±3,19	***0,03***
Hemoglobin	13,39±2,41	12,98±2,26	0,14
White blood cell	10,35±8,16	12,34±8,68	***0,03***
Platelets	233,78±94,56	221,93±98,75	0,28
Sedimentation	24,70±25,38	27,15±26,47	0,57

TSH: Thyroid Stimulating Hormone, HDL : High Density Lipoprotein, LDL: Low Density Lipoprotein

Table 3: Multiple Logistic Regression Analysis for Prediction of in Hospital Mortality.

	B	**P**	**%95 CI**
HbA1C	0,140	0,006	1,042 -1,271
Gender	0,359	0,134	0,895 -2,289
Age	0,048	0,000	1,032 -1,066
Creatinin	-0,038	0,745	0,766 – 1,210
Blood urea nitrogen	0,010	0,126	0,997 – 1,022
Glucose	0,003	0,000	1,002 – 1,004

5. Discussion

In this study, we found that in-hospital mortality occurred in 2.7% (n = 88) of the 3,207 diabetic patients who were admitted due to various diagnoses. When the data of living and deceased patients were compared, patients who died in-hospital were found to be older and possessed higher levels of HbA1c, glucose, BUN, creatine and uric acid. In our multivariate analysis, we discovered that short-term all-cause mortality rates in hospital increased with higher HbA1c levels. These results suggest that unselected T2DM patients with higher HbA1c levels are more likely to have poor outcomes when in hospital. Studies with various blood parameters (altered blood glucose, HbA1c, insulin-like growth factor 1, B-type natriuretic peptide (BNP) and N-terminal prohormone BNP, etc.) have been examined for their early prediction of mortality in diabetic patients, and these factors' associations with mortality has been demonstrated. Among these parameters, many studies have been performed on the association between Hb1Ac and mortality, both in diabetic and non-diabetic patients. In a meta-analysis, it was shown that a 1% increase in HbA1c corresponds to an increase of 1.15 times in all-cause mortality [13]. Similarly, Karen et al. showed that patients with HbA1c levels > 6 were 4.5 times more likely to endure poor outcomes than those with a HbA1c < 6 [14]. However, previous studies focused on the correlation between HbA1c and all-cause mortality to determine long-term mortality rates while some studies outlined an association between in-hospital mortality and HbA1c. Many previous studies have also found different mortality results between intensive hyperglycaemia and therefore HbA1c levels in diabetic patients. While microvascular risks and the risks of MI and death were lower in The United Kingdom Prospective Diabetes Study (UKPDS), all-cause and cardiovascular mortality were higher in the group receiving glycaemic treatment (HbA1c < 6.0%) in the Action to Control Cardiovascular Risk in Diabetes (ACCORD) study [15, 16]. Among the studies performed both with diabetic and non-diabetic patients, Liberty et al. included 1,024 consecutive patients who went to internal medicine outpatient clinics. They divided the patients into three groups as those who had diabetes, patients with glucose levels higher than 140 mg/dL but had no known diabetes (hyperglycaemic patients) and the patients with no diabetes or hyperglycaemia; these patients' mortalities were assessed after one year. The authors found that glucose levels at admission did not affect mortality, but HbA1c levels lower than 6.5% caused adverse effects in the 1-year mortality in patients with diabetes and hyperglycaemia [17]. In their meta-analysis of 46 studies, Cavero-Redondo et al. found an increase in the risk of all-cause mortality when HbA1c levels were, respectively, higher than 8.0% and 6.0% in diabetic and non-diabetic populations [18]. Li et al. found an association between high (*geq* 10% or 11% in men and women) HbA1c levels and all-cause mortality in 13,334 male and 21,927 female patients (< 6.0% in men and women) (j-type correlation). Although mortality from low HbA1c levels has not been fully understood, it was thought to occur due to hypoglycaemia attacks [19]. Nicholas et al. compared patients with HbA1c levels < 6 and those with HbA1c levels > 9 among 97,450 diabetic patients in terms of 365-day mortality, and they found that higher HbA1c values (> .9%) were associated with increased all-cause mortality [20]. In our study, similar to that conducted by Li et al., low HbA1c (< 6.5%) levels were associated with increased rates of all-cause mortality compared to normal HbA1c levels. In their study of long-term mortality in 11,205 patients with type 2 diabetes, Skriver et al. studied the relationships between mortality and HbA1c levels that were measured at the beginning of the study, the middle of the study and after 22-26 months. They found a significant correlation between HbA1c levels and a variability of HbA1c over 0.5 and long-term mortality [21]. In their study, Sluik et al. performed a cohort analysis in a group of 4,345 patients with registered diabetes diagnoses within the scope of The European Prospective Investigation into Cancer and Nutrition (EPIC). HbA1c was measured in blood samples that were stored for up to 19 years. They demonstrated that those with low HbA1c levels had a better chance of survival than those with high HbA1c levels. Most importantly, they found that this correlation between HbA1c levels and survival was linear and independent of the duration of the disease, drug usage, and comorbidities. In addition, they stated that any improvement in HbA1c levels appeared to be associated with a decrease in mortality risk [22]. Likewise, the correlation between HbA1c levels and mortality has been demonstrated in studies performed with selected patient groups. He et al. analysed 147 patients with diabetes and hepatocellular cancer in terms of the factors affecting their mortality and found that a HbA1c-based score model predicted the risk of death in patients with diabetes and hepatocellular disease [23]. Again, Huang et al., found an

association between a HbA1c levels ≥ 7% and short-term mortality in patients with gastric cancer [24]. Many previous studies have shown that high levels of HbA1c are associated with chronic complications and mortality in diabetic patients. In our study—unlike in others—we found that a high HbA1c level was associated with all-cause in-hospital mortality in diabetic patients hospitalized with various diagnoses. It is known that poor glycaemic control increases long-term mortality by causing adverse effects on the immune system, increasing oxidative stress and inflammation and damage to the cardiovascular system [25–28]. In addition, it is known from previous studies that acute stress from hyperglycaemia causes adverse impacts through platelets in diabetic patients by increasing the concentration of inflammatory cytokine or relative neuroglycopenia, which causes immunosuppression, increases infections, and heightens blood pressure [29]. In our study, HbA1c levels were higher in the patient group with mortality, suggesting that their long-term glycaemic control was poor. Furthermore, glucose values at the time of admission were higher in patients with in-hospital mortality compared to the other patients. This may suggest that acute effects of hyperglycaemia may affect in-hospital mortality in this group of patients with poor glycaemic control. To supporting this result, Mahmoodpoor et al. demonstrated that acute hyperglycaemia developed due to critical diseases or as a symptom of diabetes; this correlated to mortality while in intensive care and was associated with high HbA1c (pre-existing hyperglycaemia). In this study, we demonstrated that Hb1Ac levels suggesting impaired glycaemic control in the past three months may predict all-cause in-hospital mortality in patients with an additional diagnosis of diabetes.

6. Limitations

There were a few limitations in our study. First, the study was designed as a retrospective case control, and some selection biases may have existed. Another limitation of this study includes reliance on single glycated haemoglobin and glucose measurements at baseline measurements during the follow-up period. In addition, the duration of diabetes could not be ascertained in patients. Finally, sufficiency and effectiveness of antidiabetic therapies administered during hospitalization may have affected the clinical results. We could not analyse the efficiency of antidiabetic therapy due to missing data.

7. Conclusion

Our study shows that an elevated HbA1c level is a strong independent predictor of in-hospital all-cause mortality in unselected patients with pre-existing T2DM. Although HbA1c levels are referenced during diagnosis and in the follow-up treatment of diabetes, they can be helpful in predicting all-cause in-hospital mortality in diabetic patients. Large-scale prospective studies are required to clarify the relationship between elevated HbA1c levels and short-term outcomes.

8. Conflict of interest

No conflict of interest was declared by the authors.

9. Acknowledgments

The authors thank www.metastata.com for their contributions to statistical analysis and trial design.

10. Financial Disclosure

The authors declared that this study has received no financial support.

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