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Önsöz

Herkese Merhabalar,

On on altıncı yılımızın ilk sayısında toplam on makale yer almaktadır.

Bu sayıda katkıda bulunan gerek yazarlarımıza gerekse hakemlerimize çalışmalarından dolayı teşekkür ederiz.

Saygılarımla.

Editör

Dr. Zeynel Abidin Mısırlı

Preface

Greetings to everyone,

In this edition of our journal, we have a total of ten articles related to science and mathematics education.

Thanks to everyone for contributing and/or becoming the reviewer of our journal.

Editor

Dr. Zeynel Abidin Mısırlı

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Research Article

An Investigation of Pre-Service Elementary Teachers' Skills of Teaching Numbers Through Digital Storytelling

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Abstract – Digital storytelling is an educational method that narrates educational objectives with photographs and stories in the computer environment. In this study, pre-service elementary teachers' skills of teaching numbers through digital storytelling were examined. 98 pre-service elementary teachers participated in this descriptive research. The digital stories were evaluated with the rubric developed in the study. The analysis results revealed that pre-service teachers' skills of teaching numbers were at a moderate level. Besides, while it differed significantly according to gender, it did not differ significantly according to mathematics achievement. The analysis results also showed that the pre-service teachers have some deficiencies in teaching numbers. They designed activities related to introducing the number and the use of numbers but not the part-whole relationships of numbers in their digital stories.

Key words: digital storytelling, first-grade students, mathematics education, pre-service elementary teachers, teaching numbers.

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Introduction

Numbers appear in every aspect of our lives, and we even live with numbers in daily life. The foundations of mathematics are based on numbers, and all mathematical operations are done using numbers. Therefore, numbers are among the first mathematical concepts that students learn and constitute the first mathematics subject of primary school. Since teaching numbers is important in mathematics education, there are many studies in the literature on this subject. In different studies, integers (Battista, 1983; Fuadiyah & Suryadi, 2019), rational numbers (Behr et al., 1984; Kieren, 2020; Moss & Case, 1999), decimal numbers (Tian et al., 2020; Wearne & Hiebert, 1988), exponential and rooted numbers (Duatepe Paksu, 2010), teaching these types of numbers and students' learning difficulties about them were investigated. Studies on teaching numbers to young children are generally carried out for students with intellectual disabilities (Kaplan, 2019; Murphy et al., 1984; Van Luit & Schopman, 2000). In addition, Alptekin (2015) listed the important points and suggestions about teaching counting skills. Albayrak et al. (2019) examined the opinions of pre-service teachers on how to teach the concepts of counting and numbers. However, there is a gap in the literature about the studies examining the pre-service elementary teachers' skills of teaching numbers. Therefore, in this study, pre-service elementary teachers' skills of teaching numbers were investigated through digital storytelling.

Teaching Numbers to First Graders

As a tool that forms the basis of mathematics, numbers can be used in different meanings. Haylock and Cockburn (2008) mention three types of numbers according to the use of numbers. First is the nominal use of numbers and allows objects to be distinguished. For example, waiting for bus 3 is not waiting for 3 buses or waiting for 3th bus. The nominal use distinguishes it from the others by labeling it with the number 3. The counting use of numbers, which is called cardinal use of the number, is used to express the number of elements in a set. For example, like 3 pens. The last type of using numbers is sorting use, which refers to giving order to the objects, like the student in the third row. This is also known as the ordinal use of numbers. The comprehension of the numbers requires the students to understand these three meanings together.

Children's learning numbers also requires the development of counting skills. Counting skills are prerequisites for teaching number-related concepts, developing prediction skills, and mathematical thinking (Olkun & Toluk Uçar, 2012; Olkun et al., 2013; Muldoon et al., 2013). Van de Walle et al. (2014) stated that the counting skills of children depend on counting numbers sequentially and associating the sequence of numbers with one-to-one matching of items in a set. According to Baykul (2000), some preparatory studies should be done before teaching of numbers to children in the first grade. These are the activities of rhythms, meaningful counting, conservation of numbers, quantity comparisons based on intuition, and one-to-one matching. Accordingly, the child must first acquire back-and-forth rhythms for counting skills. The child can count rhythmically by heart, so he must learn to count meaningfully. The child should count the number names corresponding to each object. The ability to learn the concept of number also depends on gaining conservation. Comparing the number of elements of the sets made with one-to-one matching is also important and necessary for children to comprehend the size expressed by the number. Altun (2014) states that the teaching of numbers actualizes in three stages:

1. Introducing the number: Showing the number with a cluster scheme, counting the number of set elements with one-to-one matching, giving examples of different sets suitable for the number.

2. Comprehending the number: The part-whole relationships of the number, the division of the number into parts.

3. Use of number: Using the number to represent quantity and order, ordinal and cardinal meanings of number.

Olkun and Toluk Uçar (2012) listed the developmental stages of numbers in children: verbal counting, regular counting, one-to-one matching, comparison (less-more, equality), cardinal use, conservation of number. According to verbal counting stage, the child starts counting the numbers randomly in the pre-school period without understanding them conceptually. After this stage, called regular counting, the child discovers a sequence among the numbers and starts counting regularly. Then, while counting, he understands the necessity of using a number for each object in a set and performs the one-to-one matching. In this stage, classification, sorting, and comparison activities with one-to-one matching contribute to the development of the number concept. He then realizes that the number corresponding to the last object in the set represents the number of set elements; thus he understands cardinal use. Conservation is acquired by realizing that different arrangements of the elements of a set do not change the number of elements.

Understanding numbers is also closely related to the development of the number sense. The development of number sense is vital for developing mathematical thinking and forming a mathematical background in young children. Number sense is the flexible use of numbers when mentally estimating, evaluating, calculating between number representations, and relating numbers, symbols, and operations to make sense of a numerical situation (Markovits & Sowder, 1994). According to McIntosh et al. (1992), the components of number sense are the ability to understand numbers and operations and use numbers in flexible ways for mathematical judgments. Studies show that mathematical activities can improve number sense (Markovits & Sowder, 1994; McIntosh et al., 1992). Number sense is also associated with estimation and computational skills (McIntosh et al. 1997; Reys & Yang, 1998), and it is included in the mathematics curriculum concerning estimation skills (Ministry of National Education [MoNE], 2018).

Digital Storytelling

Digital storytelling combines narratives with digital media to create a story (Robin, 2008). In digital stories, students create a short video film by designing a story with their knowledge of a particular subject and combining it with painting and musicfor the aim of using these films as an educational application in the classroom (Wang & Zhan, 2010). In this digital story design, students first determine the subject, explore it, and create the story from the obtained information by combining it with multimedia elements, so a short video of a few minutes is created (Kajder, 2004). Digital stories usually last between two minutes and ten minutes in length (Foley, 2013). The reasons such as the cheaper technology tools, the more accessible information and equipment on the subject, and the easy sharing of the created stories on the web have made the use of this approach widespread (Meadows, 2003). The elements of digital stories are as follows:

Perspective: Determining the story's main idea and the author's point of view.

A striking question: A key question that will keep the audience engaged until the end of the story.

Emotional content: The emotional connection of the story content with the audience.

Good vocalization: Voiceover to help the target audience understand the content.

Music that enhances the impact: Background music that matches the story and serves the purpose of the story.

Affordability: Using enough content to tell the story without overloading it.

Speed: The rhythm of the rapid and slow pace of the story (Bull & Kajder, 2004; Robin, 2008).

The use of digital stories positively affects student performance, improves students' 21st-century skills, and increases motivation and participation in lessons (Dogan & Robin, 2008; Robin, 2008). In addition, the use of digital stories in teacher education increases pre-service teachers' knowledge, provides them interdisciplinary knowledge for problem-solving, and improves their pedagogical skills (Shelton et al., 2017; Starcic et al.2016). Pre-service teachers view using digital stories in the teaching process positively due to their advantages, such as active participation, success, motivation, creativity (Özpınar, 2017).

Studies are also carried out to use digital stories in mathematics education. Gould and Schmidt (2010) reported the positive results of using digital stories in trigonometry teaching. Albano and Pierri (2017) developed a storytelling model based on digital story problems in mathematics education and reported that this model has positive results on students' mathematics achievement. Walters et al. (2018) used digital stories as a problem-solving strategy tool for pre-service mathematics teachers. They reported that pre-service teachers evaluated this strategy as an effective tool in teaching mathematics concepts to their future students, providing participation and motivation. Besides, Niemi et al. (2018) examined the effects of using digital stories in mathematics teaching. The results showed that digital stories in mathematics lessons support the development of mathematical literacy and 21st-century skills. It enables students to work in collaborative groups actively, generate new ideas, associate them with daily life, and show high creativity, motivation, and participation. Dinçer and Yılmaz (2019) found that the digital stories developed in data analysis provide conceptual learning and middle school students showed a positive attitude towards digital storytelling. Albano et al. (2020) revealed the contributions of digital stories in mathematical processes. Bratitsis and Mantellou (2020) examined the effect of using digital stories in teaching two subtraction algorithms, and it was found that students learned these algorithms more efficiently through digital storytelling. However, it is seen that there is a gap in the studies that examine the teaching skills of pre-service teachers through digital stories from a pedagogical perspective.

Aim of The Study

This study aimed to examine the pre-service elementary teachers' skills of teaching numbers through digital stories. In the study, to determine whether gender and mathematics achievement have a significant effect on pre-service teachers' skills of teaching numbers, the effects of these variables on their skills of teaching numbers were also examined. The research questions are as follows:

1. What is the level of pre-service elementary teachers’ skills of teaching numbers through digital stories?
2. What deficiencies do they have in the stages of teaching numbers?
3. Do their skills of teaching numbers vary by gender?
4. Do their skills of teaching numbers vary by mathematics achievement?

Method

This study was carried out using the descriptive methodology. Descriptive studies obtain data to identify certain characteristics of a group. It is the most widely used method for summarizing the characteristics of individuals or groups in educational research (Fraenkel, Wallen & Hyun, 2012). This method was used to describe pre-service elementary teachers' skills of teaching numbers.

Participants

The study was carried out in the Education Faculty of a university in the Black Sea Region in the 2020-2021 academic year. Convenience sampling from random sampling methods was used to select the participants. The participants consist of 98 pre-service elementary teachers attending the Mathematics Teaching course. 72 of them are female (73.5%), 26 are male (26.5%).

Data Collection Tool

The data collection tool consists of digital stories created by pre-service teachers on teaching numbers. In the scope of the Mathematics Teaching I course, pre-service teachers were asked to create a digital story to teach first-grade students the achievement of "*Read and write numbers*" (MoNE, 2018) belonging to the first year of the Mathematics curriculum. Firstly, digital stories were introduced to pre-service teachers. How digital stories are used in the educational environment, and the design stages of them were explained. A sample of the digital story was created using the Photo Story 3 program and shared with them. Then they were asked to design a digital story following the stages proposed by Jakes and Brennan (2005). These stages are; writing, scripting, storyboarding, using multimedia tools, creating and sharing a digital story. Pre-service teachers were given 15 days to design digital stories.

All ethical rules were followed in this study. Ethics committee approval for the study was obtained with the decision of the XXX University Social and Human Sciences Ethics Committee, dated 26.02.2021 and numbered 2021/184.

Data Analysis

The digital stories designed by the pre-service teachers were analyzed with the rubric developed in the study. The rubric was developed by taking into consideration the stages of teaching numbers by Altun (2014). The rubric is included in Table 1.

Table 1. Rubric Used in The Evaluation of Digital Stories

	The stages of teaching numbers	Insufficient (0 points)	Moderate (1 point)	Sufficient (2 points)
1	Introducing the number			
2	Comprehending the number			
3	Use of number			

The three stages of teaching numbers in the rubric were evaluated using the assessments of insufficient - moderate - sufficient. The total score of the rubric ranges from 0 to 6. Total scores of pre-service teachers' skills of teaching numbers were obtained by evaluating the digital stories with the rubric. To ensure scoring reliability, after the researcher scored the digital stories, they were presented to an expert working in mathematics education. He was asked to score the digital stories by using the rubric independently. Then, the scores were compared, the different scorings were determined, and after discussion, a joint decision was made by the researcher and the expert. So the scoring was finalized. Descriptive statistics of total scores were calculated and presented in the findings. Since the total scores did not show a normal distribution (Kolmogorov-Smirnov; $p<0,05$), Kruskal Wallis and Mann Whitney U tests from nonparametric analysis methods were used in the data analysis. The first-grade mathematics course scores of the pre-service teachers were used as their mathematics achievements. The deficiencies of the pre-service teachers in the stages of teaching numbers were also determined by examining the digital stories and summarized. The data were analyzed in SPSS 17.0 program.

Findings

Pre-Service Elementary Teachers' Skills of Teaching Numbers with Digital Stories

The descriptive statistics of the pre-service teachers' total scores are presented in Table 2.

Table 2. Descriptive Statistics of Pre-Service Teachers Total Scores

	Pre-service teachers	**M**	SD	Min	Max	Skewness	Kurtosis
1	Female	3,33	1,02	1,00	6,00	0,43	1,47
2	Male	2,96	0,92	1,00	6,00	0,76	1,73
3	Total scores	3,23	1,00	1,00	6,00	0,51	1,33

When the scores of the pre-service teachers were evaluated according to the min-max score range (0-6 points), the mean scores of the female pre-service teachers (M=3.33, SD=1,02), male pre-service teachers (M=2,96, SD = 0,92), and all the pre-service teachers

(M=3.23, SD=1,00) are observed to be close to the "moderate" level. Therefore, it can be concluded that pre-service teachers' skills of teaching numbers are at a moderate level.

The Deficiencies of Pre-Service Elementary Teachers in The Stages of Teaching Numbers

When the digital stories were examined in terms of design, it was determined that pre-service teachers designed digital stories well enough; the visuals, voices, and background music were satisfying. However, there were some deficiencies in terms of teaching numbers. It is possible to list these deficiencies as follows;

In the stage of "introducing the number", it was found that the pre-service teachers generally showed the number with a cluster diagram and wrote it with symbols. They gave different examples about the numbers, thus enriching the examples related to the number will enable the child to abstract the number easily. At this stage, the most striking deficiency in the stories was counting the number of the cluster's elements without one-to-one matching. Some pre-service teachers preferred to count the number verbally rather than match the elements in the cluster one-to-one. They thought that this was sufficient for the child to comprehend the number. Therefore, it can be said that some pre-service teachers overlooked the importance of one-to-one matching. Whereas one-to-one matching is essential in understanding the number and developing the number sense. In Figure 1, examples of this stage from digital stories were presented.

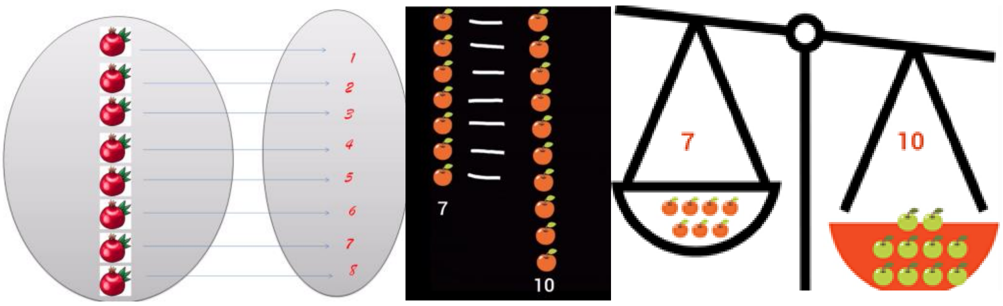

Figure 1. Examples of Activities About "Introducing The Number"

When the activities designed for the stage of "comprehending the number" were examined, it was seen that a small number of pre-service teachers designed activities for this stage. It is crucial to teach the part-whole relationships of numbers in developing the number sense and estimation skills. However, unfortunately, it has been found that pre-service teachers did not emphasize the part-whole relationships of the numbers and include this stage in their digital stories; many of them have overlooked this stage.

In the activities designed for "use of the number", the pre-service teachers created different activities according to whether the number expresses the quantity or sort. It was seen that they designed activities expressing the quantity of the number and counting objects with one-to-one matching to acquire the child the cardinal use of the number. Secondly, they designed visuals to teach the sorting meaning of the number. In Figure 2, there is an example of images belonging to the stories.

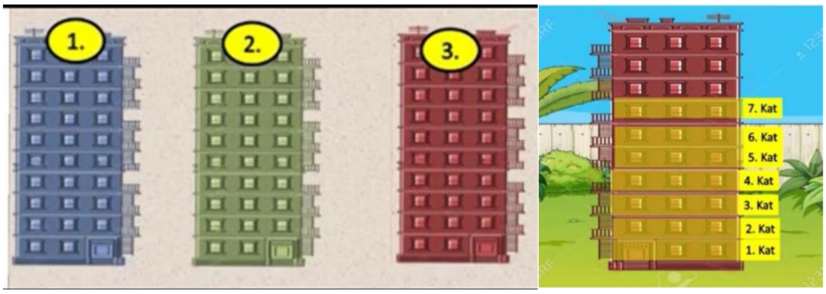

Figure 2. Examples of "Use of Number" in Digital Stories

Examination of Total Scores by Gender

The pre-service teachers' total scores of teaching numbers were compared according to their gender, and the results in Table 3 were obtained.

Table 3. The Mann Whitney U Test Results For The Examination of Total Scores According to Gender

		N	Average Rank	Sum of Rank	U	p
Total scores	Female	72	52,76	3799,00	701,00	,030*
	Male	26	40,46	1052,00		

**p<0,05*

As shown in Table 3, the analysis results showed that the total scores of the pre-service teachers differ significantly according to their gender (U = 701,00; p <0,05). Average ranks according to gender revealed that female pre-service teachers (Average rank = 52,76) have significantly a higher average than male pre-service teachers (Average rank = 40,46). Consequently, it can be said that female pre-service teachers' skills of teaching numbers through digital stories are higher than males.

Examination of Total Scores by Mathematics Achievement

The pre-service teachers' total scores of teaching numbers were examined according to their mathematics achievement levels using the Kruskal Wallis test, and the results in Table 4 were obtained.

Table 4. The Kruskal Wallis Test Results For The Examination of Total Scores According to Mathematics Achievement

	Mathematics achievement levels	N	Average Rank	df	χ^2	p	Significant difference
Total scores	AA	12	58,58	6	7,548	,273	-
	BA	11	42,73				
	BB	25	52,64				
	BC	18	52,94				
	CC	18	49,44				
	DC	7	43,00				
	DD	7	31,14				
	Total	98					

As shown in Table 4, the total scores of the pre-service teachers did not show a significant difference according to their mathematics achievement levels ($\chi^2 = 7{,}548$; p> 0,05). Therefore, it can be stated that pre-service teachers' skills of teaching numbers do not differ according to their mathematics achievements.

Discussion and Conclusion

Teaching numbers to children requires the development of counting skills and number sense. However, counting is not a simple skill for young children as it requires using of different concepts together (Akman, 2002). The development of the number sense starting from pre-school years continues with the teaching of numbers formally in the first grade. In this regard, elementary teachers have a vital role in the development of this skill. In this study, pre-service elementary teachers' skills of teaching numbers through digital stories were examined. As a result, it was found that pre-service teachers' skills of teaching numbers were at a moderate level. When the digital stories were analyzed according to the stages of teaching numbers, it was determined that pre-service had some deficiencies. It was seen that the pre-service teachers showed the numbers with clusters on different examples to introduce the number to the children. However, it was also observed that many pre-service teachers did not design activities to match the numbers with the elements of the cluster one-to-one. One-to-one matching is important in comprehending the number and developing the number sense. Olkun et al. (2013) determined that the cardinal meaning, resulting from one-to-one matching, is less developed than other principles in children. It can be argued that this result is because teachers do not give enough examples of one-to-one matching and cardinal meaning in their lessons. In the stage of comprehending the number, it was determined that many pre-service teachers did not emphasize the part-whole relationships of numbers in their digital stories.

However, they designed activities to teach the quantity or sorting meanings of the number in the last stage. In this regard, the results of the study are in line with Albayrak et al. (2019)'s research. Albayrak et al. (2019) examined the opinions of pre-service pre-school teachers on how to teach counting and number concepts. As a result, they determined that pre-service teachers have deficiencies regarding the activities that can be done in the teaching of counting and number.

When digital stories were evaluated in terms of design, it was determined that pre-service teachers created satisfying digital stories. Similarly, Shelton et al. (2017) conducted a 7-week hands-on training in which 31 primary school pre-service teachers created digital storytelling videos. As a result of the study, it was reported that all pre-service teachers created appropriate videos, liked using digital storytelling in education environments, and were interested in using it with their future students. However, in a different study, it was observed that pre-service teachers encountered technical problems and had difficulties while creating digital stories (Özpınar, 2017). In this study, the pre-service teachers did not experience any technical problems or difficulties. It has even been observed that they are enthusiastic and motivated, and they make different and creative designs with different programs. These positive results were also expressed as a result of other researches. Digital stories improve pre-service teachers' problem-solving and pedagogical competencies (Starcic et al., 2016), and they evaluate this method as an effective tool to teach mathematics concepts to their students, to provide participation and motivation (Özpınar, 2017; Walters et al., 2018). The results of different studies revealed that although teachers' perceptions of digital stories in the classroom were almost entirely positive after the workshops, most of them did not use digital storytelling in practice (Doğan & Robin, 2018).

Another result of the study was that the pre-service teachers' skills of teaching numbers through digital stories differ significantly according to gender, and female pre-service teachers had higher number of teaching skills than males. Consequently, it can be argued that women are better at teaching numbers than men. Besides, pre-service teachers' skills did not differ according to their mathematics achievement. This result showed that pre-service teachers' mathematics achievement did not affect their ability to teach numbers. It can be thought that academic success is not so effective on the teaching skills.

Recommendations

The research results revealed that although the pre-service teachers' skills of teaching numbers were at a sufficient level, they have some deficiencies in the stages of teaching numbers. Therefore, pre-service teachers should comprehend the stages of teaching numbers well during undergraduate education. The need for the studies is striking due to the limited number of studies in the literature on this subject. Therefore, studies that examine these skills of pre-service teachers and elementary teachers with different methods will fill the gap in the field and guide researchers and educators.

Sınıf Öğretmeni Adaylarının Sayıları Öğretme Becerilerinin Dijital Hikayelerle İncelenmesi

Özet:

Hızla dijitalleşen dünyada eğitimin de giderek teknolojik yöntemlere yöneldiği görülmektedir. Dijital hikaye eğitim kazanımlarının bilgisayar ortamında fotoğraflar ve hikayelerle öyküleştirildiği yeni bir eğitim yöntemidir. Bu çalışmada Sınıf Öğretmeni adaylarının ilkokul birinci sınıf öğrencilerine sayıları öğretme becerileri dijital hikayeler aracılığıyla incelenmiştir. Betimsel türde gerçekleştirilen araştırmaya Sınıf Öğretmenliği 3. sınıfta öğrenim gören 98 öğretmen adayı katılmıştır. Öğretmen adaylarının sayıları öğretmek amacıyla tasarladıkları dijital hikayeler, çalışma kapsamında geliştirilen rubrikle değerlendirilmiştir. Elde edilen puanlar betimsel olarak ve nonparametrik istatistik yöntemleriyle analiz edilmiştir. Analiz sonuçları, sınıf öğretmeni adaylarının sayıları öğretme becerilerinin orta düzeyde olduğunu, cinsiyete göre anlamlı fark gösterirken matematik başarısına göre anlamlı olarak farklılaşmadığını ortaya koymuştur. Ayrıca dijital hikayelerin analizi, öğretmen adaylarının sayıları öğretirken sayının tanıtılması ve sayının kullanım şekilleri aşamalarına yönelik etkinliklere dijital hikayelerinde yer verdiklerini ancak sayının parça-bütün ilişkilerinin kavratılmasına yönelik etkinlik tasarlamadıklarını göstermiştir. Araştırma sonuçları öğretmen adaylarının sayıların öğretim aşamalarında bazı eksikliklerinin olduğunu göstermiştir.

Anahtar kelimeler: birinci sınıf öğrencileri, dijital hikayeler, matematik eğitimi, sayıların öğretimi, sınıf öğretmeni adayları.

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Research Article

The Effect of STEM Approach in Science Education on Academic Achievement: A Meta-Analysis Study

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Abstract - In this research, a meta-analysis study was carried out to investigate the effect of the STEM approach in science education in Turkey on academic achievement of students. In accordance with the criteria determined in the meta-analysis study, the studies included in the research are at the Master's and doctoral levels, and these studies were taken from the National Thesis Center database of the Council of Higher Education. In the study, a total of 147 studies carried out between 2010 and 2020 were examined and 31 Master's and 3 Doctoral thesis studies fit the criteria of inclusion were included in the meta-analysis. The sample of a total of 34 studies consisted of 1962 students at the 4th, 5th, 6th, 7th and 8th grade levels. Analyses were made with the help of Comprehensive Meta-Analysis V2 statistical program. The random effects model was used because the structure was heterogeneous when the studies were analysed and combined. As a result of the research, the effect size of the STEM approach in science education on students' academic achievement was calculated as 1.420. So, this value has a large effect size. In this context, it has been concluded that the STEM approach in science education has a positive and large effect on increasing the academic achievement of students. The effect size on academic achievement differed according to the grade level, and the highest effect was observed in the studies at the 4th grade level.

Keywords: STEM, science, academic achievement, meta-analysis

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Introduction

The fact that science and technology change and develop in a way that competes with time has made it necessary to be in a state of constant renewal in the age we live in. Countries that are able to adapt to these changes and developments and therefore constantly innovate and develop their needs in this direction have always managed to be one step ahead of other countries in all fields, especially in economy. The economic well-being of a society is directly related to the education system of that society. Because, powerful countries are referred to as countries that can both use and develop these technologies without ignoring the changes in science and technology by keeping their education systems in the foreground. In the globalizing world, the economic competition of countries, rapidly developing science and technology will affect our lives in the future more than today (MEB, 2006). Therefore, the ongoing varies and advancements in science and technology affect the education field, and changes in the field of education also affect science and technology (Selvi & Yıldırım, 2017). These innovations in the current era have also increased the need for qualified manpower. In this process, it is expected that education will serve to meet the needs of employment, manufacturing, innovation and qualified workforce, as in many sectors (Bybee, 2010; Dugger, 2010). Countries with high skilled labor standards are always one step ahead of other countries in other fields, especially in economy. One of the main causes for this situation is that the countries in question realize the importance of the interaction between science, technology and education, develop their education policies in line with their needs, and especially constantly update their science and mathematics education programs in pursuance of the needs of the developing age (Selvi & Yıldırım, 2017). It is of great importance that individuals acquire 21st century skills such as innovation, problem solving, group work, communication, critical thinking and research, especially in order to keep up with the rapidly developing science and technology in our age (Aydın, Saka, & Guzey, 2017). Therefore, the education of individuals who will be employed in the future is also a very important issue. Although it is purposed to raise persons who are adequate in the fields of science, technology, engineering and mathematics in order to take a place in the global economic race, it is stated that education has a serious task in raising these individuals (Akgündüz, 2016; Bybee, 2010; OECD, 2017).

STEM is a concept obtained by bringing together the acrostics of the words Science, Technology, Engineering, Mathematics (Moomaw, 2013). It is an approach put forward by the United States of America (USA) in the early 1990s to integrate the disciplines of science, mathematics, engineering and technology (Sanders, 2009). STEM education is an interdisciplinary teaching approach that combines the fields of science, technology, engineering and mathematics and ensures that these disciplines are taught in relation to each other (Roberts, 2012; Wang, Moore, Roehrig, & Park, 2011).

STEM education mainly purposes to provide students with some skills that individuals need to learn and develop in the current age (Bybee, 2013; Thomas, 2014). These skills, which are called 21st century skills; are referred to as learning and innovation skills (creativity and innovation, problem solving, communication, critical thinking, collaboration); information, media and technology skills (information literacy, media literacy, “Information, Communication and Technology” literacy); life and career skills (flexibility and adaptability, assertiveness and self-management, social and intercultural skills, leadership and responsibility, productivity and accountability) (Partnership for 21st Century Learning, 2007).

STEM education is very important for Turkey's economic competitiveness (Corlu, Capraro, & Capraro, 2014). As a matter of fact, the results of Turkey's international exams such as PISA (Programme for International Student Assessment) and TIMSS (Trends in International Mathematics and Science Study) reveal Turkey's need for STEM education. According to the 2015 TIMSS results, it is seen that Turkey's science and mathematics achievement is lower than the scale average at all levels, while according to the 2018 PISA results, Turkey's science and mathematics achievement is below the average of OECD countries (Okulu, 2019). Having to compete with these countries at every point, Turkey's results from international exams are seen as a problem (Acar, 2018; Aydeniz, 2017; Çolakoğu & Gökben, 2017; Elmalı & Balkan Kıyıcı, 2017; Gülhan, 2016; Okulu, 2019).

Many countries such as USA, England, Singapore and South Korea attach great importance to STEM education for being successful in PISA / TIMSS exams. When the education systems of the countries that are successful in these exams and are in the first place are examined, it is seen that most of these countries integrate STEM education with their curriculum (Yıldırım, 2018). It is aimed to strengthen STEM education in the MEB 2015-2019 Strategic Plan (MEB, 2015). In the STEM Education Report prepared by the Ministry of National Education, the following were suggested; STEM education should be given to all students, STEM education action plan can be developed together with institutions/organizations such as universities, TUBITAK and TUSIAD, STEM centers can be established and in-service trainings can be given to teachers there, and the content of the courses in the curriculum can be reduced gradually. Also, it is emphasized that transition to stage STEM education can be made (MEB, 2016). It is seen that science, engineering and entrepreneurship practices were included in the 2018 Science Curriculum in a different way from previous years (MEB, 2018).

In the recent past, a remarkable increase has been observed in studies on STEM education in our country (Aysu, 2019; Gülseven, 2020; Ozan 2019; Özaslan, 2019; Soysal, 2019). Studies conducted to measure the effectiveness of these studies and to evaluate the studies in general as a whole are rare. In this respect, it is thought that the meta-analysis study obtained from studies measuring the effect of STEM approach on science education in our country will contribute to the relevant literature. It is thought that this study will be useful in terms of describing the results of master's and doctoral thesis studies conducted between 2010-2020 and in accordance with the research criteria and evaluating the situation as a whole.

In this study, it is aimed to reveal the effectiveness of the STEM approach on the academic achievement of students in science education. In this respect, it is aimed to examine the studies conducted in our country in line with the STEM approach between 2010 and 2020 through meta-analysis. In the direction of this main purpose, answers to the following questions were sought.

I. How is the effect of STEM approach used in Science education on academic achievement of students?

II. In terms of grade level, is there a remarkable difference between the effect sizes of the STEM approach used in science lessons on the academic achievement of students?

Method

Research Model

Due to the rapid increase of studies conducted today, even experienced experts have difficulty in following these studies. Therefore; a meta-analysis method was developed in order to help to understand the information, to gather the results of more than one study, to reach a general conclusion and to enable the data to be analysed quantitatively (Walker, Hernandez, & Kattan, 2008). Meta-analysis is a method used to reach a generalization from the results of these studies by evaluating the researches conducted for a specific purpose or subject as a whole (Büyüköztürk, Kılıç Çakmak, Akgün, Karadeniz, & Demirel, 2016). Its' difference from other literature review methods is that it is based on statistical methods in bringing together, integrating and analysing research findings (Durlak, 1995). Meta-analysis is simply the analysis of other analyses. It brings together the results of other studies in a coherent way (Cohen & Manion, 2001; Wolf, 1986).

With this method; individual studies carried on at several times, in different places and with different sample groups can be combined to reach more general, safer and more valid results. This result is statistically stronger than an analysis of any single study due to an increased number of subjects, diversity in any subject or cumulative effects and results (Dinçer, 2021).

Given the ever-increasing amount of new information, the use and value of this method will likely increase. To achieve this, those are needed: variables, outcomes and goals should be well defined; to be able to access to appropriate and well-documented studies; in identifying and selecting the research, prejudgement and bias should be evaluated, heterogeneity should be determined, and a sensitive analysis should be performed (Rosenthal & Dimatteo, 2001).

As in every study, there are some steps to be followed for conducting the study in a healthy way, to review all the necessary steps in a planned manner and to do it in a sequence while doing meta-analysis. If the determined steps are followed, more distance can be taken by spending less time.

Steps of Meta-Analysis

The stages of meta-analysis can be listed as determining the aims and objectives of the research, literature review, coding, calculation of effect size, statistical model selection and analysis, and finally results and comments.

Determining the Purpose and Objectives of the Research

First of all, the problem of the research should be determined and hypotheses suitable for the problem should be established (Camnalbur, 2008).

Literature Review

In a meta-analysis study, it is recommended to use many of the literature review methods together when conducting a resource research on any subject (Şahin, 2005).

Coding Studies

One of the considerable issues in the coding process is the creation of a general coding system that can cover the data found in all the studies to be used in the study and the creation of a special coding system that can show the unique characteristics of the studies (Şahin, 2005).

Calculation of Effect Sizes

Effect size is defined as a value used for calculation of relation strength and its direction in a study. Developed by Cohen in 1988, this element is defined as the heart of meta-analysis (Kaşarcı, 2013).

Statistical Model Selection and Analysis

Since the determination of the statistical model related to the research results may change, model selection is of great importance in order to obtain reliable results and reach a general judgment. This model to be used also affects how the statistical analysis will be done and how the results will be interpreted (Camnalbur, 2008).

Results and Findings

After performing aforesaid processes meta-analysis studies are interpreted and made into report.

Data Collection

In this meta-analysis study, the subject of which has the effect of STEM approach on academic achievement in science education, the theses published on the research subject were searched in the database of the National Thesis Center of the Council of Higher Education in order to determine the studies included in the research. When the studies acquired as a result of the scans were investigated, thesis studies were preferred to be included in the analysis and coding was made. Some of the studies were not comprised in the analysis because they were not experimental studies and some did not contain enough data to analyze. As a result, the sample of the meta-analysis study consists of 34 postgraduate thesis studies examining the effect of STEM approach on academic achievement in science education.

Criteria in Study Selection

The criteria adhered to in study selection were examined under two headings as inclusion and exclusion criteria.

Inclusion Criteria

Experimental and quasi-experimental studies that contain sufficient data (sample sizes, averages and standard deviations of the groups) to calculate the effect size investigating the effect of the STEM approach on academic achievement in science education in the theses for which publication permission was granted in the National Thesis Center were included in the analysis. Apart from this, the criteria determined for the studies included in the study can be listed as follows.

- The study must have been done between 2010-2020
- It must be a PhD/Master's thesis,

• In real experimental and quasi-experimental designs, pretest-posttest model with control group must be used,

• It must include sample size, mean and standard deviation values,

• It must have clearly investigated the effect of the STEM approach on student achievement/academic achievement,

• The studies must have taken place in Turkey and the language must be Turkish.

In some studies, more than one data was taken because there was more than one experimental group or achievement test.

Exclusion Criteria

The fact that a study is not included in the meta-analysis in the current study is due to the fact that the study is not within the research limits or does not have the necessary statistical data for meta-analysis (Lipsey & Wilson, 2001; Wolf, 1986). Therefore, studies that did not meet the inclusion criteria were excluded from studies to be used for meta-analysis.

Characteristics of the Study (Dependent and Independent Variables)

There are cause-effect relationships in dependent and independent variables (Büyüköztürk, Kılıç Çakmak, Akgün, Karadeniz, & Demirel, 2016). In meta-analysis studies, the effect that creates the effect sizes is the independent variable, and the resulting effect size is the dependent variable (Cohen, 1988). The 34 studies reached were carefully examined in regards to content and data. The characteristics that are thought to have an impact on the study were taken as the independent variable. In the general research problem, the independent variable can be expressed as the teaching approach. For theses researched in terms of academic achievement; grade level is the independent variable, and the dependent variable is the academic achievement level.

Coding Studies

A clear and detailed coding system should be developed for the studies included in the meta-analysis. The coding system used in the research consists of three parts. The first part is the “work identity”. This section contains information such as the identification number of the study, the name of the study, the name of the author, and the year of the study in order to identify the study. The second part is the “study content”. In this section, information such as the course and education level in which the STEM approach is applied are given. The third part is “study data”. In this section, information about the sample size, means and standard deviation values obtained from the experimental and control groups in the studies were determined. The data in the studies were processed in pursuance of the coding form. In this study, the coding form created in accordance with the purpose of the study includes study code, year, type of thesis, test types used, sample numbers of control and experimental groups, mean and standard deviation data.

Data Analysis

The analysis of the study was made under two headings. First, descriptive statistics were made and reported. Then, the data were analyzed using the meta-analysis method. The operation efficiency technique, which is used in cases where arithmetic means that cannot be obtained from the same scale, are used while performing the analysis (Cohen, 1988; Lipsey & Wilson, 2001). This technique was developed by Glass (1976) and is used more in psychology, social sciences and educational sciences. The purpose of this is to reveal the difference between the mean of the experimental and control groups in experimental studies (Hunter & Schmidt, 1990). Transaction effect meta-analysis uses the standardized effect size denoted by the symbols “d”, “g” or “ES”. The effect size is found by dividing the difference between the mean of the experimental group and the control group by the total standard deviation. There are different coefficient classifications in the literature for effect size classification (Cohen, 1988; Thalheimer & Cook, 2002).

According to Cohen (1988), if the effect size is at the level of 0.20 - 0.49, it is defined as small; if it is at the level of 0.50 - 0.79, it is medium; if it is at the level of 0.8 and above, it is defined as large. According to Thalheimer and Cook (2002) it is interpreted as large at - 1.09 level, very large at 1.10-1.44 level, excellent effect at 1.45 level and above.

The Comprehensive Meta-Analysis (CMA) program was used in the analysis of the data. Analyses were made by choosing the format in which standard deviation, mean and sample sizes were entered as data in the CMA program. The significance level of the analyses was determined as 0.05. First, the effect sizes of individual studies that met the inclusion criteria in the meta-analysis were calculated. Hedges's g coefficient was used to calculate the effect sizes. Then, heterogeneity test was performed to determine the model to be applied. The random effects model used in heterogeneous studies was chosen, concluding that the study was heterogeneous, as both the p value was significant and the Q value was larger in the range where the df value coincided with the X^2 critical table comparison. The overall effect size was computed according to the random effects model. After these procedures, the analysis is largely completed. The next step is to prepare the necessary graphics and tables. The publication bias test of the subsequent meta-analysis is performed with different statistical methods. In publication bias, the funnel plot is evaluated first. Since funnel plot interpretation is a highly subjective process, publication bias statistics are also needed. One of them is Rosenthal's Protected N statistic. Here, Classic fail-safe N statistic expresses how many studies with zero effect value should be included in the analysis in order for the p value to be greater than the alpha value, that is, to remove the significance. It is also called the fault protection number. In addition, Orwin's fail-safe N statistic computes the number of studies required to bring this effect to zero by looking at the average effect size. In other words, while the critical value for Rosenthal is the p value, it is the effect size for Orwin (Dinçer, 2021).

Findings

In this section, the descriptive statistics of the studies included in the research as a result of the literature review and the findings related to the individual and combined effect sizes obtained by the meta-analysis method are given.

The main purpose of the meta-analysis study is to calculate the effect size of the STEM approach in science education on the academic achievement of students. In this context, studies that met the inclusion criteria were analyzed and interpreted by including the findings related to these analyzes.

Findings Related to Descriptive Statistics

As a result of the literature review, 147 studies examine the effects of the STEM approach in science education on the academic achievement of students between the years 2010-2020 were reached. 34 studies were included in the study because they contained sufficient statistical data and met the inclusion criteria.

Table 1. Proportions of Theses Included in the Study

Publication Type	Studies Reached	Studies Included into Meta-Analysis	Studies Excluded	Proportions of Studies Included (%)
Master's	130	31	99	23,85
PhD	17	3	14	17,65
Total	**147**	**34**	**113**	**41,50**

When Table 1 was examined, as a result of the literature review, a total of 147 studies, 130 master's theses and 17 doctoral theses, were reached. 23.85% of master's theses, 17.65% of doctoral theses and 41.50% of all accessed studies were included in the analysis.

Descriptive statistics related to the studies about the effect of STEM approach in Science education on student's academic achievement are given in tables.

Table 2. Frequencies of Theses related to Study Types

Publication Type	Frequency (f)	Percentage (%)
Master's	31	91,17
PhD	3	8,83
Total	**34**	**100**

There are 31 (91.17%) master's thesis studies and 3 (8.83%) doctoral studies included in the meta-analysis study within the scope of academic achievement variable. In addition, a total of 5 data were obtained from 2 master's thesis studies and 2 data from 1 doctoral thesis study, and 39 data from 34 studies in total were included in the analysis. The total number of samples in these studies was 1962 people, including the experimental and control groups.

Table 3. Distribution of Theses by Years

Publication Year	Frequency (f)	Percentage (%)
2010	-	-
2011	-	-
2012	-	-
2013	-	-
2014	1	2,94
2015	-	-
2016	-	-
2017	2	5,88
2018	9	26,47
2019	18	54,94
2020	4	11,77
Total	**34**	**100**

It is observed that the theses included in the meta-analysis study within the scope of the academic achievement variable were made mostly in 2019, with 18 (54.94%) theses.

Table 4. Distribution of Theses by Grades

Grade Level	Frequency (f)	Percentage (%)
Grade 4	2	5,88
Grade 5	5	14,71
Grade 6	9	26,47

Grade 7	12	35,29
Grade 8	6	17,65
Total	**34**	**100**

It attracts attention that 12 (35.29%) of 34 theses included in the meta-analysis within the scope of the academic achievement variable were studies with 7th grade students. The least number of studies is the 4th grade level with 2 studies with a rate of 5.88%.

Table 5. Distribution of Sample Numbers of Theses

Sample Number	Frequency (f)	Percentage (%)
1<N<50	19	55,88
50<N	15	44,12
Total	**34**	**100**

The sample number of 34 thesis studies (the sum of the sample numbers of the experimental and control groups) that examined the effects of the STEM approach in science education on the academic achievement of students in science lessons included in the research was 1962. While the studies with a sample number of up to 50 people consisted of 19 thesis studies with 55.88%; The number of studies with more than 50 samples was determined as 15 (44.12%).

Findings Related to Effect Size

In this section, in order to test the sub-problems, the individual and general effect sizes of the data (number of samples, standard deviation and arithmetic mean) taken from the studies included in the research are calculated with the CMA program and the publication biases of the studies in the research sample were tested.

The publication bias is given in Table 6 by calculating the individual and general effect sizes of 39 data obtained from 34 studies that met the inclusion criteria in the meta-analysis study on the effect of the STEM approach in science education on the academic achievement of students.

Table 6. Findings on Individual Effect Size at 95% Confidence Interval of Studies

No	*Study Code*	*Hedges's g*	*Lower Limit*	*Upper Limit*	*p*
1.	CEYLAN,2014A	0,898	0,355	1,440	0,001
2.	CEYLAN,2014B	0,859	0,318	1,399	0,002
3.	PARLAKAY, 2017	0,732	0,231	1,233	0,004
4.	SARICAN, 2017	0,408	-0,178	0,995	0,172
5.	GAZIBEYOGLU, 2018	0,576	0,029	1,122	0,039

6.	DOGANAY, 2018	2,206	1,429	2,982	0,000
7.	DEDETURK, 2018A	1,060	0,727	1,392	0,000
8.	DEDETURK, 2018B	3,167	2,699	3,635	0,000
9.	KARCI,2018	0,649	0,085	1,213	0,024
10.	NAGAC, 2018	0,298	-0,288	0,884	0,319
11.	ACAR, 2018A	1,446	0,784	2,107	0,000
12.	ACAR, 2018B	1,571	0,924	2,218	0,000
13.	KOCA, 2018	1,326	0,573	2,079	0,001
14.	CALISICI,2018	0,930	0,318	1,542	0,003
15.	HIGDE, 2018	2,768	1,949	3,587	0,000
16.	IRAK, 2019	0,387	0,119	0,654	0,005
17.	NECCAR, 2019	-0,253	-0,886	0,381	0,434
18.	CIMENTEPE, 2019	1,652	0,984	2,320	0,000
19.	OZAN, 2019	0,895	0,010	1,779	0,047
20.	BUYRUK, 2019	0,288	-0,244	0,820	0,289
21.	DOGAN,2019	0,616	0,185	1,048	0,005
22.	CETIN,2019	4,981	3,732	6,230	0,000
23.	SEN, 2019	0,813	0,087	1,539	0,028
24.	KURT, 2019	1,141	0,334	1,947	0,006
25.	AKKAYA 2019	2,495	1,678	3,312	0,000
26.	TASCI, 2019A	3,226	2,436	4,015	0,000
27.	TASCI, 2019B	2,074	1,430	2,718	0,000
28.	TASCI, 2019C	2,460	1,771	3,148	0,000
29.	BUYUKBASTIRMACI, 2019	6,933	5,789	8,078	0,000
30.	SOYSAL, 2019	0,843	0,363	1,324	0,001
31.	BAHSI, 2019	0,915	0,198	1,632	0,012
32.	AYSU, 2019	2,675	2,005	3,345	0,000
33.	OZLEN, 2019	0,003	-0,560	0,565	0,992
34.	KAYABAS, 2019	0,932	0,305	1,559	0,004
35.	IZGI, 2020	1,251	0,653	1,849	0,000
36.	OZASLAN, 2019	0,698	0,220	1,176	0,004
37.	OZTURK, 2020	2,446	1,705	3,188	0,000
38.	GULSEVEN, 2020	1,053	0,535	1,570	0,000
39.	GUVEN, 2020	0,445	-0,173	1,064	0,158

It is seen in Table 6, the individual effect size, lower-upper limits and p values of 39 data obtained from a total of 34 studies were calculated. When the hedges's values in the comparison are examined, only Neccar’s (2019) study has a negative effect size. The reason for this can be explained as the result of the research against the experimental group. Apart from this, when the lower and upper limits are examined, the lowest lower limit is -0.886; the highest upper limit was calculated as 8,078.

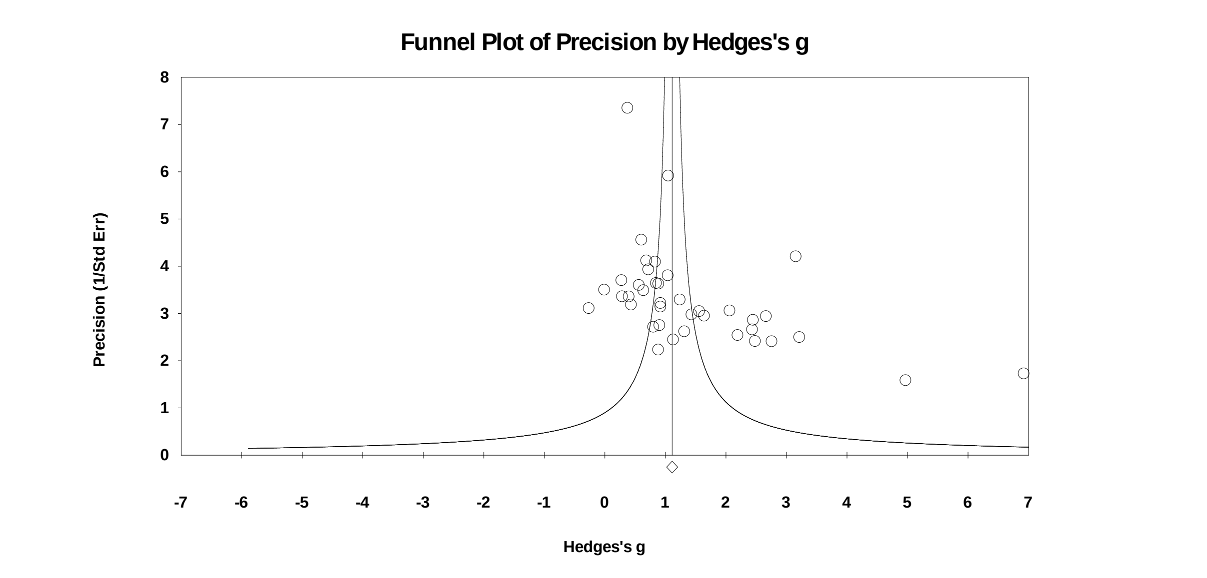

Figure 1. Distribution Funnel Plot of Effect Sizes According to Hedges' g Value Related to Studies

It is also necessary to calculate the overall effect size of the studies for which individual effect size calculations are made. It is great importance to control the heterogeneity of studies before calculating the overall effect size. Because the selection of the model to be used is determined at this step. In this context, the individual effect size funnel plot in Figure 1 needs to be interpreted. When the funnel graph is examined, individual studies are shown as a circle and the overall effect level is shown as a diamond. Individual studies are expected to be between the slope lines in the graph. However, it is seen that some studies are outside the slope lines. The fact that most of the individual studies are located within the slope line gives the interpretation that the study has a heterogeneous structure. However, this comment alone is not sufficient. Therefore, it is necessary to look at the Q and p values in order to interpret the heterogeneity situation objectively (Dinçer, 2021).

Table 7. Heterogeneity Test of Studies

Heterogeneity				***Tau-squared***			
Q-value	df (Q)	P-value	I-squared	TauSquared	StandardError	Variance	Tau
443,651	38,000	0,000	91,435	0,917	0,273	0,075	0,958

When Table 7 is examined, the Q value was computed as 443,651 as a result of the heterogeneity test. This value was found to be between 49,802-55,708 at the 95% significance level of 38 degrees of freedom in the X^2 table. Accordingly, the fact that the Q value is higher than the critical value range in the X^2 table (X^2=49,802-55,708 for df=38) shows that the studies included in the meta-analysis have a heterogeneous structure. If the result of the heterogeneity test is significant, the random effects model should be used (Dinçer, 2021).

The overall effect sizes of the fixed and random effects models of the studies included in the meta-analysis are given in Table 8.

Table 8. General Effect Sizes for the First Sub-Problem

Model	*N*	*Standard Error*	*Hedges's g*	*%95 Confidence Interval*		*Z-value*	*p-value*
				Lower Limit	*Upper Limit*		
Fixed	*39*	0,047	1,112	1,021	1,203	23,871	0,000
Random	39	0,163	1,420	1,101	1,738	8,738	0,000

The overall effect size was computed primarily according to the fixed effects model and was measured as 1.112 with a standard error of 0.047. However, as a result of the heterogeneity test obtained from the studies included in the meta-analysis, the model to be used in the calculation of the overall effect size was chosen as the random effects model.

As a result of the analysis of the random effects model, the overall effect size value was computed as 1,420 with a standard error of 0.163. It is seen that the effect size has a lower limit of 1,101 and an upper limit of 1,738 in the 95% confidence interval. The positive value of the overall effect size indicates that this effect is in favor of the experimental groups using the STEM approach. In addition, the calculated z value of 8,738 was found to be statistically remarkable with p=0,000. A P value less than 0.05 shows that there is a remarkable difference between the groups. According to the findings obtained in this direction, it can be interpreted that the STEM approach in science education has a positive and large effect on increasing the academic achievement of students (Dinçer, 2021).

The forest graph showing the distribution of Hedges's g effect sizes of the studies taken within the scope of the study and examining academic achievement is given in Figure 2.

Study name	Hedges's g	p-Value	Relative weight	Relative weight
CEYLAN.2014A	0.898	0.001	2.68	
CEYLAN.2014B	0.859	0.002	2.68	
PARLAKAY. 2017	0.732	0.004	2.69	
SARICAN. 2017	0.403	0.172	2.62	
GAZIBEYOGLU. 2018	0.576	0.039	2.68	
DOGANAY. 2018	2.208	0.000	2.45	
DEDETURK. 2018A	1.080	0.000	2.79	
DEDETURK. 2018B	3.167	0.000	2.71	
KARCI.2018	0.649	0.024	2.64	
NAGAC. 2018	0.298	0.319	2.62	
ACAR. 2018A	1.448	0.000	2.55	
ACAR. 2018B	1.571	0.000	2.57	
KOCA. 2018	1.328	0.001	2.45	
CALISICI.2018	0.930	0.003	2.60	
HIGDE. 2018	2.768	0.000	2.42	
IRAK. 2019	0.387	0.005	2.82	
NECCAR. 2019	-0.253	0.434	2.59	
CIMENTEPE. 2019	1.852	0.000	2.55	
OZAN. 2019	0.895	0.047	2.38	
BUYRUK. 2019	0.288	0.289	2.67	
DOGAN.2019	0.618	0.005	2.74	
CETIN.2019	4.981	0.000	2.00	
SEN. 2019	0.813	0.028	2.51	
KURT. 2019	1.141	0.008	2.43	
AKKAYA 2019	2.495	0.000	2.42	
TASCI. 2019A	3.228	0.000	2.45	
TASCI. 2019B	2.074	0.000	2.55	
TASCI. 2019C	2.480	0.000	2.54	
BUYUKBASTIRMACI. 2019	6.933	0.000	2.10	
SOYSAL. 2019	0.843	0.001	2.70	
BAHSI. 2019	0.915	0.012	2.51	
AYSU. 2019	2.875	0.000	2.55	
OZLEN. 2019	0.003	0.992	2.64	
KAYABAS. 2019	0.932	0.004	2.59	
IZGI. 2020	1.291	0.000	2.61	
OZASLAN. 2019	0.698	0.004	2.71	
OZTURK. 2020	2.446	0.000	2.49	
GULSEVEN. 2020	1.053	0.000	2.65	
GUVEN. 2020	0.445	0.158	2.60	
	1.420	0.000		

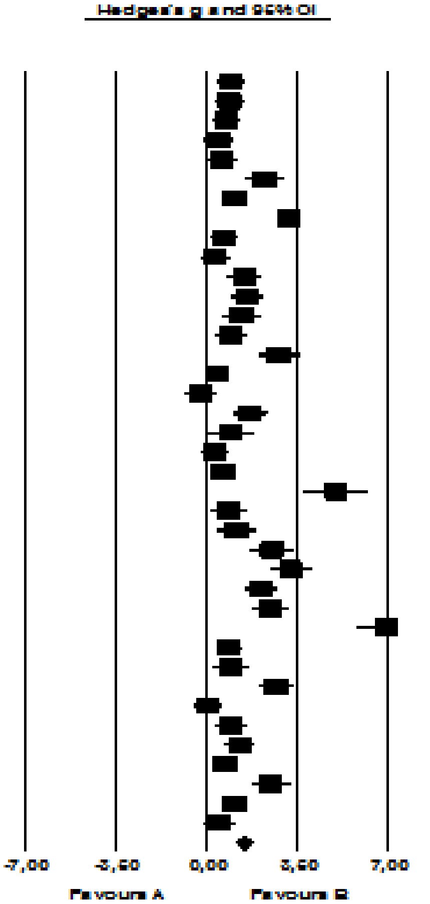

Figure 2. Forest Graph and Study Weights of Studies Related to the First Sub-Problem

When Figure 2 is examined, the parts represented as filled squares in the forest graph represent the individual effect sizes. The size of these squares is directly proportional to the number of samples. In addition, the horizontal lines passing through the squares represent the working ranges. The scale of the forest graph can be changed according to the effect size. Although the effect size value was generally concentrated between 0-1, the scale was kept wide in order to reveal the whole picture, since the effect size of a study was calculated as 6,933 (-7, +7). It is seen that the rhombus that expresses the overall effect size (1,420) in the forest graph is positive and a value greater than 1. This indicates that the STEM approach in science education has a large impact on academic achievement.

The Funnel Diagram of Studies Related to the First Sub-Problem (Publication Bias Graph) is given in Figure 3.

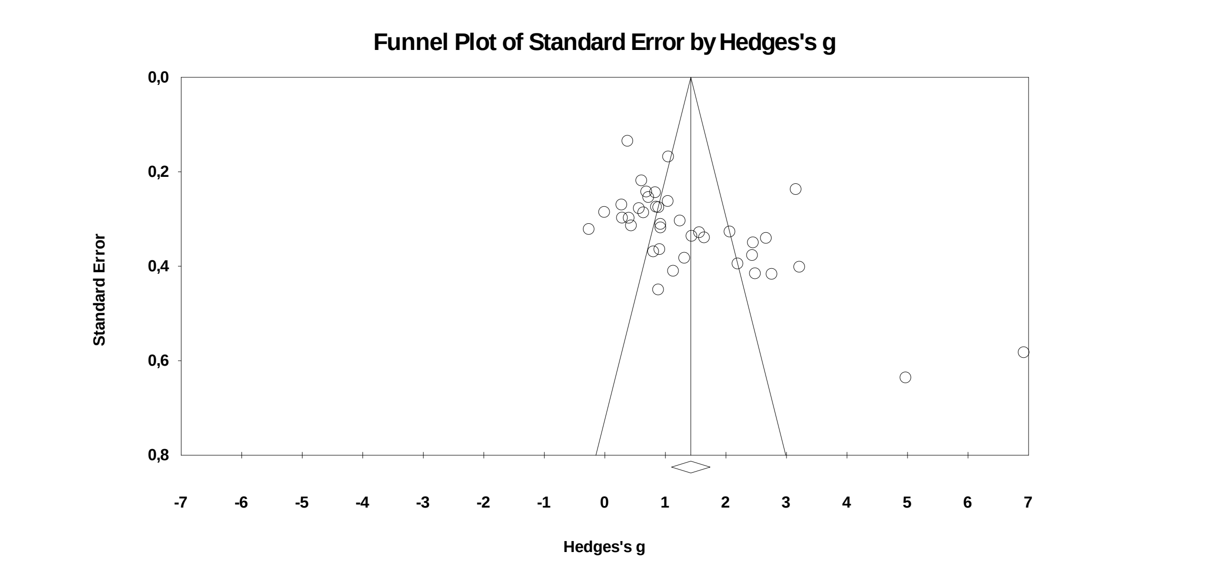

Figure 3. Funnel Diagram of Studies Related to the First Sub-Problem (Publication Bias Graph)

When the publication bias graph is examined, the effect sizes are seen on the horizontal axis, and values such as sample size or standard error are seen on the vertical axis. The line in the middle of the graph represents the overall effect value. Studies with a large sample size are generally clustered at the top of the graph and around the overall effect size. Studies with a small sample size are generally collected at the bottom of the funnel plot. It is expected that all individual studies will be scattered symmetrically and within the funnel lines to avoid publication bias. Studies scattered outside the funnel may cause publication bias. However, since Funnel graphs are generally evaluated subjectively, it is necessary to look at other publication bias statistics regarding publication bias (Dinçer, 2021).

Table 9. Classical Fail-Safe N Analysis of the First Sub-Problem

Rosenthal's Analysis of Protection Number	
Z value	26,151
p value	0,000
Alpha value	0,050
Z value for Alpha	1,959
N	39
Number of studies required to invalidate research	6905

When Table 9 is examined, the error protection number (fail safe N) obtained from the meta-analysis study was calculated as 6905 according to Rosenthal's method. This value is essentially the obtained p=0.000 significance value; It refers to the number of studies with an effect size value of “0” that should be included in the meta-analysis in order to increase the p>0.05 value, that is, to remove the significance. In this context, 6905 non-significant studies should be comprised in the meta-analysis in order to invalidate the findings and significance obtained from the meta-analysis study made from the data of 39 studies. Since this number is quite high, it can be interpreted that the meta-analysis study carried out is reliable and the publication bias is low (Dinçer, 2021).

Orwin's fault protection number (Orwin's Fail-Safe N Analysis) data are given in Table 10.

Table 10. Orwin's Fail-Safe N Analysis of the First Sub-Problem

Orwin's Fault Protection Count Analysis	
Hedges' g in observed studies	1,112
Criterion for an insignificant Hedges's g	0,100
Average Hedges' g for Lost Studies	0,000
Number of Lost Runs required to bring Hedges' g to less than 0.1	395

According to Orwin's error protection number method, the average effect size obtained from the current study was calculated as 1.112, and in order to decrease the effect size value of the study to an insignificant value, it was necessary to include in the meta-analysis; There should be 395 studies with an effect size value of "0". Since it is very difficult to reach this number in the literature, it can be interpreted that the publication bias of the meta-analysis study conducted according to the findings obtained in this method is low and reliable (Dinçer, 2021).

Moderator Analysis Related to Grade Levels

In this section, it has been investigated whether there is an important difference between the effect sizes of the STEM approach in science education in terms of grade level on the academic achievement of students. In this context, the effects of the STEM approach in science education in terms of grade level in terms of students' academic achievement were determined as comparative effect sizes. Comparisons were grouped into 4th, 5th, 6th, 7th, and 8th graders. The individual and general effect sizes of the studies regarding the grade levels, the lower and upper limits of the 95% confidence interval, and the p values are given in Table 11.

Table 11. Individual and Overall Effect Sizes for Grade level Moderators, Lower and Upper Limits at 95% Confidence Interval, and p Values

Grade Level	***Study Code***	***Hedges's g***	***Lower Limit***	***Upper Limit***	***p-value***
Grade 4	ACAR, 2018A	1,446	0,784	2,107	0,000
Grade 4	ACAR, 2018B	1,571	0,924	2,218	0,000
Grade 4	OZTURK, 2020	2,446	1,705	3,188	0,000

Fixed		1,772	1,380	2,164	0,000
Random		1,798	1,208	2,388	0,000
Grade 5	PARLAKAY, 2017	0,732	0,231	1,233	0,004
Grade 5	KARCI,2018	0,649	0,085	1,213	0,024
Grade 5	IRAK, 2019	0,387	0,119	0,654	0,005
Grade 5	OZAN, 2019	0,895	0,010	1,779	0,047
Grade 5	GUVEN, 2020	0,445	-0,173	1,064	0,158
Fixed		0,507	0,307	0,707	0,000
Random		0,507	0,307	0,707	0,000
Grade 6	SARICAN, 2017	0,408	-0,178	0,995	0,172
Grade 6	DEDETURK, 2018A	1,060	0,727	1,392	0,000
Grade 6	DEDETURK, 2018B	3,167	2,699	3,635	0,000
Grade 6	NAGAC, 2018	0,298	-0,288	0,884	0,319
Grade 6	NECCAR, 2019	-0,253	-0,886	0,381	0,434
Grade 6	CIMENTEPE, 2019	1,652	0,984	2,320	0,000
Grade 6	CETIN,2019	4,981	3,732	6,230	0,000
Grade 6	KURT, 2019	1,141	0,334	1,947	0,006
Grade 6	AKKAYA 2019	2,495	1,678	3,312	0,000
Grade 6	AYSU, 2019	2,675	2,005	3,345	0,000
Fixed		1,457	1,275	1,640	0,000
Random		1,710	0,901	2,519	0,000
Grade 7	GAZIBEYOGLU, 2018	0,576	0,029	1,122	0,039
Grade 7	DOGANAY, 2018	2,206	1,429	2,982	0,000
Grade 7	KOCA, 2018	1,326	0,573	2,079	0,001
Grade 7	HIGDE, 2018	2,768	1,949	3,587	0,000
Grade 7	BUYRUK, 2019	0,288	-0,244	0,820	0,289
Grade 7	DOGAN,2019	0,616	0,185	1,048	0,005
Grade 7	SEN, 2019	0,813	0,087	1,539	0,028
Grade 7	BUYUKBASTIRMACI, 2019	6,933	5,789	8,078	0,000
Grade 7	KAYABAS, 2019	0,932	0,305	1,559	0,004
Grade 7	IZGI, 2020	1,251	0,653	1,849	0,000
Grade 7	OZASLAN, 2019	0,698	0,220	1,176	0,004
Grade 7	GULSEVEN, 2020	1,053	0,535	1,570	0,000
Fixed		1,084	0,910	1,257	0,000
Random		1,534	0,892	2,175	0,000
Grade 8	CEYLAN,2014A	0,898	0,355	1,440	0,001
Grade 8	CEYLAN,2014B	0,859	0,318	1,399	0,002
Grade 8	CALISICI,2018	0,930	0,318	1,542	0,003
Grade 8	TASCI, 2019A	3,226	2,436	4,015	0,000
Grade 8	TASCI, 2019B	2,074	1,430	2,718	0,000
Grade 8	TASCI, 2019C	2,460	1,771	3,148	0,000
Grade 8	SOYSAL, 2019	0,843	0,363	1,324	0,001
Grade 8	BAHSI, 2019	0,915	0,198	1,632	0,012
Grade 8	OZLEN, 2019	0,003	-0,560	0,565	0,992
Fixed		1,169	0,969	1,368	0,000

Random		1,330	0,739	1,921	0,000
Fixed	***Overall***	1,112	1,021	1,203	0,000
Random	***Overall***	0,806	0,637	0,976	0,000

When Table 11 is examined, the individual and general effect sizes of the grade level moderator were calculated. Actually, to make an interpretation, it is necessary to decide on the model selection by evaluating the heterogeneity test findings within the group (Dinçer, 2021). Table 12. includes heterogeneity test findings related to grade level.

Table 12. Heterogeneity Test Findings Related to Grade Level

Grade Level	***N***	***Standard Error***	***Heterogeneity***				***General Effect***	***%95 Confidence Interval***	
			Q-value	***df (Q)***	***P-value***	***I-squared***		***Lower Limit***	***Upper Limit***
Grade 4	3	0,200	4,484	2	0,106	55,399	1,772	1,380	2,164
Grade 5	5	0,102	2,573	4	0,632	0,000	0,507	0,307	0,707
Grade 6	10	0,093	162,560	9	0,000	94,464	1,457	1,275	1,640
Grade 7	12	0,088	145,011	11	0,000	92,414	1,084	0,910	1,257
Grade 8	9	0,102	68,724	8	0,000	88,359	1,169	0,969	1,368
Total WihtinFixed			383,352	34	0,000				
Total BetweenFixed			60,299	4	0,000				
Total BetweenMixed			32,237	4	0,000		0,806	0,637	0,976

When the heterogeneity test findings related to the grade level moderator of the study in Table 12 were analysed, firstly, the importance level of the within-group heterogeneity test in the fixed effects model was calculated as $p<0.05$. Apart from this, the statistical value of Q was found to be 383,352 and the df degree of freedom was 34. The critical value of the $\chi 2$ table at the df=34 level and at the 95% importance level is between 43,773 and 49,802 values. It was monitored that the statistical value of Q calculated in the heterogeneity test ($p<0.05$) was higher than the critical value determined in the chi-square distribution at 34 degrees of freedom, and it is significant since $p<0.05$. In the light of this result, it is deduced that the studies have the same widespread effect within themselves. However, studies can be interpreted as heterogeneous because they have a higher distribution than expected. In addition, the p value was calculated according to the betweengroup heterogeneity test according to the mixed effects model. The importance level of the test was calculated as $p=0.000$. In addition, when the Q statistical value of the study was examined, it was calculated as 32,237 and the degree of freedom was 4. The critical value of the $\chi 2$ table at the 95% significance level is 9,488. It was observed that the calculated Q statistical value was 32,237 ($p<0.05$), exceeding the critical value determined in the chi-square distribution at 4 degrees of freedom. According to both models, there is a statistically important difference between the subgroups. As a matter of fact, since the source of the variance between the groups is investigated, it is necessary to report according to the mixed effects model (Dinçer, 2021). In this case, it was concluded that the academic achievement effect sizes of the STEM approach in Science Education made an important difference according to the grade level moderator created and the academic achievement of the studies conducted at the 4th grade level was higher than the other grade levels. In Table 13, the overall effect sizes for the grade level moderator are given.

Table 13. Random Effects Model General Effect Sizes Related to Grade Level Moderator

Grade Level	*Number of Study*	*Effect Size*	*Lower Limit*	*Upper Limit*	*p-value*
Grade 4	3	1,798	1,208	2,388	0,000
Grade 5	5	0,507	0,307	0,707	0,000
Grade 6	10	1,710	0,901	2,519	0,000
Grade 7	12	1,534	0,892	2,175	0,000
Grade 8	9	1,330	0,739	1,921	0,000
General		0,806	0,637	0,976	0,000

According to the Random Effects Model in the moderator of the grade level, the effect size of the 4th grade level has the largest effect size and its value was calculated as 1,798. It is followed by the Grade 6 level with an effect size of 1,710. This value is followed by the effect size of the 7th grade level of 1,534. Afterwards, 8th and 5th grade levels come with effect sizes of 1,330 and 0.507. According to the scale of Cohen (1988), 4th, 6th, 7th, and 8th grade level effect size values express a large level of effect; Grade 5 level effect size corresponds to medium effect size according to the same scale. According to the Thalheimer and Cook (2002) scale, the effect size of the 5th grade level is medium and the remaining grade levels indicate a very large level.

In terms of grade levels, the individual effect sizes of the STEM approach in science education on academic achievement are given in Figure 4. effect sizes are positive at all grade levels, except for one study at grade 6.

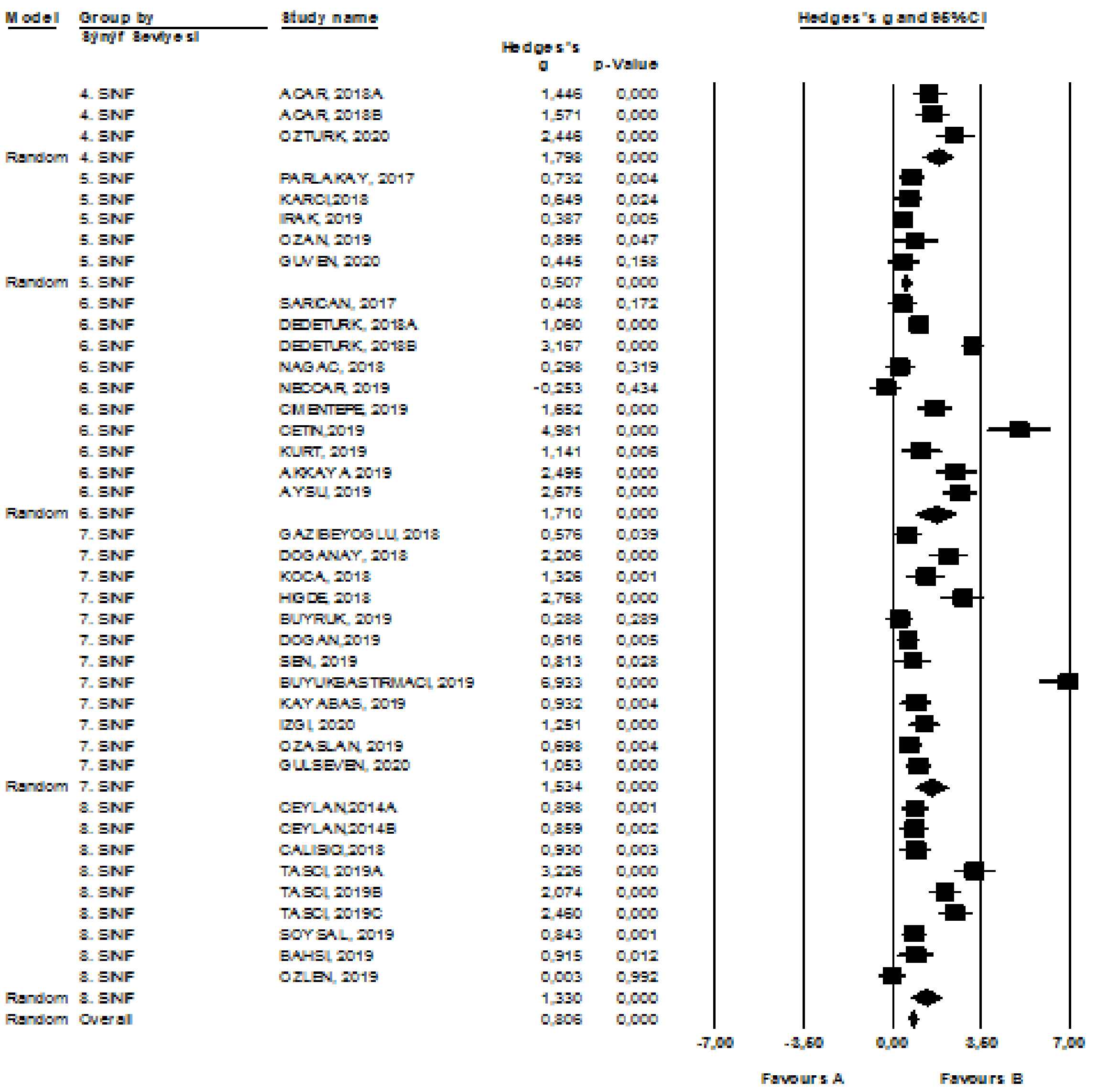

Figure 4. Distribution of Effect Sizes and Forest Graph for Grade Level Moderator

Discussion

In this section, the descriptive statistical findings of the study and the findings related to the sub-problems are discussed and interpreted.

In order to determine the effect of the STEM approach in Science Education on the academic achievement of students, a total of 147 studies, 130 master's and 17 doctoral studies, were carried out between 2010-2020. However, a total of 34 studies, including 31 master's and 3 doctoral studies, that met the inclusion criteria were included in the research. 23.85% of master's theses and 17.65% of doctoral theses; 41.50% of the theses reached in total were included in the meta-analysis. When the rates of inclusion of the studies reached in the meta-analysis were examined, it was seen that both master's and doctoral theses were included in the study at similar rates in total. However, it has emerged that quantitative studies at the doctoral level are insufficient and it is necessary to conduct studies at this level. When the grade levels of the studies included in the research are examined, it is seen that the research was conducted at the 7th grade level at most. When the Science Curriculum is examined, it is thought that the existence of many units and sub-topics in different learning areas suitable for organizing STEM activities in the 7th grade Science Curriculum may cause the studies to be concentrated at this grade level.

Considering the rates of these studies, it is striking that the rate of master's theses made within the scope of academic achievement variable is 91.17%. It is of great importance in terms of demonstrating the effectiveness of the STEM approach in science education on academic achievement, as the large number of qualified data to be included in the research in meta-analysis studies will ensure that the overall effect size obtained as a result of the analysis is more accurate. In this regard, the need to conduct doctoral-level studies on the STEM approach has emerged. In addition, among 34 studies within the scope of the research; In Ceylan (2014), Doğanay (2018), Acar (2018) studies, two different experimental and control groups; As Taşçı (2019) applied three different achievement tests, a total of 39 data were obtained. It is striking that mostly a single experimental and control group and a achievement test in one area were used in the studies. In addition, it is seen that more than half of the 34 theses examining the achievement variable were studied in 2019. This means that the interest in STEM education has increased recently and the Science, Engineering and Entrepreneurship Practices in the Science Curriculum, which was renewed in 2018 and revised in 2019, are thought to be extremely effective. At the same time, when the sample numbers and grade levels of the theses included in the research were investigated, it was observed that the studies consisting of 39 data from 34 studies consisted of a total of 1962 people and corresponded to 50 students on average. Since the studies included in the meta-analysis are experimental studies carried out with a quantitative approach, the sample size is expected to be 50 students on average; it is thought that they were chosen as the experimental and control groups during the application was effective in the emergence of this situation. It is seen that most of the studies included in the research consisted of 6th and 7th grade students. It is thought that the renewed Science Curriculum was effective in the emergence of this situation.

When the findings related to the effect sizes were examined, 38 positive studies and 1 negative study were found out of 39 data in the 95% confidence interval. It is expected that studies with a positive effect size will be more numerous than studies with a negative effect.

The fact that a study has a negative effect size shows that the study is on behalf of the control group (Wolf, 1986). It is thought that this situation may be caused by operator error. Apart from this, the effect size of the remaining 38 studies is on behalf of the experimental group. The study with the largest effect size in individual studies belongs to the study of Büyükbastırmacı (2019), while the smallest effect belongs to the study of Neccar (2019). While 27 studies among 39 data obtained from 34 studies had large effect size; 5 studies had a medium effect size, 5 studies had a small effect size, and finally 2 studies had a negligible effect.

The effect sizes of the individual studies were first combined according to the fixed effects model with the help of the CMA program, and the overall effect size was computed as 1.112 at the 95% confidence interval. As a matter of fact, as a result of the heterogeneity test, it was determined that the Q value was higher than the critical value in the X^2 table, so the studies had a heterogeneous structure. In this context, the calculations were made according to the random effects model. As a result of the analyses made according to this model, the overall effect size was calculated as 1,420, and based on this finding, it can be said that the STEM approach in science education has a positive and large effect on the academic achievement of students.

It is stated that 6905 studies are needed to remove the significance level of 39 studies combined with the meta-analysis method (Rosenthal Method), and it is impossible to reach this number; in addition, it can be said that the findings obtained from the analysis results are reliable as the general effect size is large and at least 395 studies are needed to reduce this effect to an insignificant value like 0.100 (Orwin Method).

In his meta-analysis study, Saraç (2018) stated that 25 of 27 individual data on the effect of STEM education applications in science and mathematics courses on the academic achievement of students had a positive effect and 2 had a negative effect. In addition, it was concluded that the overall effect level was 0.442 and moderate, and that the applied STEM education practices had a positive effect on the academic achievement of the students. Yücelyiğit and Toker's (2020) meta-analysis study on Early Childhood Education STEM studies stated that all 12 individual data had a positive effect on the extent to which STEM activities applied to preschool children affect learning and language development, and expressed the overall effect level. It was calculated as 0.556. As a result of the research in question, it has been concluded that this effect has a moderate effect and that the STEM activities applied have a positive effect on the language development and learning of the students. In the study of Ayverdi and Öz Aydın (2020), the effect of STEM education on academic achievement was investigated by comparing STEM researches conducted in our country and abroad. In the study, 38 studies were examined, and it was revealed that the individual effect sizes calculated in the data analyzed from the studies conducted in our country were mostly medium and large. The effect sizes of studies conducted abroad were generally calculated as small and large. It is seen that these studies are relatively consistent with the findings on the effect of the STEM approach in science education on academic achievement.

As a result of the literature review, the number of meta-analysis studies applied with the STEM approach is negligible. However, there are individual studies that will show the positive effect of the STEM approach in science education on the academic achievement of students (Ceylan, 2014; Parlakay, 2017; Sarıcan, 2017; Gazibeyoğlu, 2018; Doğanay, 2018; Dedetürk, 2018; Karcı, 2018; Acar, 2018; Koca, 2018; Çalışıcı, 2018; Hiğde, 2018; Irak, 2019; Çimentepe, 2019; Ozan, 2019; Buyruk, 2019; Doğan, 2019; Çetin, 2019; Şen, 2019; Kurt, 2019; Akkaya, 2019; Taşçı, 2019; Büyükbastırmacı, 2019; Soysal, 2019; Bahşi, 2019; Aysu, 2019; Özlen, 2019; Kayabaş, 2019; İzgi, 2020; Özaslan, 2019; Öztürk, 2020; Gülseven, 2020; Güven, 2020).

The studies in the research, 3 of them were studied at the 4th grade level, 5 of them at the 5th grade level, 10 of them at the 6th grade level, 12 of them at the 7th grade level and 9 of them at the 8th grade level. According to the heterogeneity test results, it is heterogeneous within and between groups. The effectiveness of the STEM approach in science education differs according to the grade level. The highest impact value is at the 4th grade level, and the lowest impact value is at the 5th grade level. However, when the effect size values are examined according to the Cohen (1988) scale, at the 4th, 6th, 7th and 8th grade levels; it is seen that it is moderately effective at the 5th grade level. This situation is thought to be due to the fact that the general effect sizes of the studies at the other grade level are higher compared to the 5th grade level in terms of grade level.

Results and Suggestions

When the effect size values of the studies included in the meta-analysis are classified, the effect size value is at least moderate in most of the studies. In addition, the direction of the effect size of the majority of the studies included in the meta-analysis is positive. The effect size of the STEM approach in science education on academic achievement is large, significant and positive. Therefore, it can be said that the STEM approach in science education is quite effective in increasing the academic achievement of students.

When the effect of STEM approach in science education on academic achievement according to grade level was examined, it was concluded that it had a positive effect on all grade levels and that there was a statistically important difference between grade levels in the heterogeneity test. Considering the effect size values according to the grade levels, it was seen that the lowest effect size value was at the 5th grade level, and the highest effect size value was at the 4th grade level. It has been concluded that while the effect of STEM approach in science education on the academic achievement of the 5th grade students is moderate, it has a wide effect on all other grade levels.

In our country and abroad, there have been many studies examining the STEM approach on students, pre-service teachers and teachers recently. In these studies, the effects of STEM approach on variables such as academic achievement, attitude towards science course, scientific process skills, motivation, conceptual understanding, career interest, and permanence were examined. In addition, there are also studies in which STEM education is examined according to independent variables such as gender, age, education level, grade level and course type and different results are obtained. When the results in the literature were examined, a meta-analysis was needed to gather the studies examining the effect of the STEM approach on existing variables under a single roof and to reveal the trends in this research area. In this respect, this study, which examines the effect of STEM approach on academic achievement in science education, is thought to shed light on future studies.

In the light of the findings obtained as a result of the analyses made within the scope of the research, some suggestions were made for researchers and practice.

According to the results of the research, it has been seen that the STEM approach in science education has a positive and large effect on the academic achievement of the students. Therefore, it can be recommended that the STEM approach be widely used at all levels of education. In this direction, teachers should be made aware of the STEM approach and in-service training should be given in this direction. As a result, it is thought that the STEM approach will increase the quality of education and training.

In meta-analysis studies, as the quantity increases, that is, when more publications are reached, the quality also increases in direct proportion. As a matter of fact, the large number of publications is important in terms of giving more reliable results of the research. It has been observed that the working history of the STEM approach in our country is not very old; there has been an increase in the number of studies conducted in recent years. Increasing studies in this direction will reveal the effect of the STEM approach on academic achievement in a more generalizable and clear manner.

Researchers have difficulty in accessing the theses that are not accessible in the National Thesis Center of the Council of Higher Education. The solution of this problem may be effective in the emergence of more qualified studies.

The fact that most of the theses included in the research are master's theses reveals the lack of studies done at the doctoral level. In the future, the number of studies at the doctoral level may be increased. In addition, cross-country comparisons can be made by researchers.

In the studies included in the meta-analysis, it was observed that the researchers mostly conducted experimental studies at the primary school level. For these reasons, the number of studies carried out in other education levels can be increased.

The characteristics of the studies to be included in the meta-analysis and the fact that the statistics needed to compute the effect size are not given enough or in accordance with the meta-analysis affects the quality of the meta-analysis studies. As a matter of fact, the lack of a standard in the presentation of the data in the studies complicates the work of the researchers. In this context, it is recommended that researchers show sensitivity to include both descriptive and statistical data in a more systematic and complete manner in their studies.

In the current study, only the studies conducted for the science course were included in the research. Since the STEM approach is an approach that is directly related to many disciplines, meta-analysis can be carried out in many areas.

Fen Eğitiminde STEM Yaklaşımının Akademik Başarıya Etkisi : Bir Meta-Analiz Çalışması

Özet:
Bu araştırmada Türkiye'de fen eğitiminde STEM yaklaşımının öğrencilerin akademik başarıları üzerindeki etkisini incelemek amacıyla meta analiz çalışması yapılmıştır. Yapılan meta analiz çalışmasında belirlenen ölçütlere uygun olarak araştırma kapsamına alınan çalışmalar Yüksek Lisans ve Doktora düzeyinde olup bu çalışmalar Yüksek Öğretim Kurulu Başkanlığı Ulusal Tez Merkezi veri tabanından alınmıştır. Araştırmada, 2010-2020 yılları arasında gerçekleştirilmiş, toplam 147 çalışma incelenmiş ve dâhil edilme kriterlerine uygun olan 31 Yüksek Lisans ve 3 Doktora tez çalışması meta analiz kapsamına alınmıştır. Toplam 34 adet çalışmanın örneklemini 4., 5., 6., 7. ve 8. sınıf seviyesindeki toplam 1962 öğrenci oluşturmaktadır. Analizler Compreh ensive Meta Analysis V2 istatistik programı yardımıyla yapılmıştır. Çalışmalar analiz edilip birleştirildiğinde yapının heterojen olmasından dolayı rastgele etkiler modeli kullanılmıştır. Araştırma sonucunda, fen eğitiminde STEM yaklaşımının öğrencilerin akademik başarısına etki büyüklüğü 1.420 olarak hesaplanmıştır. Bu değer, yapılan sınıflandırmaya göre akademik başarı değişkeni açısından geniş düzeyde bir etki büyüklüğüdür. Bu bağlamda fen eğitiminde STEM yaklaşımı öğrencilerin akademik başarısını arttırmada olumlu yönde ve geniş düzeyde bir etkiye sahip olduğu sonucuna ulaşılmıştır. Akademik başarıya olan etki büyüklüğü sınıf seviyesine göre farklılaşmış olup en yüksek etki 4. sınıf seviyesindeki çalışmalarda gözlenmiştir.
Anahtar Kelimeler: STEM, fen bilimleri, akademik başarı, meta analiz.

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Research Article

Why Do Pre-Service Teachers Who Believe In The Necessity Of Proof Have Low-Level Self-Efficacy With The Proof?

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Abstract – This research aimed to examine the opinions of pre-service mathematics teachers towards proof. The students received low scores in the items containing the self-evaluation of the proof in the quantitative dimension, a qualitative form was applied to obtain more comprehensive information. Therefore, quantitative and qualitative research approaches were used together along with mixed methods. Although all students expressed their opinions on the necessity of proof, it was concluded that they had difficulties in the proof process due to the difficulty level of proof and their lack of mathematical knowledge in their self-evaluation. And also, a great majority of pre-service teachers stated that they intended to use alternative learning approaches instead of proof in their future careers. Although proof is seen as a necessary process, pre-service teachers' proof self-efficacy levels are low and their thoughts on using proof in their professional life are insufficient. Therefore, it can be recommended to include proof in pre-graduate education and to eliminate the students' lack of basic knowledge.

Key words: proof, mathematics education, proof self- efficacy.

Introduction

The crucial factor determining the degree of difficulty in mathematics is “precision”. Mathematics, unlike other fields of science, requires absolute precision (King, 1992). It can be argued that the situation that gives this certainty to mathematics is proof because mathematical objects gain certainty as they are proven. Showing the accuracy of mathematical knowledge is an essential activity for mathematics (Almeida, 2000). Bell (1976) sees proof as a process that takes place as a result of sequential steps and adds that there should be categories of verification (showing the truth of a proposition), explanation (showing why the proposition is true), and systematization (organizing propositions and theorems in an inductive system). Similarly, Baki (2008) saw mathematical proof as a process and defined this process as verification, explanation, and abstraction.

The source of the proof is Euclid's book “Elements” which was published in the 4th century BC. According to Euclid, the process of proving consists of four stages; verifying the result, convincing others, finding a result, and placing the results in a deductive system (Almeida, 2003). There are two purposes in mathematical proof. The first of these is to show that the hypothesis is brought to an end with logical steps, and the second is to understand why and how to go from assumptions. Therefore, proof provides an understanding not only that a theorem is true, but also why it is true (Doruk & Kaplan, 2013). Considered in this way, it can be said that the proof is both convincing and exploratory, and can contribute to the systematization of mathematics by reconciling the results obtained with it (Hanna & Barbeau, 2009).

In a mathematical proof, there should be definitions, theorems, and operational processes, and these structures should reveal the mathematical-logical predictions hidden in the expressions. Therefore, the purpose of mathematical proof can be expressed as proving the truth or falsity of what is claimed in every situation (Baki, 2008) as well as showing the relationship between verifications (Lee, 2002). Thus, it can be ensured that students predict facts that are not seen in a real-life situation or event, and that they can reach positive results by testing their accuracy and falsity (Zaslovsky & Peled, 1996). Therefore, it is possible to say that mathematical proof is important and necessary for relational, permanent, and meaningful learning. For this important process to come to the fore in mathematics teaching steps, it is necessary to know the opinions and self-evaluations of both teachers and pre-service teachers in terms of an effective roadmap.

There are various studies regarding the notion of mathematical proof. In one of these studies, for instance, Moralı et al. (2006), who investigated the opinions of pre-service teachers on proof, stated that the opinions of the participants on proof were not fully formed. According to them, this stems from the fact that pre-service teachers do not know the importance of proof in teaching mathematics. In another study, Saeed (1996) investigated the relationship between students' understanding of some concepts related to mathematical proof and their attitudes towards mathematics. As a result of the research, he discovered that many students at the university level had difficulties in learning and teaching proof. Another study by Kotelawala (2007) aimed to reveal teachers' attitudes and beliefs towards proof in mathematics classrooms.

Kotelawala found that teachers' past concerns and difficulties with proof affected their use of proof in their classrooms. In another study, Nordström (2002) reported that, although students exhibited positive attitudes towards proof, they had great difficulties in proving many intermediate-level statements.

Considering the studies carried out, teachers and pre-service teachers should understand the importance of proving and having proof skills and include proof processes in their professional lives. As stated by Weber (2004), one of the aims of mathematics is to make students acquire the ability to prove. It is, therefore, thought that the attitudes and self-evaluations of mathematics teachers and prospective teachers towards proof are also likely to affect their use of proof in their lessons and their teaching methods. In this respect, it is important for people who will take part in the education task to focus on proof in mathematics education starting from the primary education level in terms of the development of students' mathematical thinking. And thus, it is effective in developing positive attitudes towards mathematics in students (Moralı et al., 2006). For this reason, knowing the opinions and self-evaluations of people who will undertake the education task will be beneficial in interpreting the mathematics education processes of the students transferable.

For this reason, within the scope of this research, first of all, pre-service teachers' views on proof, their awareness of the importance of proof, and their self-evaluations on proof skills were discussed. In the data obtained, a difference was determined in the levels of evaluating the necessity of proof, and the self-evaluation levels of the pre-service teachers, and it was aimed to determine the pre-service teachers' definitions of proof, the stages they had difficulty in, and their preferences to include proof in mathematics lessons in their future careers. The subject of proof was analyzed not only as an attitude towards proof but also as a self-evaluation of proof skill and related factors.

It is thought that pre-service teachers' views on proof are an important determinant of their ability to benefit from proof in their classrooms, their mathematics education, and their future careers because knowing pre-service teachers' views on proof and self-evaluation can provide a roadmap for proof skills. When the existing literature is examined, students' attitudes and beliefs towards proof were determined in several studies (Kotelawala, 2007; Mingus & Grassl, 1999; Saeed, 1996; Üzel & Özdemir, 2009). In addition, there are also various studies on proof self-efficacy in the literature. For example, in one study, Regier and Savic (2020) explored how fostering mathematical creativity may impact student self-efficacy for proof. In another study, the relationship between persistence and self-efficacy in proof construction was investigated (Selden & Selden, 2014). In another study by Shonge and Mudaly (2021), the aim was to develop and investigate preliminary validity evidence for a new instrument for measuring self-efficacy for mathematical proof in high school students. In another study, Viholainen et al. (2019) examined university students' motivation and self-efficacy beliefs about proof and proving. Although there are similar studies involving different variables related to self-efficacy with the proof, there is no study that deals with the relationship between students' opinions and self-evaluations and related factors. It is thought that this study will contribute to the literature.

Method

The research was designed only in a quantitative dimension at the first stage, and the scope of the research was expanded based on the difference between the opinions of the pre-service teachers on the necessity of proof and the self-efficacy levels of proof skill, according to the data obtained at the first stage. The in-depth analysis was carried out in a mixed research model in which quantitative and qualitative approaches were used together with open-ended scale application.

In this study, a non-random convenient sampling method was adopted in the selection of the study group. The study group consists of 34 pre-service mathematics teachers studying in a state university's primary education mathematics teaching program. Although the number of participants is small, the sample can be considered quite representative. The spring semester was waited for the application of the scales in the research because the pre-service teachers had taken the courses of Abstract Mathematics, General Mathematics, Analysis-I, and Analysis-II, in which they knew of the existence and use of the proof process.

The quantitative dimension of the research was carried out with a descriptive survey. In the qualitative dimension, the document analysis of the answer codes given to the opinion, self-evaluation, and design questions for proof was made and the mixed method was used. The 5-point Likert-type "Opinion Scale for Proof" developed by Moralı et al. (2006) was used in the research. The reliability of the scale was determined as 0.80 and factor analysis was performed to determine its construct validity. The scale has seven factors; personal proof proficiency, the importance of proving, the effect of proof on understanding the theorem, self-perceptions about proving, general views on proving, examples, theorem perspectives, and views on the relationship between problem-solving and mathematical proof.

In the evaluation of the quantitative scale, the arithmetic mean of the scale items was used and it was tried to determine the direction of the score distribution (high, medium, low). The attitudes of the sample were also examined in terms of the factors that emerged as a result of the factor analysis performed with the help of the SPSS 25.0 program, and it was observed that opposite average values were obtained especially between some factors in the scale. Contrary to the importance of proof, the effect of proof on understanding the theorem, and the fact that their general views on proving reflect a high mean score, the pre-service teachers have a low mean score, especially regarding their proof proficiency and self-perceptions about proof. To understand the reason for these contradictory results obtained for proof and to obtain a detailed result, a qualitative form that contained 3 open-ended questions was applied as a second step after the quantitative scale was applied. As a result, data were collected with a questionnaire containing both the statements and open questions.

Findings and Discussions

Within the scope of the research, firstly, the "Opinion Scale for Proof" was applied to obtain the opinions of the pre-service teachers about proof, and then a qualitative scale application consisting of open-ended questions was applied for in-depth analysis due to the contrast between the average scores obtained among the sub-factors of the scale. Table 1 presents the descriptive statistical values of the answers given by the pre-service teachers to the "Opinion Scale for Proof".

Table 1
Descriptive Statistics Values of Pre-service Teachers' Opinions on Proof

	Min.	Max.	Mean	Std. Deviation
1. A mathematical proof is about facts and explaining facts.	3.00	5.00	4.23	0.65
2. When a mathematical result is proven, I believe it to be true.	4.00	5.00	4.64	0.48
3. Seeing a result illustrated by an example does not always help me understand why that conclusion is true.	3.00	4.00	3.55	0.50
4. Proof is indispensable for theoretical mathematics.	3.00	5.00	4.38	0.65
5. In mathematics, we can only understand whether something is true with the help of examples.	1.00	3.00	2.32	0.68
6. I do not understand why we need to do the proofs. All the results we see in the	1.00	5.00	2.38	1.15

lesson were proven before without a doubt by famous mathematicians.				
7. Proofs sometimes involve strategies that are not very clear.	2.00	5.00	3.52	0.78
8. In mathematics, if a result is clearly true, there is no point in proving it.	1.00	5.00	2.17	0.99
9. I like doing mathematical proofs.	1.00	5.00	2.20	0.91
10. I am confident in my ability to prove myself.	1.00	5.00	1.85	0.98
11. Working through the stages of proof helps me understand why something is true.	2.00	5.00	3.64	0.81
12. Seeing different proofs of a theorem helps me understand it better.	2.00	5.00	4.00	0.85
13. A mathematical proof also depends on other mathematical results.	2.00	5.00	3.94	0.81
14. I usually have a hard time understanding the proofs.	1.00	5.00	3.97	0.99
15. Proving is, in a sense, problem-solving.	1.00	5.00	3.55	0.99
16. Only professional mathematicians can do mathematical proofs.	1.00	3.00	1.94	0.81
17. I think knowing the theorem (or proposition) is more important than proving it.	1.00	4.00	2.58	0.74
18. Dealing with proofs is very boring.	1.00	5.00	3.94	0.95
19. Although I generally understand what a theorem means, I find it difficult to understand its proof.	1.00	5.00	4.02	0.71
20. I can only understand proof when the teacher does it in the classroom.	1.00	5.00	3.97	0.99

In this scale, which consists of 7 factors, it was determined that there is an opposite correlation between the mean score values of some factors that are directly or indirectly related to each other. The first factor is the students' personal proof proficiency (items 14, 18, 19, and 20 of the scale), the second factor is the students' views on the importance of proving (items 6, 7, 8, and 17), the third factor is the students' views on the effect of proof on understanding the theorem (11, 12, 13 and 16), the fourth factor is students' self-perceptions towards proving (items 9 and 10), the fifth factor is students' general views on proving (items 1, 2 and 4), the sixth factor is students' perspectives on examples and theorems (items 3 and 5) and the seventh factor is students' views on the relationship between problem-solving and mathematical proof (item 15). Considering these sub-factors ("personal proof proficiency" and "self-perceptions towards proving"), the mean scores of these items (9, 10, 14, 18, 19, 20) of the preservice teachers showed that they had low self-efficacy in proof.

Since the self-evaluation of the personal proof proficiency was low, the reasons for this were investigated more comprehensively with open-ended questions. In the qualitative scale applied, the students were asked what it means to prove, the parts that they have difficulty in proving, and their preferences for proving in their future careers.

Table 2 shows the answers given to the question "What does mathematical proof mean to you? Please explain."

Table 2
Statements of Pre-Service Teachers about Mathematical Proof

Expression categories	f	Some sample expressions from participant opinions
Meaningful Learning	18	*"Explaining with proof provides more meaningful learning, so it has an important place for mathematics."* *"It makes sense of mathematics. It makes me understand that the formulas I learned came from somewhere, not just a written formula."*
Permanent learning	11	*"Rather than memorizing the subject, I think one of the basic steps of permanent learning is proof."* *"Proof is important both for better understanding of a subject and for keeping the formulas in mind as we can recreate the formulas in moments of forgetting."*
Causality Relationship	5	*"The human brain has an inquiring nature and seeks reasons in mathematics, not memorization. To present these reasons, the proof is important."* *"Proof takes us to the basics of knowledge. It makes us understand where the information comes from, it makes us think."*

In Table 2, the answers of the pre-service teachers on what proof means are grouped under three categories. The vast majority of pre-service teachers stated that proof is a process that provides meaningful learning (f=19). As can be seen in the sample expressions in the table, the answers of all pre-service teachers about the concept of proof point to the necessity and importance of proof. The question was asked to determine whether students' positive opinions about the functionality and importance of proving on a quantitative scale would be repeated on a qualitative scale, and it was observed that the data derived from both qualitative and quantitative scales were consistent.

Table 3 contains data on student answers to the open-ended question "Are there any stages that you have difficulty in proving, and if so, what are they?"

Table 3
Statements of Pre-Service Teachers about Situations in which They Have Difficulty in Proving

Expression categories	f	Some sample expressions from participant opinions
Structure of the proof	15	*"I generally have no difficulty in proving, but the content of some proofs can be difficult.* *"In some proofs, I cannot relate some steps to each other, so it can be difficult."* *"Proofs that are complex and require some acceptances are difficult for me."*
A Mixed Process	10	*"Since proof involves some complicated steps, constant proofing in class can scare the student. It would be better if the proofs were chosen for some specific topics."* *"In some subjects, making a demonstration can be easier and more effective than proof."*
Mathematical Knowledge Level	9	*"Whether I am successful or unsuccessful in proving is related to my mastery of the subjects."* *"I think being able to prove is very high level and requires knowing a lot about different subjects related to mathematics. So, even though it is very important, I cannot always succeed.*

According to the data in Table 3, the situations that pre-service teachers had difficulty in proving were grouped under three categories. The point that students frequently expressed was that some proofs included more difficult steps due to their nature (f=15). This situation is named under the category of content/structure of the proof. The second category was the expression of proof as a complex process in general (f=10). In the answers under this category, it was stated that it is generally difficult to prove and situations such as showing examples, making demonstrations, and solving problems provide an easier learning process than proof. Another category is related to basic mathematical knowledge level. The pre-service teachers who responded under this category associated their inadequacy in proving with their lack of knowledge rather than the proof process and showed mastery of mathematics as a prerequisite for proving. In that case, it can be argued that pre-service teachers divide the self-evaluation of proof into internal factors (level of mathematical knowledge) and external factors (difficulty of proof, seeing proof as a high-level process).

Table 4 contains data on student responses to the open-ended question "When you become a teacher, would you consider using proof in your lessons, please explain".

Table 4

Pre-Service Teachers' Views on Using Proof in their Lessons

Expression categories	f	Some sample expressions from participant opinions
Conditional Yes Opinion (I Use But…)	27	*"I do not think of using it all the time, I want to make some subjects permanent by giving examples from daily life, not by proof.* *"Proving all the time can scare the student. Students may see the mathematics course as a lesson in which every subject must be proven."*
Unconditional Yes Opinion	7	*"I use proof because learning proof is important."* *"I am thinking of using proof because I think knowing where the theorems come from is more memorable than memorizing the theorems."* *"Yes, I would consider using it. Even if it is high-level for students, I try to think about where the information I give comes from and how I can show it."*

In Table 4, after the general opinions and self-evaluations of the pre-service teachers on proof and their ideas about their future career choices were examined, these ideas were gathered in two categories; "conditional yes" and "unconditional yes". Although the majority of pre-service teachers (f=27) stated that proof is important in mathematics education, they stated that they thought of using proof only under certain conditions in their professional life. As a justification for this, they put forward reasons such as the fact that it is difficult for students to use proof all the time, and demonstration by example is more understandable. Pre-service teachers who said "unconditional yes" (f=7) stated that they intended to use proof in their lessons because it is important and useful to prove without specifying any conditions.

In general, pre-service teachers expressed a positive opinion on the importance of proving on a quantitative scale and similarly repeated their positive views on the effectiveness of proof on a qualitative scale. However, when the self-assessments of the pre-service teachers about proving proficiency were examined, it was determined that the pre-service teachers found their proficiency levels to be insufficient. It was also determined that the inadequacy of the pre-service teachers in self-evaluation in making proofs is related to the difficulties arising from the structure of the proof and their lack of basic mathematical knowledge, and in connection with these difficulties, the pre-service teachers also stated that they would not always use proof in their professional life, but only under certain conditions.

Conclusions and Suggestions

When the scores of the students are examined, the average score of the answers given by the participants about the necessity of proof and the importance of proof indicates that the pre-service teachers have positive opinions about the importance of proof and its contribution to the learning process. The average scores of the items (9, 10, 14, 18, 19, 20) related to proof self-efficacy in the scale showed that the students had low proof self-efficacy. However, the average score of the items of the sub-factors in which proof is evaluated as a cognitive skill and students making self-assessments about proof proficiency generally shows that pre-service teachers find their proficiency levels lacking. This shows that although students consider proof important in the learning process, they have difficulty in performing it and they see themselves as inadequate in proving. With this result, to understand the underlying reason for this more clearly, after the quantitative scale, the students were applied a qualitative scale and were asked what it means to make a proof, the parts that they had difficulty in proving, and their preferences for proving in the future. Most of the pre-service teachers expressed their views on what proof meant, and said that proof is a process providing meaningful learning. They also mentioned the relationship of proof with permanent learning and causality. And also, the answers of all pre-service teachers in the study to the concept of proof pointed to the necessity and importance of proof. All pre-service teachers who participated in the research expressed positive views on the functionality and importance of proving on a quantitative scale, and they similarly repeated their positive views on the effectiveness of proof in the open-ended question in the qualitative scale, and thus consistent results were obtained in the quantitative-qualitative scale data. Similarly, the results of another study showed that the students were highly motivated to learn to understand and construct proofs, but they were more uncertain about their proving skills (Viholainen et al., 2019).

As the second question in the qualitative scale, the situations in which the pre-service teachers had difficulties in proving were examined, and the answers were gathered in three categories; the structure of the proof, the fact that it is a complex process, and the lack of basic mathematical knowledge. The most frequent opinion of the students was that some proofs included more difficult steps due to their nature. The second most frequently answered category was the expression of proof as a complex process in general. In the answers in this category, it was stated that it is generally difficult to prove, and alternative methods such as showing examples, demonstrations, and problem-solving offer more understandable learning systematic than proof. Another category was determined as the level of basic mathematical knowledge, and the pre-service teachers in this category associated their inadequacy in proving with the existing knowledge deficiencies rather than the proof itself and showed mastery of mathematics as a prerequisite for proving. When these results are evaluated, it is possible to argue that pre-service teachers divide the self-evaluation of proof into internal factors (level of mathematical knowledge) and external factors (difficulty of proof and seeing proof as a high-level process).

Although the importance of proof for mathematics was emphasized in different studies, it was determined that university students and mathematics teachers were unsuccessful in proof (Cusi & Malara, 2007; Doruk & Kaplan, 2015; Ko & Knuth, 2009; Weber, 2001). In Nordström's (2002) study, it was stated that although students exhibited positive attitudes towards proof and learning proof, they had great difficulties in proving many intermediate-level statements. Within the scope of the current research, pre-service teachers' self-evaluations about proof skills were examined and it was determined that they had insufficient self-efficacy beliefs in this regard.

After the general opinions and self-evaluations of the pre-service teachers about proof, their ideas about their future career choices were also examined. These opinions were gathered under the categories of "conditional yes" and "unconditional yes", and the pre-service teachers who answered "unconditional yes" stated that they thought of using proof in their lessons without specifying any conditions. Those who answered "conditionally yes" stated that they found it appropriate to use proof only if the subject was suitable and there were no alternative teaching practices and that it would be difficult for students to prove constantly. Considering that the vast majority of pre-service teachers gave the "conditional yes" answer, it can be thought that this situation is also related to their low self-efficacy beliefs about proof skills.

As a result, in the current study, the prominent situation is that although proof is considered important, having an incomplete self-efficacy belief, especially because the proof process is described as difficult, negatively affects the pre-service teachers' preferences for using proof in their professional lives. Mathematics is considered difficult because it is precise rather than abstract, and, unlike all other fields, requires absolute precision (King, 1992). It can be said that the concept that gives this certainty to mathematics is proof and that the pre-service teachers' self-insufficiency beliefs about proof skills are based on the precision aspect of mathematics. However, although mathematics is known as a science of precision, and thus, of proof, the role of proof is not generally reflected in curricula (Reiss, Heinze & Klieme, 2002), and it has become a priority to focus more on it in mathematics education (Schabel, 2005).

It is thought that, in addition to the attitudes towards proof, the proving skills and proof self-efficacy beliefs are also important determinants in the ability of pre-service teachers to benefit from proof in their mathematics learning processes and in their future professional life. And also, in connection with the intrinsic factors (insufficient mathematical knowledge level) in the self-evaluation of pre-service teachers, it can be suggested to determine the knowledge deficiencies with a test application that covers all the conceptual knowledge that pre-service teachers should have according to their grade level and to carry out a compensatory learning process that focuses on these deficiencies. More radical applications can be suggested for the external factors, namely the structure of proof and seeing it as a complex process, which is another issue in pre-service teachers' beliefs about their inadequacy of proof skills. As a result, it is thought that it would be beneficial to give more place to proof teaching at every grade, both in curricula and in classroom studies, to make students acquire the habit of making proof, therefore meaningful learning, self-efficacy belief in proof skill and the preference of making proof in their professional life.

İspatın Gerekliliğine İnanan Öğretmen Adayları İspatla İlgili Neden Düşük Düzeyde Özyeterliğe Sahiptir?

Özet:

Bu araştırma matematik öğretmen adaylarının ispata yönelik görüşlerini incelemeyi amaçlamaktadır. Araştırmanın nicel boyutunda, öğretmen adayları ispat öz değerlendirmesine yönelik maddelerden düşük puan aldığı için daha kapsamlı bilgi elde etmek amacıyla nitel bir ölçek uygulanmıştır. Bu nedenle nicel ve nitel araştırmanın birlikte uygulandığı karma yöntem yaklaşımı kullanılmıştır. Öğretmen adaylarının tamamı ispatın gerekliliği doğrultusunda görüş bildirmesine rağmen ispatın zorluk derecesi ve matematiksel bilgi eksikliklerinden dolayı ispat sürecinde zorlandıklarını belirtmişlerdir. Ayrıca öğretmen adaylarının büyük çoğunluğu mesleki hayatlarında ispat yerine alternatif öğrenme yaklaşımlarını kullanmayı amaçladıklarını belirtmişlerdir. İspat gerekli bir süreç olarak görülse de öğretmen adaylarının ispat öz-yeterlik düzeylerinin düşük ve meslek hayatlarında ispatı kullanma düşüncelerinin yetersiz olduğu sonucuna ulaşılmıştır..Bu nedenle lisans öncesi eğitimde de ispata yer verilmesi ve öğrencilerin temel bilgi eksikliklerinin giderilmesi önerilebilir.

Anahtar kelimeler: ispat, matematik eğitimi, ispat özyeterliliği.

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Statements Of Publication Ethics
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Araştırma Makalesi/ Research Article

Special Education Teachers And Technology: A Metaphor Analysis

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Abstract – In this study conducted within the scope of phenomenology, a metaphor as the participants' perceptions towards technology and the rationale of this metaphor were analyzed. Study data were obtained from 164 special education teachers. The metaphors that special education teachers stated for technology were grouped into four categories: (1) Process, (2) Material, (3) Necessity, and (4) Approach. It has been observed that special education teachers' perceive technology as a process by attributing its developing, changing, and producing. In the material category, metaphors such as tools, computers, and encyclopedias are frequently expressed. In the category of necessity, it was emphasized that the teaching process is challenging the teacher and the student, and this challenge is in areas such as following up-to-date developments, receiving additional training, participating in certificate programs, and acquiring personal hobbies. The category of approach included expressing technology as a way of life, a philosophy or a belief.

Key words: instructional technology, special education, technology integration, metaphor.

Introduction

In the world, there are various individuals with different physical, cognitive, affective, or even unidentified problems who need more help than other students in their learning process. Special education can be considered a structure that gathers such individuals under an umbrella. Special education is the whole of the educational services offered to enable students with different characteristics than average individual characteristics to continue their lives independently. Special education is an adaptable form of education based on different needs according to abilities and is adopted by students who cannot adapt to the normal education system (Florian, 2008). Special education provides services for individuals who need special education, and the learning abilities of these individuals are different from the norms (above or below) to the extent that they require special education. Individuals who need special education need different educational arrangements than other students due to both physical and mental problems that create difficulties in learning processes (Lamsa et al., 2018). Special education requires special teaching strategies to facilitate the learning and skill acquisition of individuals with learning difficulties, communication, behavior, development, or other problems (Cifuentes et al., 2016). It has been stated that students who need special education and have a highly heterogeneous population require knowledge and ability, expertise, and unwavering commitment to evaluate, plan, collaborate and provide effective interventions (Mastropieri et al., 2011). It is also very important that these students are adopted and supported by the family, teacher, and even society. Students in need of special education experience chronic school failure.

Special Education and Technology

Current pedagogies/approaches in special education emphasize the necessity of technology integration in special education classes to support teaching processes (Bağlama et al., 2017; Saddler-Smith & Smith, 2006). Integrating technology into classrooms serving students in need of special education seems to have a positive effect on their academic progress, emotional development, behavioral goals, and learning motivation (Okolo & Diedrich, 2014). Technology is used to reduce the limitations brought about by the difficulties these students face and can help them increase their ability to perform learning tasks and their participation in learning (Zhang, 2000). The integration of computer-aided tools and software applications improves the basic life skills of students in special education and can provide them with knowledge and learning experiences in different learning areas (Cumming et al., 2014; Drigas & Kostas, 2014). Technology-based learning activities allow students to experience individualized learning without the need for continuous teacher participation (Thomas et al., 2019). When computer-assisted tools are integrated into learning environments in special education, they help students with different class levels and different special education needs to gain knowledge in a wide variety of learning areas (Starcic & Bagon, 2014). Alzrayer and Banda (2017) stated that the correct use of computer-assisted tools in special education can encourage students who need special education to participate in learning activities and increase their confidence in learning. It is an important finding that technology, which is an important factor in improving the learning outcomes of students with special needs, increases and improves communication and information sharing between special education personnel and parents (Siyam, 2018).

The fact that technology is constantly and rapidly changing does not mean that technology can be integrated into the classroom immediately, unfortunately, it is accepted and used by students and teachers. Although numerous projects and investments have been made in educational technologies around the world, it is seen that there are still limitations in integrating technology into the learning process (Buabeng-Andoh 2012). Technology integration is long-term and includes many interrelated factors. Teachers play a key role in effectively integrating technology into teaching processes (Teo, 2011). To meet the unique teaching needs of students in need of special education, educators must find answers to a range of contemporary workforce challenges, including special and additional courses for these students (Leko et al., 2015). In this context, it is vital to secure and maintain a solid workforce of knowledgeable, durable, determined, talented, and effective private educators (Belknap & Taymans, 2015).

Teachers find it difficult to integrate technology, especially when there is a lack of appropriate training, insufficient time, scarcity of available tools and resources, and lack of technical support (Saddler-Smith & Smith, 2006). The technological Pedagogical Content Knowledge (TPACK) model defines the types of knowledge required to integrate technology into teaching in a way to meets the mentioned logic (Koehler et al., 2013). The model is defined as the knowledge of teachers about how to use technology effectively to facilitate learning in certain content areas using appropriate pedagogical approaches (Brantley-Dias & Ertmer, 2013). However, in the TPACK model, it is also emphasized that technological knowledge alone is not enough to enable teachers to successfully integrate technology. Instead, effective technology integration depends on complex interactions between technological, pedagogical, and content knowledge (Koehler et al., 2013). Due to professional development focused on TPACK, teachers' confidence in integrating technology, changing pedagogical practices, and improving learning outcomes can also be supported (Koh et al., 2017). In the literature, there are studies that examine the views of private teachers on technology use and the factors that encourage or prevent their use of technology. In the study conducted by Almeida and colleagues (2016), urban and rural education teachers' computer use, self-efficacy perceptions, and the effect of teacher preparation programs on their performance of working with students who need special education at low frequency are examined. The study results show that rural teachers rely more on technology to access resources and materials than their counterparts in urban settlements, but also rely on more technical and complex software. Another striking finding in the study was that there was a negative relationship between self-efficacy and taking a computer training course. It is very important to realize that having only technological knowledge is necessary but not sufficient within the scope of technology integration.

Aslan (2018) analyzed special education teachers' attitudes towards assistive technologies and collected data from 251 special education teachers in this context. According to the data of the study, the participation of special education teachers in training regarding age, gender, professional seniority, and assistive technologies does not affect the attitudes of special education teachers towards assistive technologies. However, it is reported that special education teachers' attitudes towards assistive technology affect the department they graduated from and their taking lessons towards assistive technology. Special education teachers with visual impairment in the graduation field have higher attitude scores towards assistive technology than the teachers who graduated from the departments of hearing, vision, and mental disability. Another remarkable finding of the study is that teachers who take courses on assistive technology have higher attitude scores than those who do not. Kutlu and colleagues (2018) also examined the views of special education teachers on the use of assistive technology, similar to the study by Aslan (2018). 211 special education teachers participated in the study. According to the data of the study, among the factors that prevent special education teachers from using assistive technologies are the complexity of technological equipment, the cost of this equipment, lack of technological equipment, and insufficient knowledge of assistive technologies. Another finding of the study is that education support, technical support, and budget support are among the support strategies for the use of assistive technologies. Evgin and colleagues (2020), on the other hand, discussed special education teachers and technology issues together with a similar study and examined the opinions and experiences of teachers. Different findings were obtained in this study, in which eight special education teachers participated. Data were collected on why special education teachers use assistive technologies, the benefits of these technologies, the difficulties they encounter in the process of using them, and the teachers' opinions and suggestions regarding the use of technology in the special education process. When the findings of the study are evaluated in general terms, it is seen that the teachers find the use of technology use, but they encounter difficulties during use. The study emphasizes the importance and necessity of using technology in special education. Bağlama and colleagues (2017) investigated the views of special education teachers working at a special education institute on the use of technology in teaching mathematics to students with special needs. Although teachers perceived their technology competencies as high, most of the teachers in the study argued that there is a need for more in-service training on using technology in mathematics teacher processes. In the mixed-method study conducted by Allsopp and colleagues (2009), the perceptions of special education teachers towards technology integration were examined. Stress, workload, and lack of appropriate teacher training were reported to be major barriers to technology integration.

With the necessity of technology integration in special education classrooms, what is meant to be expressed is not that the teacher uses technology, but that they use technology by the objectives and needs. While applying technology-enhanced teaching, teachers who monitor the effectiveness of teaching and use their observations to plan future lessons are described as "successful" (Kennedy & Deshler, 2010). Thanks to teachers' experiences with technology, teachers develop their knowledge and belief in technology. Students will also be able to benefit from technology by integrating technology with course content and using technology effectively (Wang et al., 2012). Considering these and similar reasons, they must be encouraged to use computer-aided tools to update their teaching approach and help students who need special education in their learning (Russak, 2016). Special education teachers need different strategies to teach and it is very important and necessary to implement the most appropriate strategy, whether special education or general education processes. The more appropriate strategies the teacher chooses, the more benefits the student's benefit (Hess et al., 2008). Special education teachers do not have a chance to stay away from teaching processes surrounded by technology. Therefore, it can be said that coping with these processes will support the planning of more efficient teaching processes.

Exploration of technology with metaphors

Special education teachers and visionaries need to take advantage of the driving forces of the 21st century strategically and systematically to turn transformative models into reality (Fullan, 2011). Furthermore, teacher training needs to be designed to meet the needs of prospective teachers and provide hands-on activities focused on the ability of technology to influence students' learning (Siyam, 2019). Teacher educators must adopt a much-needed transition from the 20th century to the 21st-century teacher development practices (Brownell et al., 2010).

Teachers' positive perceptions of technology are an important factor for technology integration (Ertmer, 1999), but it is not enough (Bauer, 2013). Being aware of the contributions of technology can be a step towards a good integration process. Before supporting teachers to use technology, what needs to be done is to see how teachers make sense of technology. Studies that improve our understanding of dynamics such as the thoughts, beliefs, and experiences of special education teachers regarding the use of educational technology can guide professional development efforts to encourage the effective use of technology in the context of special education (Anderson & Putman, 2020). At this point, "metaphor" can be considered as an assistive data collection tool. Metaphors further increase the quality of the learning process by establishing strong connections between the student's past learning and personal experiences and newly learned concepts and creating vivid images (Arslan & Bayrakçı, 2006).

It is also seen that metaphors are frequently used in educational studies to examine different phenomena. In their metaphor study, Karaçam and Aydın (2014) examined middle school students' perceptions of the concept of technology. In the study in which a total of 191 students studying in the city of Ankara participated, 68 different metaphors related to the concept of technology were obtained. These metaphors are grouped under the conceptual categories of useful, evolving, constantly changing, rapidly evolving, necessary, beneficial and harmful, limitless and infinite, and rapidly spreading. Korkmaz and Ünsal (2016) examined the perceptions of preschool teachers towards the concept of technology. 76 preschool teachers working in Gaziantep province participated in the study. A total of 57 metaphors were obtained from preschool teachers. The metaphors produced are grouped under the categories of positive, negative, eternity, a living being, need, and life. Durukan and colleagues (2016) examined pre-service teachers' perceptions of technology through metaphor analysis. 53 teacher candidates participated in the study. The teacher candidates produced a total of 118 metaphors. The metaphors of the teacher candidates are primarily divided into positive, negative, and neutral main categories. Under the positive main category, metaphors related to the concept of technology, development, progress, being a source of information, facilitation, renewal, need, change, being eternal, entertaining, reaching, enlightening, producing, communicating, being interesting, being a major, being useful grouped under subcategories. Under the main category of negative, technology metaphors are grouped under addiction and harmful sub-categories. In the Neutral main category, technology has been included with features such as good or bad. Şahin (2019) examined the teachers' views on the use of information technology through metaphor analysis. 13 teachers participated in the study. The metaphors stated by the teachers were grouped under three main themes: (1) useful tool, (2) depth, development and change, and (3) two-way effect (useful-harmful). Özyurt and Badur (2020) investigated primary school students' perceptions of technology with the help of metaphor analysis. 346 primary school students participated in the study. The metaphors obtained from the students were grouped under 10 themes: equipment, structure, imagination, research, harmful, profession, change and development, educational tool, game, and affective. It is seen that the concept of technology has been studied by different researchers and that both similar and some different views and themes have been put forward. As can be seen from the studies, common meanings occur in the perceptions of different groups towards technology. In this context, it can be easily stated that technology affects every audience from similar angles. Although the potential of technology in special education is known, it is considered important to determine/know the perceptions of special education teachers towards technology. In this context, this study can be a step in interpreting the current situation in special education processes and evaluating special education and technology policies.

Purpose of Study

This study aims to examine the perceptions of special education teachers towards the concept of technology through metaphors. For this purpose, the following questions were sought:

1. What are the metaphor perceptions that special education teachers have towards the concept of technology?
2. Under which conceptual categories can the metaphors developed by special education teachers be collected?

Method

Research Model

This research was designed with the phenomenology approach, which is one of the qualitative research methods. Phenomenology research is an inquiry strategy applied to reveal human experiences about a phenomenon defined by the participants (Creswell, 2007). In factual science work, the focus is on evaluating experiences and understanding the essence of these experiences (Miller, 2003; Rose et al., 1995). It can be defined as a focus (Yıldırım & Şimşek, 2011). In this direction, the focus of the research process is how special education teachers conceptualize their thoughts on technology with the help of metaphors.

Metaphors can make communication more economical and efficient and can fill word gaps and motivate semantic change (Colston & Gibbs, 2017). Metaphor is one of the most important tools of trying to partially understand what we do not fully understand, our emotions, aesthetic experiences, our moral practices, and our spiritual consciousness (Lakoff & Johnson, 2003). Metaphor is also a tool of explanation and persuasion (Thibodeau et al., 2017). Botha (2009) states that metaphor can be used as a way of discovery in education and with its creative, innovative, and interactive role, it will provide similarities between the student's previous understanding of an unknown subject and new knowledge acquisition.

Study Group

164 special education teachers participated in the data collection process of the research. The provinces where special education teachers work can be listed as follows: Adana *(n = 11)*, Ankara *(n = 22)*, Amasya *(n = 3)*, Bursa *(n = 16)*, Çanakkale *(n = 4)*, Elazığ *(n = 10)*, Erzurum *(n = 7)*, Eskişehir *(n = 8)*, Gaziantep *(n = 6)*, Istanbul *(n = 21)*, İzmir *(n = 19)*, Kayseri *(n = 6)*, Muğla *(n = 10)* , Sinop *(n = 1)*, Sivas *(n = 1)*, Tekirdağ *(n = 2)*, Şanlıurfa *(n = 5)*, Osmaniye *(n = 2)*, Konya *(n = 4)*, Antalya *(n = 5)*, and Mersin *(n = 1)*. The demographic characteristics of the participants are given in Table 1.

Table 1 Demographic information of the participants

		f	%
Age	30 -	38	23.2
	31-40	73	44.5
	41-50	39	23.8
	51 +	14	8.5
Gender	Female	96	58.5
	Male	68	41.5
Education S.	Graduate	143	87.2
	Master	16	9.8
	PhD	5	3.1
TOTAL		164	100

In order to communicate with the special education teachers who will participate in the study, informative e-mails were sent to the administrators of the special education and rehabilitation centers in each province and the e-mail addresses of the special education teachers who wanted to participate were requested. Participants consist of special education teachers whose e-mails were reported to the researcher. That is, the participants were included in the study with the convenience sampling method.

Data Collection

The views of the participants were collected using an interview form consisting of open-ended questions developed by the researchers. (*"What would it be if you wanted to explain technology with a living or non-living being?"* and *"Can you explain why?")*

Analysis of Data

Content analysis consisting of coding, finding themes, coding data, and organizing according to themes was used in the analysis of the data. In content analysis, data that are similar to each other are organized by bringing together within the framework of certain concepts and themes, and the data are interpreted (Yıldırım & Şimşek, 2013). In addition, descriptive statistics (frequency distributions and percentage expressions) were used to report the data. The metaphors and justifications of the metaphors developed by the participants were resolved in four stages (Figure 1).

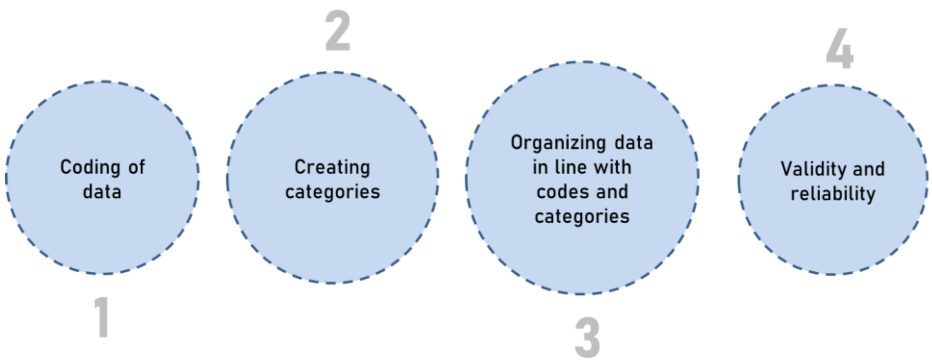

Figure 1 Steps followed in analyzing metaphors

At the stage of coding the data, a code number was assigned to each participant to be shown as "K1, K2, ... K164". The answers given by the participants to the first and second questions of the interview form were collected under the headings of "metaphor" and "explanation". While conceptualizing / coding the data, it was tried to find a concept that could best reflect the meaning in that section. During the category creation phase, the metaphor perceptions produced by all participants for the first and second questions in the interview form were examined in terms of their common characteristics of the technology. To create the categories, the codes were brought together, examined, similarities and differences were determined, and it was checked whether the data under the resulting category constitute a meaningful whole. At the stage of organizing the data according to codes and categories, after detailed coding was done by the researcher, and after the determination of the categories that could bring together the relevant codes, the researcher created a system where he could organize the data he collected and carried out the editing process according to this system. To ensure internal validity, students' perceptions of technology were first defined with direct quotations and then interpreted. To ensure the consistency of the findings, the consistency of the concepts that constitute the themes among themselves and with other themes was evaluated and it was checked whether they formed a meaningful whole. The findings were analyzed by two special education experts and one education specialist and found to be realistic. To ensure external validity, the method of the research has been tried to be defined in detail. In addition, the findings were supported by the literature, thus providing a variety of literature. After listing the data obtained in the study, first, the researcher and two experts were coded separately and coding categories were developed. The reliability coefficient between coders was calculated as 94% according to the formula "Reliability = (Compromise / [Compromise + Disagreement]) * 100" given by Miles and Huberman (1994). If this value is over 90, it shows that the encoding has very high reliability. The consensus of the coders was completed with the result of joint work on the different data interpreted in 6%.

Findings

164 special education teachers produced 152 valid and 95 different metaphors in total. 95 metaphors were analyzed together with their justifications and gathered under four categories. These categories are defined as process, material, requirement, and approach (Figure 2). In this section, metaphors of each category and other information obtained are given respectively.

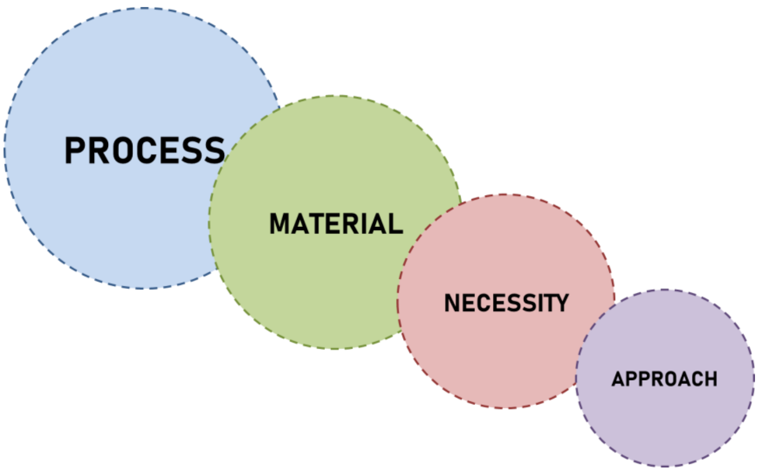

Figure 2 Categories Obtained after Analyzing Metaphors

Metaphors of the "Process" Category

The metaphors expressed by special education teachers who participated in the study and produced valid metaphors were mostly included in the process category. The process category was named this way because of metaphors such as change, development, adventure, growing up, and the common reasons for metaphors. Valid metaphors stated by special education teachers regarding the process category are given in Table 2.

Tablo 2 Metaphors related to the process category

Metaphor name	f	%	Metaphor name	f	%
Change	7	21.9	Run	1	3.1
Development	6	18.8	Fatigue	1	3.1
Topicality	4	12.5	Resistance	1	3.1
Grow	3	9.4	Refresh	1	3.1
Keep up with the times	2	6.3	Information source	1	3.1
Life	1	3.1	Develop	1	3.1
Adventure	1	3.1			
Stairs	1	3.1			
Move forward	1	3.1			
TOTAL	-	-	-	32	100

As seen in Table 2, 14 different metaphors in total have been included in the process category. These; metaphors change, development, actuality, growing up, keeping up with the times, life, adventure, ladder, progress, running, resistance, refreshing, and information source. Special education teachers mostly expressed the metaphor of technology change. K18 filled out the open-ended interview form as follows: *"Technology is change. Because its existence is due to the change of humanity and needs. For this reason, it can never remain the same and has to change. It is the best concept that corresponds to change.*

Technology was defined by six special education teachers with the metaphor of development. Preferring the development metaphor, K38 filled out the open-ended interview form as follows:

> *"Technology is a development process. It does not contain anything that does not develop. It both develops within its own development cycle and develops everything that is caught in this cycle."*

Technology was defined by four special education teachers using the metaphor of actuality. Using the metaphor of actuality, K3 filled out the open-ended interview form as follows:

"Technology is the most up-to-date and latest version of everything. If it is not up to date, it is not possible for it to stay in our lives. Although it has different features in up-to-date, its main feature is that it must be up-to-date and keep up to date.

The process category includes metaphors that describe technology as a living construct without borders. In this sense, it draws attention that we are faced with a concept that goes beyond any tool.

Metaphors of the "Material" Category

The metaphors expressed by special education teachers who participated in the study and produced valid metaphors were placed under the secondary majority category of material. The material category is named this way because of the common reasons that technology is expressed as a tool or equipment. Valid metaphors stated by special education teachers regarding the material category are given in Table 3.

Table 3 Metaphors related to the material category

Metaphor name	f	%	Metaphor name	f	%
Tools	8	29.7	Store	1	3.7
Computer	4	14.8	Lecture notes	1	3.7
Internet	3	11.1	Bag	1	3.7
Book	3	11.1			
Information source	2	7.4			
Encyclopedia	1	3.7			
Guide	1	3.7			
A4 paper	1	3.7			
Notebook	1	3.7			
TOTAL	-	-	-	27	100

As can be seen from Table 3, a total of 12 different metaphors have been included in the material category. These; tools-equipment, computer, internet, book, information source, encyclopedia, guide, notebook, lecture note metaphors. Special education teachers mostly expressed the tool-equipment metaphor for technology. K26 filled out the open-ended interview form as follows:

"Technology is actually the definition of tools in my classroom. Because the tools and equipment are no longer just paper or pen as before. My computer, my mouse, my overhead projector, everything, but all tools and gadgets describe technologies in my class.

Technology was defined by four special education teachers using computer metaphors. Preferring the computer metaphor, K17 filled out the open-ended interview form as follows:

"Technology is actually computers. I access the exam, content, management, follow-up from there. With the computer, I can manage all my technological processes and needs."

Technology was defined by three special education teachers using the internet metaphor. Using the internet metaphor, K90 filled out the open-ended interview form as follows:

"The internet is actually the biggest, unlimited and infinite technology. With it, I reach out to other needy students. I share with them. I work in collaboration with their teachers. Technology equals internet. Because if it weren't for that big network, I don't think we could do anything."

Technology has been defined by three special education teachers using a book metaphor. Using the book metaphor, K143 filled out the open-ended interview form as follows:

"It's a printed book or technology for me. I have many books in my library and I could find whatever I wanted. Before, of course… Now the library is not enough and now I have an unlimited library. Technology is a great book that offers me this opportunity."

The material category actually explains the tools and devices that show the physical equivalent of technology and are used in the teaching process.

Metaphors of the "Necessity" Category

The metaphors expressed by special education teachers who participated in the study and produced valid metaphors were numerically included in the third majority category of necessity. Valid metaphors stated by special education teachers regarding the requirement category are given in Table 4.

Table 4 Metaphors related to the necessity category

Metaphor name	f	%	Metaphor name	f	%
School	3	13.6	Health	1	4.5
Money	2	9.1	Work	1	4.5
Medicine	2	9.1	Shelter	1	4.5
Breath	2	9.1	Getting a certificate	1	4.5
Food	2	9.1	Getting additional training	1	4.5
Innovation	2	9.1	Have a hobby	1	4.5
Puzzle	1	4.5			
Water	1	4.5			
Exam	1	4.5			
TOTAL	-	-	-	22	100

As can be seen from Table 4, a total of 15 different metaphors have been included in the requirement category. These; school, money, medicine, breathing, food, innovation, riddle, water, test, health, work and shelter are metaphors. Special education teachers most often expressed the school metaphor for technology. K49 filled out the open-ended interview form as follows:

"Technology is a school that has to go and finish, and actually never ends. There are so many things we do not know about. Teacher, administrator, lesson, exam, friendship. It's a great door. It is also quite mysterious. Isn't that the same in school for a child who is just starting out? It's like school for me and will always be. Will I graduate, that place is discussed, of course ..."

Technology was defined by both special education teachers with the metaphor of money. Preferring the money metaphor, K51 filled out the open-ended interview form as follows:

"The definition is money. Without money, we cannot meet most of our needs. Even if there is no technology, unfortunately, we are about to be unable to meet any needs in educational processes. This situation does not make me very happy. Is that what to improve? This is unfortunately…"

Technology was defined by two special education teachers using the drug metaphor. Using the drug metaphor, K82 filled out the open-ended interview form as follows:

"There is a medicine. I wanted to resemble him. I need it to relieve my pain and to continue my daily life in good health. If I slow down the learning process of my students or they fall behind, I will have a headache and technology will ease my pain.

Technology was defined by two special education teachers using the metaphor of breathing. Using the metaphor of breathing, K75 filled out the open-ended interview form as follows:

"Technology is like breathing. Breathing occurs naturally, but most importantly, you cannot live without it. It is in such critical places that we cannot move even one step further without technology. It is actually that important and necessary. "

The necessity category includes metaphors that show that technology is an inseparable part of life.

Metaphors of the "Approach" Category

The metaphors expressed by special education teachers who participated in the study and produced valid metaphors were numerically placed under the fourth majority approach category. Valid metaphors stated by special education teachers regarding the approach category are given in Table 5.

Table 5 Metaphors related to the approach category

Metaphor name	f	%	Metaphor name	f	%
Way of thinking	2	13.3	Administration	1	6.7
Fashion	2	13.3	Parenthood	1	6.7
Mind	1	6.7	Accent	1	6.7
Style	1	6.7	Philosophy	1	6.7
Acceptance	1	6.7	Brain	1	6.7
Child	1	6.7	Art	1	6.7
Alive	1	6.7			
TOTAL	-	-	-	15	100

As can be seen from Table 5, a total of 13 different metaphors have been included in the approach category. These; The way of thinking, fashion, style, mind, acceptance, child, living, brain, art, management, parenting, accent, philosophy are metaphors. Special education teachers expressed the most common way of thinking metaphor for technology. K44 filled out the open-ended interview form as follows:

“Technology reflects one's way of thinking. How you use it, what you use it for, what you see it as is entirely up to you. The way you think about technology also shows how it exists in your life."

Technology was defined by two special education teachers with the metaphor of fashion. Preferring the fashion metaphor, K51 filled out the open-ended interview form as follows:

“Technology is a fashion. It changes, is followed and applied by some. But a small part is not interested at all. But even if they are not interested, that fashion infiltrates their lives somehow. "

Technology was defined by a special education teacher with the metaphor of philosophy. Using the philosophy metaphor, K82 filled out the open-ended interview form as follows:

“He can think of technology as a philosophy. It has its own concepts, processes and validities. And technology can only be explained within these limits. "

The approach category includes metaphors showing that technology is actually a superstructure related to a way of thinking.

It can be seen from the tables that present metaphors to the categories above and the opinions of special education teachers; technology is perceived from four different aspects:

process, product, requirement or approach. The metaphors of special education teachers regarding technology as a whole are shown in Figure 3.

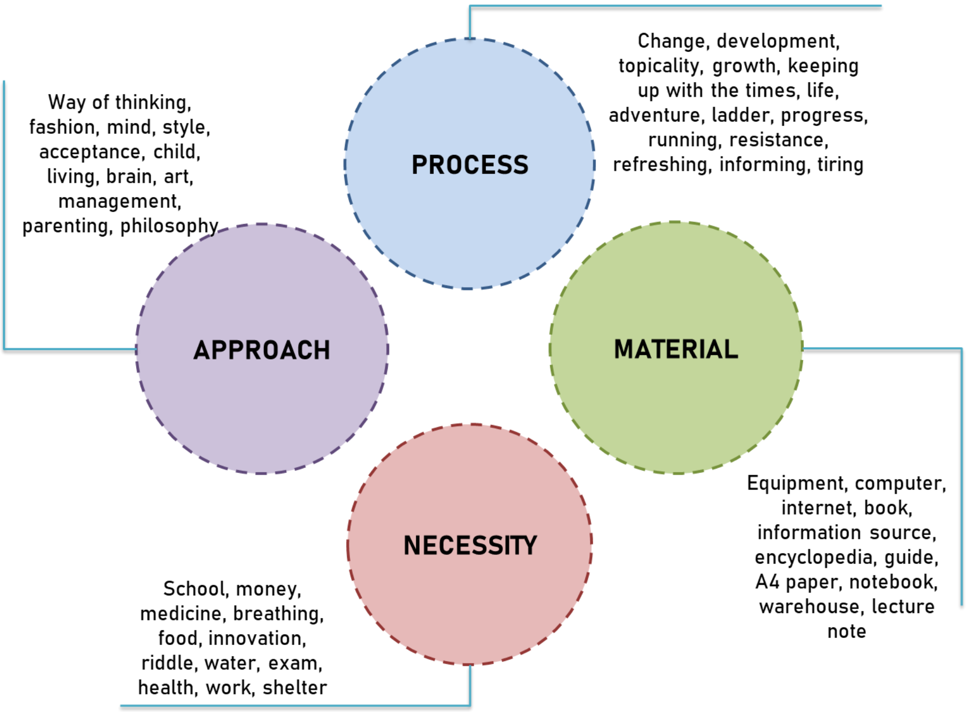

Figure 3 Categories and metaphors related to the concept of technology

Conclusion, Discussion and Suggestions

In this study, the perceptions of special education teachers towards the concept of technology were examined through metaphors. In line with the data obtained from the research, 164 special education teachers produced 95 valid and different metaphors. These metaphors produced by special education teachers were discussed and reported in four categories: (1) Process, (2) Material, (3) Necessity, and (4) Approach. When the metaphors obtained from 164 special education teachers are analyzed, it is seen that the process category is in the first place numerically. Process category has been a category expressing that technology is an ongoing, progressive and dynamic structure. Under this category, metaphors of change, development, actuality, growing, developing, keeping up with the age, life, adventure, ladder, progress, running, resistance, refreshing, and getting informed were included. Many studies are examining the views of students and teacher candidates on the concept of technology using metaphors. And the findings of the studies are in line with the findings of this study. In this study, he frequently stated that technology is a development process and that it also improves the elements associated with it, and this category is supported by the findings of different studies (Durukan et al., 2016; Ergen & Yanpar-Yelken, 2015; Gök & Erdoğan, 2010; Göksu & Koçak, 2020; Karaçam & Aydın, 2014; Kobak &Taşkın, 2012; Koç, 2013; Kurt & Özer, 2013; Şahin, 2019; Özyurt & Badur, 2020). Special education teachers emphasized that technology corresponds to the expression of "change" and stated many metaphors on this theme. It is seen that technology is transferred as the change in different studies trying to determine the perceptions towards technology with metaphors (Durukan et al., 2016; Ergen & Yanpar-Yelken, 2015; Gök & Erdoğan, 2010; Karaçam & Aydın, 2014; Kobak & Taşkın, 2012; Şahin, 2019; Özyurt & Badur, 2020). In addition, categories such as keeping up with the age (Göksu & Koçak, 2020), life (Korkmaz & Ünsal, 2016; Karakoyun, 2017), advancement (Durukan et al., 2016), supporting or facilitating learning (Durukan et al., 2016; Göksu & Koçak, 2020), which are among the characteristics of special education teachers, are supported by the literature. The findings showed us that technology has a meaning that can never be defined as a static and rigid structure. Technology is the most up-to-date solution that helps to meet the needs in life. In this context, the development, change, dynamic structure, etc. of teachers and students. It is not surprising that they perceive technology with expressions. What matters is whether teachers and students progress in parallel or not, in addition to a structure that progresses continuously. When the category of material is examined, it is seen that metaphors of equipment, computer, internet, book, information source, encyclopedia, guide, notebook, warehouse, lecture note are specified. The perception of technology as a concrete tool has been reported in different studies (Ergen & Yanpar-Yelken, 2015; Şahin, 2019; Özyurt & Badur, 2020). What do the findings revealed by the material category and the findings in the literature with similar meaning tell? In a teaching environment that keeps pace with the times, whatever you touch is a technology. It is an impossibility to want to exclude technology from such a teaching environment.

When the necessity category is examined, it draws attention that technology is perceived as a need. Under this category, the metaphors of school, money, medicine, breathing, food, innovation, riddle, water, exam, health, work, and shelter were frequently expressed by special education teachers. It was reported that technology was expressed in different metaphors in the necessity category (Çavaş et al.,2019; Durukan et al., 2016; Ergen & Yanpar-Yelken, 2015; Koç, 2013; Korkmaz & Ünsal, 2016;). Humanity is now trying to integrate itself into the world of technology rather than trying to add technology to its own life. It can be easily said that there is necessary and continuous cooperation between humanity and technology. It is not up to us to adapt technology to our whims, on the contrary, we have to adapt to technology (Feenberg, 2009). Technology, which is said to come from human hands, dominates nature and now it also dominates humanity (Günay, 2017).

When the approach category is examined, the focus is on perceiving technology as a lifestyle rather than a process, product, or necessity. It was determined that the metaphors of thinking style, fashion, style, reason, acceptance, child, living, brain, art, management, parenting, accent, philosophy were used under this category. In the study conducted by Çavaş and colleagues (2019), technology is expressed similarly with the art metaphor. However, no results of a different study supporting this finding were encountered. Technology is a philosophy above all else. There are many different opinions about which philosophical approach the technology is more suitable for or whether it requires a new perspective. Instrumentalism (instrumentalism) is a standard modern view that states that technology is a tool of the human species (Feenberg, 2009). From a deterministic point of view, it is defined as a rationally created tool that serves universal human needs (Feenberg, 2009). There are many more philosophies that consider technology as an approach, a way of thinking, and all of them try to analyze the nature of technology with its effects on human beings.

With the analysis of special education teachers' views, it has been seen that; technology is both a necessity for them and they are exposed to it as a material. While they perceive technology both as a process, they can define it with an even broader perspective and accept it as a philosophy. Although we know that there are numerous contributions to the process by integrating technology into special education processes, it is very important to support teachers at this point. Special education teachers need to recognize the 21st century and systematically benefit from the features of the 21st century to keep up with the times and act in parallel with technology. Teacher educators must adopt a much-needed transition from the 20th century to the 21st-century teacher development practices (Brownell et al., 2010). In the teacher education process, it should be designed to meet the needs of all teacher candidates and to provide applied activities that focus on the ability of technology to affect students' learning (Siyam, 2019).

In this study, we saw how close special education teachers are to technology. The importance of technology providing interactive, effective and productive learning environments for special education students and making these environments more accessible and controllable by special education teachers should also be considered. Their belief that both themselves and their students can develop in technology shows the necessity of integrating technology more into the field of special education. Technology, which offers a wide variety of material formats, has the potential to be diversified according to the qualifications and needs of students who need special education. Technology is an important factor that spreads to all areas of life and makes learning processes more comfortable, accessible and interactive. Planning the learning processes in an integrated manner with technology also means not missing the age. In short, although there is no evidence that special education teachers' technological perceptions and attitudes are negative, it is seen that they are in a close position to technology. Studies need to be done in the relevant contexts, taking this position into account. The more teachers are supported, the more accurately their learning processes will integrate with technology. At the end of these processes, again, our students will be the winner - what kind of an aim we are.

Özel Eğitim Öğretmenleri Ve Teknoloji: Bir Metafor Analizi

Özet:

Bu çalışma teknolojinin özel eğitim bağlamındaki yerine odaklanmış ve özel eğitim öğretmenlerinin teknolojiye yönelik algıları metafor analizi aracılığı ile incelenmiştir. Çalışma verileri 164 özel eğitim öğretmeninden elde edilmiştir. Olgu bilim kapsamında yürütülen bu çalışmada katılımcıların teknolojiye yönelik algıları olarak bir metafor ve bu metaforun gerekçesi çözümlenmiştir. Özel eğitim öğretmenlerinin teknolojiye yönelik belirttikleri metaforlar (1) Süreç, (2) Materyal, (3) Gereklilik ve (4) Yaklaşım olmak üzere dört kategoride toplanmıştır. Özel eğitim öğretmenlerinin ilk sırada gelişen, geliştiren, değişen, değiştiren, üreten, büyüyen özelliklerini atfederek teknolojiyi süreç olarak algıladıkları görülmüştür. Materyal kategorisinde araç gereç, bilgisayar, ansiklopedi gibi metaforlar sıklıkla aktarılmıştır. Gereklilik kategorisinde ise, öğretim sürecini, öğretmeni ve öğrenciyi zorlayan ve bu zorlamanın güncel gelişmeleri takip etmek, ek eğitimler almak, sertifika programlarına katılmak, kişisel hobiler edinmek gibi alanlarda olduğu vurgulanmıştır. Yaklaşım kategorisi ise teknolojinin bir yaşam biçimi, bir felsefe ya da inanç olarak ifade edilmesini kapsamıştır.

Anahtar kelimeler: öğretim teknolojisi, özel eğitim, teknoloji entegrasyonu, metafor.

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Araştırma Makalesi / Research Article

Mathematical Modeling in Primary School: Students' Opinions and Suggestions on Modeling Activities Applied as a Teaching Experiment

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One of the main purposes of teaching mathematics is to enable students to solve real-life problems and relate mathematics to real-life situations. As a way of facilitating the teaching of mathematics, it should be applied in lessons with mathematical modeling activities of real life problems. The aim of this study is to determine the views of primary school 4th grade students who have experienced mathematical modeling activities for 9 weeks and their suggestions for future modeling activities.The participants of the study are 12 students selected by purposive sampling method among 69 students attending the 4th grade of a public school in Konya in the 2019/2020 academic year. As a result of the study, students; In addition to positive opinions such as increasing the interest of mathematical modeling activities in the lesson, increasing their success in mathematics lessons and improving their social skills, they also expressed negative opinions such as long questions, problems in group work and insufficient time.

Keywords: Mathematical Modelling, Model Building Activities, Primary School, Student Opinions, Teaching experiment

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Introduction

In recent years, developments in the field of education have brought about changes in education programs. The primary school mathematics curriculum was also affected by the updated education programs. Especially with the changes made in 2005 and after, there are significant differences in the mentioned program. One of these changes is the expression of mathematical modeling in the 2009 primary school mathematics curriculum (MEB, 2009). This change is seen in the 2015 and 2018 programs as the contribution of mathematical modeling applications to teaching in the "issues to be considered in the implementation of the programs" (MEB, 2015 and 2018).

One of the most important applications of science, technology, engineering, mathematics and design, which are the components of STEM+A applications, which is the last reflection of science and technology in education, is undoubtedly mathematics. It is thought that the use of mathematics in daily life and the transfer of mathematics to students in a concrete way will increase the success of the students in this course. One of the general aims of the mathematics course is to develop students' problem solving skills. It is seen that the problems frequently used in mathematics lessons are not sufficient for students to associate mathematics with real life situations (Erbaş et al., 2016). This inadequacy can be overcome with problems involving daily life situations, and students can develop positive attitudes towards mathematics. Particularly in the primary school period, teaching methods that support students to develop positive attitudes towards being able to succeed in should be employed. In this way, students will be able to achieve mathematics and will enjoy this lesson by developing a positive attitude towards mathematics. Students who develop positive attitudes towards mathematics will also be able to use mathematics in their daily lives. The results obtained in PISA (2015 and 2018) and TIMSS (2015 and 2019) exams reveal that 4th and 8th grade students in our country have difficulty in making open-ended questions that relate mathematics to daily life. This difficulty can be overcome by using mathematical modeling activities in the lessons.

The national council of mathematics teachers evaluated mathematics education within the framework of principles, standards and processes in its report (NCTM, 2000). When we examine the 2018 mathematics curriculum, it is seen that the learning areas have a similar scope to the content section in the report published by NCTM, and that it coins with the principles and process standards in the NCTM report in the “Turkey Qualifications Framework” (TYÇ), which is the main basis for the implementation of the program. Mathematical competence is the development and application of mathematical thinking to solve a range of problems encountered in daily life. It is important that students gain mathematical competence, especially in the primary school period. We need to convey mathematics, which is found in all areas of our lives, to students, especially in the primary school period, by concretizing and making the verbal problems to be solved meaningful. The biggest difficulty encountered in mathematics education when students work with daily life problems is that students cannot transfer the knowledge they learned during the problem-solving phase to daily life (Altun, 2018, p.125). In cases where we cannot achieve this transfer, even if the students solve the problem correctly by using their procedural knowledge from the given information, they cannot associate this solution with daily life.

Associating mathematics with daily life and making the solution of verbal problems addressed to students meaningful by associating them with daily life is important for the permanence of learning. In this context, the most important factor that determines the adequacy of student success in solving problems in daily life is the competence of students in applying their mathematical knowledge to daily life situations (Greer, 1993). In order to relate mathematics to daily life and to solve daily life problems, students should be able to transfer knowledge from mathematics to daily life, develop and use original strategies and models, make logical predictions about the solution, and evaluate the accuracy of the result in the context of daily life (Chacko, 2004, p. 96).

One of the applications that relate mathematics to daily life situations is mathematical modeling activities. (Lesh & Doerr, 2003; English & Watters, 2005). Explaining the concepts of model and modeling before explaining mathematical modeling will help to understand this concept. Models are schemas defined by abstraction or generalization that occur in the mind during the problem solving process (Kertil, 2008). In other words, the model is the whole of the products formed in the mind during the problem solving phase. Considering the definitions of model and modeling, model and modeling in mathematics is defined as making complex systems mathematically meaningful through mathematical expressions (Lesh & Doerr, 2003). The model is the whole of the conceptual structures existing in the mind and the external representations of these structures in order to interpret and understand complex systems and structures (Doruk, 2010). As can be understood from the definitions made, the model; It is a product that will help solve the problem by going through different mental processes in the problem solving process of a situation we will encounter in real life. While we describe the model as the product that has emerged, we can evaluate the modeling as the process that occurs in the emergence of this product. In general terms, mathematical modeling is defined as a process that includes the analysis of a real-life problem with mathematical methods by transferring it to the world of mathematics (Borromeo Ferri, 2006; Bukova Güzel, 2016; Maaß, 2006). Many researchers define mathematical modeling as a process that involves the analysis of a real-life problemby transferring it to the world of mathematics and using mathematical methods (Borromeo-Ferri, 2006; Bukova Guzel 2016; Maaß, 2006). There are two prominent elements in the definitions of mathematical modeling. The first is the consideration of mathematical modeling as a process, and the second is the relationship between the real world and the mathematical world. Mathematical modeling refers to problem solving processes associated with real life, which can be interpreted differently by each student, as opposed to open-ended, routine word problems that do not contain a single correct answer. According to Maaß (2006), in modeling activities, students need to make sense of the real-life situation and express it in mathematical language, analyze and interpret the information given about the situation, select the necessary data and associate the solution with the real-life situation within the framework of these data. He also states that modeling activities are more effective than traditional word problems for the discovery of mathematics in real life and the mathematical development of students (Maaß, 2006). According to Bukova Guzel (2016), in the solution process of mathematical modeling activities, the real life situation is expressed by mathematizing, the problem and the factors affecting the solution of the problem are specified, and the answers regarding the solution of the problem are tried to be reached based on assumptions. Lesh and Doerr (2003) define mathematical modeling as a process, which they see as a phase of Model Eliciting Activities (MEAs). In this context, MEAs are non-routine problems in which students are asked to produce solutions that can be based on more than one assumption in order to generalize the model they have created, and where different possible solutions are re-evaluated, by working in groups, students propose different solutions to real-life problems.

As a result of the radical changes made in primary school mathematics curriculum after 2005 in Turkey, a problem-solving-oriented approach is adopted and students are expected to reflect their problem-solving situations in real life. It will be possible for students to have the problem solving skills expected from them and to transfer them effectively to daily life situations with the classroom environment to be created and the textbooks prepared in this direction. However, it is seen that classrooms cannot be fully transformed into an environment where these skills can be gained (Uğurel, et al., 2011). In addition, it is thought that how students perceive mathematical modeling activities, what they experience during the application and their views on these activities are important in creating a problem-solving-based classroom environment. It is possible to talk about different components of these classroom environments. Running non-routine mathematical problems associated with real-life situations is one of the important elements of these components. One of the teaching methods applied in this context is the mathematical modeling activities that have become widespread in Turkey for the last 20 years. There are many studies on the planning, implementation and evaluation of mathematical modeling activities (Doruk, 2010; English & Watters, Eraslan, 2011; 2005; İncikabı, 2020; Kaiser & Brand, 2015; Kertil, 2008; Tekin Dede, 2015; Tekin, et al., 2011; Thomas & Hart, 2010 and Şahin, 2019). However, it has been determined that the number of studies evaluating students' opinions about mathematical modeling experiences is limited in number, and there is no study in which primary school students' views on mathematical modeling are taken. It is thought that this study will contribute to the field with this aspect.

When the national literature was examined, it was seen that the studies evaluating the reflections of mathematical modeling and modeling activities were carried out with teachers, prospective teachers, undergraduate and secondary education students. (Bilen & Çiltaş 2015; Deniz & Akgün, 2014; Deniz & Akgün, 2016; Eraslan, 2011; Güder, 2013; Işık & Mercan 2015; Karalı, 2013; Pilten, et al., 2016; Şahin & Eraslan, 2019; Tekin Dede & Bukova Güzel, 2013; Tekin, et al., 2014; Tutak & Güder, 2014; Urhan & Dost, 2016). In these studies, researchers used mathematical modeling; They concluded that they developed a positive attitude towards mathematics, contributed positively to mathematics learning, and that it would be beneficial to perform them together with project-performance tasks. In addition, in the studies where teachers' opinions were, they stated that the situations such as that the teachers did not take sufficient equipment for the application of mathematical modeling, that the students in the classroom were not accustomed to modeling activities and that the application environments were not suitable caused difficulties.

Özdemir and Üzel (2012) took the opinions of 14 students attending the 6th, 7th and 8th grades in learning environments based on mathematical modeling. As a result of the three-month training, most of the students expressed a positive opinion about the teaching process. While some students stated that the study was enjoyable, others expressed their negative thoughts due to exam anxiety. In addition, the students stated that they experienced a different classroom environment and that the study was positive in terms of effective learning of mathematics. Eraslan (2011) in his study, in which primary school mathematics teacher candidates' views on model building activities and their effects on mathematics learning were examined, stated that pre-service teachers' mathematical modeling activities have limitations as well as that they can be used at different levels from primary education to higher education and that they can contribute positively to mathematics teaching.

Tekin Dede and Bukova Güzel (2013), on the other hand, took the opinions of 17 teachers in their study in which they examined the model building activity design processes. the opinions of the teachers were evaluated before and after the modeling activities, and as a result of the study, the teachers stated positive opinions about the use of model building activities in the lessons; however, it is difficult to prepare such activities; therefore, they stated that the diversity of activities should be increased. In addition, the teachers who stated that they could use these activities at the beginning or at the end of the subject, within the scope of term papers or projects, stated that they would decide on the frequency of use depending on the suitability of the subject and time. Tekin et al., (2014) stated in the study that 21 primary school pre-service teachers' opinions on model-building activities were obtained, that mathematical concepts were made more concrete, that mathematics was associated with daily life, and real-life situation was better understood with these activities. . Urhan and Dost (2016) took the opinions of nine mathematics teachers about the use of modeling activities in lessons. As a result of the study, besides positive views such as making connections between mathematical subjects of modeling activities, associating mathematics with daily life and increasing motivation towards mathematics; They reported negative opinions such as modeling activities are not suitable for use in mathematics teaching, the education system is not suitable for modeling applications, and teachers are lacking in modeling activities.

When the international literature is examined, Thomas and Hart (2010) took the opinions of 16 primary school teacher candidates about mathematical modeling. Besides the negative thoughts of the pre-service teachers such as the lack of a certain process to be followed as a result of the flexibility of the solution of modeling activities, the difficulty of its implementation and its limitations; They stated that it would be interesting to have more than one correct answer and that students could develop different thinking skills. Kang and Noh (2012) stated that they contributed to the development of high-level thinking skills of students with mathematical modeling activities in the study in which they obtained the views of pre-service teachers through modeling activities. Soon and Cheng (2013), in their study with pre-service teachers, stated that model building activities would contribute positively to students' mathematics learning.

When the relevant literature is examined, it has been determined that the opinions and evaluations of mathematics teachers, graduate students, high school and secondary school students about mathematical modeling are taken, but there is no study about the opinions of primary school students. Within the scope of this study, students were asked to make a preliminary study in order to determine their real-life situations. A classroom environment focused on solving mathematical modeling activities appropriate to the context of the preliminary study was created. In this classroom environment, students' mathematical modeling competencies were determined and their opinions on the implementation of these activities and suggestions for future activities were taken. For this purpose; following research question were formulated:

1) What are the students' ways of solving mathematical modeling activities?
2) What are the positive and negative situations that students encounter while solving mathematical modeling activities?
3) What are the positive and negative opinions of students about mathematical modeling activities?
4) What are the students' perspectives towards the mathematics lesson after the mathematical modeling activities?

Method

This study is a qualitative study aiming to reveal the opinions of primary school 4th grade students about the mathematical modeling activities they experienced for nine weeks and the suggestions of the students for the application of mathematical modeling activities. The nine-week teaching activities were designed with a teaching experiment since they can be changed and renewed according to student learning throughout the research, are carried out with consecutive teaching sessions, and the researcher is in the role of a teacher (Steffe, 1991; Steffe & Thompson, 2000). Teaching experiment is a dynamic method designed primarily to explore and understand students' mathematical activities (Steffe & Thompson, 2000). In other words, as a method of teaching experiment, it is aimed to examine the development of students in a certain process and as a result of these examinations; It is the renewal of continuing education departments in a way that will contribute to student learning. The pilot implementation of the activities was carried out in the first term of the 2019-2020 academic year, and the main implementation was carried out in the 2nd term. Three of the modeling activities ("Which vehicle shall we go to the picnic with?", "Big Foot" and "Weather" (Appendix 1) (Doerr and English, 2003) of the modeling activities used in the applications were adapted from the relevant literature, and the other six ("Kermes", "Migratory Birds"), "Who Runs?" (Appendix 2), "Which Battery Vehicle Should We Rent?", "Uncle Farmer Hüseyin" (Appendix 3) and "How Should We Build the Barge?" were developed by the researcher. The features that mathematical modeling activities should have were used in the preparation of these activities (Lesh et al., 2000; Lesh & Caylor, 2007; English, 2009). During the preparation of the activities, the opinions of the teachers of the classes where the main application and the pilot application will be made were taken. to the examination of two Turkish teachers in terms of linguistic compatibility. The schedule of e and semi-structured interviews is presented in Table 1.

Table1. *Calender of Activities and Semi-Structured Interviews*

Pilot Study	*Main Study*	*Applied Activity*	*Activity Type*
01.10.2019	05.02.2020	Which Vehicle Shall We Go to the Picnic With?	Pre-Clinical Interview
03.10.2019	07.02.2020	Kermes	1. Mathematical modeling group application
10.10.2019	14.02.2020	Migratory Birds	2. Mathematical modeling group application
17.10.2019	21.02.2020	Who Runs?	3. Mathematical modeling group application
22.10.2019	26.02.2020	Which Battery Vehicle Should We Rent?	Intermediate- Clinical Interview
24.10.2019	28.03.2020	Big Foot	4. Mathematical modeling group application
31.10.2019	06.03.2020	Uncle Farmer Hüseyin	5. Mathematical modeling group application
07.11.2019	27.04.2020	How Should We Build the Barge?	6. Mathematical modeling group application
14.11.2019	04.05.2020	Weather	Final Clinical Interview
18/19.11.2019	07/08.05.2020	Semi-Structured Interview	

Research Group

This study was carried out with six students, each selected according to the criterion sampling method, among 33 students attending 4-H class (pilot application class) and 36 students attending 4-K class (main application class) of a public school in Konya. While determining these students, it was sought to get a score of 70 and above from the achievement evaluation form (Appendix 4) developed by the researcher, and to be able to express themselves well and have developed social adaptation skills at the end of the interviews with the classroom teachers. The real names of the students participating in the study were not used, each of them was given different code names. In order not to reveal the identities of the participants, the name of the school they studied was not mentioned in the study. Information about the participants is given in Table 2. below.

Table 2. *Information About the Participants*

Participant	*Number of Sibling*	*Date of birth*	*Mother Education Status*	*Father Education Status*	*Having a Computer at Home*	*Availability of Internet at Home*	*Years in the Same Class*
Eren	1	21.10.2009	Master	Master	Available	Available	4
Serra	2	07.01.2010	Master	Master	Available	Available	4
Selin	1	08.02.2010	University	Associate Degree	Available	Available	4
Mert	3	31.01.2010	High school	Associate Degree	Available	Available	4
Kerim	2	04.07.2010	Master	University	Available	Available	4
İpek	1	13.11.2009	University	Master	Available	Available	3
Ali	3	09.04.2010	University	University	Available	Available	4
Burak	4	22.10.2010	High school	High school	Absent	Absent	3
Cem	1	10.02.2010	University	Master	Available	Available	4
Duygu	-	15.03.2010	University	University	Available	Available	4
Ece	2	27.06.2010	Master	University	Available	Available	4
Gonca	1	18.08.2010	Master	Master	Available	Available	4

According to Table 2 presented above, it is seen that the number of siblings of the students is usually one or two and their birth dates are close to each other. However, Burak, from the participant students, differs negatively in terms of education level of his parents and having internet connection and computer at home. Two of the students have been in the same class for three years, while the other ten students have been in the same class for four years.

Data Collection

The data of the study were obtained by using semi-structured interview technique, which is a flexible questioning technique, in order to reveal students' opinions and suggestions about mathematical modeling activities at the end of the activities applied to the students for nine weeks (Güler, Halıcıoğlu, & Taşgın, 2015). In the preparation of the interview questions, national and international studies related to the subject in the literature were examined. There were studies that asked the opinions of teachers, novice teachers and secondary school students about modeling activities, but since there was no study that took the opinions of primary school students about mathematical modeling activities, questions specific to the research were formed. In these questions, students' perspectives on mathematical modeling activities and suggestions for future modeling activities were tried to be revealed. Two experts from mathematics education and qualitative research were consulted for the validity and reliability of the interview questions. In line with these views, “What do you think about the problem you have solved?” instead of “ What do you think about the contribution of mathematical modeling activities to mathematics teaching?” and “Were there any difficulties you experienced during the mathematical modeling activity? If so, can you share what these challenges are? instead of “Can you tell me the positive and negative situations you encountered during mathematical modeling activities?” questions were asked.

In order to ensure the reliability of the interview questions, a pilot application was made to a student who participated in the modeling activities. No problems were encountered in the pilot application and it was determined that the questions were suitable for the purpose of the study. Semi-structured interviews, which were made after the activities carried out as a pilot application in the first semester, were held at the school where the students were studying. However, the semi-structured interviews held in the second term were held online through the Zoom program due to the Covid 19 outbreak. An interview environment was created in which students could feel comfortable and open-ended questions were asked to the students. Open-ended questions help the researcher's flexible approach to the subject and prevent important variables related to the researched subject from being overlooked (Yıldırım & Şimşek, 2013). Some of the questions asked to the students in the interviews are presented below. The research questions are basically as follows, but the questions of the interview were differentiated in line with the answers from the students and "additional questions" were asked.

1-What kind of way did you follow for the solution of mathematical modeling activities?

2-What do you think about the contribution of mathematical modeling activities to mathematics teaching?

3-Could you tell us the positive and negative situations you encountered during the mathematical modeling activities?

4- Were the modeling activities and coursework suitable for your liking and preferences?

5-Do you evaluate group work and individual work during the activities?

6-Can you evaluate your view towards mathematics as a result of mathematical modeling activities?

For the interviews, first of all, the classroom teachers were contacted and the meeting schedule was created in accordance with the curriculum of the students. The meeting schedule and total interview duration are given in Table 3.

Table 3. *Meeting Calender and Total Call Time*

Participant	*Date Time*	*Duration*
Eren	18.11.2019/09:15	36’
Serra	18.11.2019/10:00	25’
Selin	18.11.2019/11:10	32’
Mert	19.11.2019/09:00	27’
Kerim	19.11.2019/09:45	33’
İpek	19.11.2019/11:00	37’
Ali	07.05.2020/14:00	28’
Burak	07.05.2020/15:00	22’
Cem	07.05.2020/16:00	31’
Duygu	08.05.2020/14:00	24’
Ece	08.05.2020/15:00	27’
Gonca	08.05.2020/16:00	30’

Analysis of Data

Content analysis was used in the analysis of the data obtained in the study. The main purpose in content analysis is to bring together data in similar expressions within the framework of certain concepts and themes and present them with relevant explanations in a way that the reader can understand (Yıldırım & Şimşek, 2013). Before the data analysis, the audio recordings of the interviews were transcribed verbatim. These raw data are classified under a list of codes and themes created to make sense for the reader. Then, these themes were summarized and presented in an explanatory framework under three categories (Miles & Huberman, 1994). In order to ensure the reliability of the data obtained from the interviews, the coding and categorization process was repeated 15 days later by the researcher. Subsequently, the data collected by two field experts who have doctorate degrees in education and are experienced in qualitative research were analyzed in different places, coded and categories were created (Yıldırım & Şimşek, 2013). Afterwards, two experts and researchers who analyzed the data came together, discussed and re-evaluated on the points where the difference of opinion occurred. The differences between the agreed common themes were eliminated and a consensus was achieved in the codes and categories created in this way.

Findings

When the data obtained in this study were examined, the views of the students on the use of modeling activities in the teaching process were determined under 3 themes. These themes include the students' positive opinions about the application of mathematical modeling activities, their negative opinions about the application of mathematical modeling activities, and the "recommendations for how and application of modeling activities" to be applied in the future, based on the students ' own experiences regarding the application of mathematical modeling activities in the teaching process.

Under the theme of "Students' Positive Opinions about Mathematical Modeling Activities"; The sub-themes "Contributes positively to mathematics learning", "Contributes positively to individual development", "Group work affects teaching positively" and "Will to take an active role" were formed. Under the theme of "Students' Negative Opinions about Mathematical Modeling Activities"; The sub-themes of "Unusual question structure", "Negative that will affect classroom management" and "Problems arising from conventional education practices" were formed. Under the theme of "Students' Suggestions for the Implementation of Mathematical Modeling Activities"; The sub-themes of "Suggestions for the studies carried out before the application", "Suggestions for the application", "Suggestions for the presentation" and "Suggestions for associating with other courses" were created.

While creating sub-themes, codes with direct quotations obtained from student opinions were included. In the sub-theme "Contributes positively to mathematics learning"; There are codes for increasing interest in the lesson, associating mathematics with real life and developing a positive attitude towards mathematics. In the sub-theme "Contributes positively to individual development"; There are codes for increasing success in mathematics, the belief that problems can be solved, and ensuring permanent learning. In the sub-theme “Group work affects teaching positively”; development of social skills and belief codes that problems can be solved. In the sub-theme “willingness to take an active role”; The codes of expressing oneself better, fulfilling responsibility and helping each other are included. In the sub-theme “The question structure is unusual”; There are codes for questions being too long, not being able to associate solutions with real life, differentiating the correct answers according to the people, and not understanding the questions. In the sub-theme "Negative situations that will affect classroom management"; There are codes for not being used to group work, having problems, having noise in the classroom, and late answering of the questions by the teacher. In the sub-theme of "problems arising from conventional education practices"; There are codes for not asking these questions in the exams and for the course duration to be short. In the sub-theme "Suggestions for the studies carried out before the implementation"; There are codes to help us solve the activities and if we do it together as a group. In the sub-theme of "Suggestions for implementation"; more time should be given and we should choose the groups. In the sub-theme “Suggestions for presentation”; Everyone should submit and we should not be interrupted are included in the codes. In the sub-theme "Suggestions for associating with other courses"; It can be used in other courses and there are codes that will be difficult to use in other courses. The themes and sub-themes created from the findings obtained from the semi-structured interviews with the students are presented in Table 4.

Table 4. *Themes, Sub-Themes and Codes Obtained From Student Opinions*

Theme	*Sub-Theme*	*Code*
Students' Positive Opinions about Mathematical	Makes a Positive Contribution to Mathematics Learning	Increasing Interest in the Course Relating Mathematics to Real Life Developing Positive Attitudes Towards Mathematics

Modeling Activities	Contributes Positively to Individual Development	Increasing Success in Mathematics Lesson
		Belief That Problems Can Be Solved
		Ensuring Permanent Learning
	Opinions on the Positive Effects of Group Work on Teaching	Development of Social Skills
		The Belief That Problems Can Be Solved With Group Work
	Active Mission Request	Expressing Yourself Better
		Fulfilling Responsibility
		The Importance of Solidarity
Students' Negative Opinions about Mathematical Modeling Activities	Unusual Question Structure	The Questions Are Too Long
		Inability to Relate Solutions to Real Life
		Differentiation of Correct Answers According to People
		Not Understanding Questions
	Adverse Situations Affecting Classroom Management	Unaccustomed to Group Work, Having Problems
		Having Noise in the Classroom
		Late Answering of Asked Questions by the Teacher
	Problems Caused by Traditional Educational Practices	Not Asking These Questions in Exams
		Short Lesson Duration
Students' Suggestions for the Application of Mathematical Modeling Activities	Recommendations for Studies Before Implementation	Help us Solve Events
		Let's do it together as a group
	Implementation Recommendations	We must choose the groups
		More Time Should Be Given
	Presentation Suggestions	We must not be interrupted
		Everyone Should Submit
		It will be difficult to use in other lessons

Suggestions for Associating with Other Courses	Can Be Used In Other Lessons

Students' Positive Opinions about Mathematical Modeling Activities

The positive opinions of the students about Mathematical Modeling Activities were grouped under four: "It contributes positively to mathematics learning", "It contributes positively to individual development", "group work affects teaching positively" and "willingness to take an active role". Students expressed different opinions under these themes, and the statements regarding these opinions of the students are given below, through direct quotations:

Makes a Positive Contribution to Mathematics Learning

The students expressed their views with expressions such as "increasing interest in the lesson", "associating mathematics with real life", "developing a positive attitude towards mathematics" under the theme of "It contributes positively to mathematics learning". The statements regarding these views of the students are given below with direct quotations.

Increasing Interest in the Course

> *Selin: I didn't like math very much before, I used to say it's so annoying, but after this activity started, my favorite subject was mathematics.*
>
> *İpek: Normal math classes were a bit boring for me. Lessons with mathematical modeling sounded more fun.*
>
> *Kerim: I did it with love. I was keeping a diary in the 2nd grade, I wrote there as my favorite subject is mathematics. I already loved mathematics, and I started to like it more because you made it fun by adding fun, and it was nice for me to have a contribution to my group… That's why I did it with pleasure.*
>
> *Serra: Modeling activities are fun, I didn't realize how the time passed because they were more fun. The questions in the other test books were boring, so I was bored while solving them. Normal test books had simple but boring questions, here there were difficult but fun questions. I mean, it fit my criteria, it would be nice if it was a little easier, but it was a lot of fun, it was fine.*

Some of the students stated that the modeling activities were fun and they enjoyed doing the activities with the expressions "my favorite lesson was mathematics", "the lessons with modeling are more fun" and "difficult but fun questions". The students, who stated that the test books were boring, stated that the time passed quickly and the activities were suitable for their tastes and criteria.

Relating Mathematics to Real Life

> *Gonca: It seemed to me that these questions could really come up, so it made sense. I liked it very much, for example, the stork question can be encountered both in the spring and it was very entertaining for me, at the same time I learned something. The running race challenged me a bit. We didn't know whether we would choose the winner or the one with less time. While doing it in the group, Ece says "Let's choose the first one" Cem says "Let's choose another one". There was a bit of discussion, but we finally found the right way.*

> *Duygu: I think the travel problem was difficult, but we had such a problem, we didn't know where to go, it was just like that. Also, the benevolent big-footed foot was very different. As everyone's feet were different, their height was also different. Personally, I loved the math class, but now I love it even more. Because the questions in the test may not be real sometimes, but your questions were real, and I understand better the situations that can be real.*

> *Cem: I loved the math class, but sometimes it was boring. While solving the questions, I could not fully understand what it meant, 3 more than 5 minus or something did not make sense to me. But now it lies in your logic and the solution proposals have a justification. It's more fun when you explain why we did it this way. If I did math for a day, I wouldn't get bored in these activities.*

It was observed that the students associated mathematics with real life during the implementation of the modeling activities, with the statements "it may appear in the spring months" and "now lies in your logic, there is a justification for the solution proposals". It was observed that the students were more willing to approach the activities carried out with examples from real life situations.

Developing Positive Attitudes Towards Mathematics

> *Mert: It has a positive effect on my math, and it's more fun than normal math. I wish we could continue this process, but we cannot. If we did, I would like to solve a few more questions.*

> *Serra: I used to not be good at math, I didn't like it very much, it seemed like it was very challenging. But after the modeling activities, I loved it and my blood warmed up.*

> *Burak: I did not like mathematics very much, I was also afraid that I would be wrong. Indeed, sometimes it was like that, I made a lot of mistakes in mathematics in the trials. But now I always start with math, I have some confidence in myself.*

> *Ece: I used to love mathematics, but after modeling activities, I started to like it more. Besides, the questions were good, I knew what it was, I could guess the result. After these activities, math started to seem simpler to me because I understood what I was doing.*

As can be understood from the statements above, some students stated that after the model building activities, "I liked mathematics after the modeling activities, my blood warmed up", "now I always start with mathematics, I have a little confidence in myself" and "mathematics has started to seem simpler to me". It appears that they have begun to receive

Contributes Positively to Individual Development

Students stated that modeling activities contributed to their individual development. Under the theme of positive contribution to individual development, his views were expressed as "Increasing success in mathematics course", "Development of affective skills" and "Ensuring permanent learning". The statements regarding these views of the students are given below with direct quotations.

Increasing Success in Mathematics Lesson

> *Ali: We had difficulties with these questions at first, but we can overcome these difficulties with group work. I think these activities are fun, you learn without realizing it, I think my math has improved. In particular, I gained practicality while doing transactions, and they gave us a lot.*

> *Selin: Test books also help me learn, modeling activities also help me learn. But as we solved these questions, we were preparing for the future. What we learned made more sense.*

> *Mert: This process helped me improve my math and taught me how to do math questions. It was fine for me to learn, and it helped us all improve in math. Yes, we could think of everything more detailed and faster in mathematics.*

Students expressed that their success in mathematics lessons increased with expressions such as "I gained practicality while doing the operations", "We were preparing for the future" and "This process helped me to improve my mathematics". In addition to this, it was observed during the research that the students developed positive attitudes towards mathematics and their achievements increased with their own expressions.

Belief That Problems Can Be Solved

> *Cem: I was excited at first, then it started to feel fun. Our relationships with our friends became stronger, and we learned mathematics without realizing it.*

> *Duygu: When I first read the questions, I was thinking how are they such different questions? It got easier as we solved it, we drew graphs, we drew tables, it was not at all like the questions we covered in the lessons, but I like it.*

> *Ece: The activities application was very nice and fun. It was difficult at first, but as we did, we got used to it. Sometimes there were problems within the group, but we were solving them by talking. Everyone can get offended even in the normal class, but here a way to agree is sought.*

Selin: In normal test books, there were simple but boring questions, here there were difficult but fun questions. I mean, it fit my criteria, it would be nice if it was a little easier, but it was a lot of fun, it was fine.

Above; It is understood from the expressions "our relations with our friends have become stronger", "we had problems but we were solving them by talking" and "everyone can get offended in the classroom, but a way of understanding is sought here", it It is understood that the students' affective skills towards mathematics improved during the modeling process. The students stated that this development was more evident as the process progressed, and that they had difficulties in the activities in the first weeks, but they adapted in the following weeks.

Ensuring Permanent Learning

Burak: These activities were very good for learning mathematics. We started to learn more meaningfully, sometimes we had difficulties, but this made us think and think… As we thought, everyone came to different ideas and solutions.

Ece: These questions are very different and fun, while solving other questions, the same things happen like a machine. I may not remember the questions we solved in the lessons, but your questions always stay in my mind, I always remember the procedures and stuff. So you're arguing here, different things happen. Sometimes what you think is right isn't right, but it's fun.

Gonca: It's not like the others, so the math is the same, you're actually doing the same, different models can be encountered in real life. But others find it a little difficult, it would be more understandable if they were like this. Our teacher Harun was telling us that we needed to solve the test immediately. There is no test here, we learn by understanding a little more, it is fun.

It is understood that the students stated that learning is more permanent from the statements above, "we started to learn more meaningfully", "but your questions always stay in my mind" and "there is no test here, we learn with a little more understanding". In addition, students stated that modeling activities were more memorable, unlike routine math problems.

Opinions on the Positive Effects of Group Work on Teaching

The students who participated in the study stated that the group work applied in the modeling activities contributed positively to the teaching of mathematics. Under the theme that group work affects teaching positively, his views were expressed as "development of social skills" and "the belief that problems can be solved with group work". The students' views on this theme are presented with the direct quotations given below.

Development of Social Skills

> *Serra: Modeling activities were good for my love of mathematics, group work was also good. It also made a difference to the school, it was our last year, I got closer with İpek and so on. It had such advantages and disadvantages.*

> *İpek: At first, Serra and Kerim had a little fight, but as they progressed, they started not to fight and got along better. I was a little more shy in the first weeks, I wasn't solution-oriented, to be honest, but as I progressed towards the end, I produced more ideas and my shyness went away a bit. The activities are good fun, besides, we generate ideas so that our shyness is relieved and we have fun, we do group work.*

> *Cem: We never played with them. We were always hanging out with boys and girls separately. I became better friends with them in this group. Then we did better work as a group than individually.*

The statements of some students, whose opinions were taken, "I got closer with İpek", "I got a little bit more shy" and "I became better friends with them in the group" reveal that the students' social skills improved and they established better relationships with their friends during the modeling process.

The Belief That Problems Can Be Solved With Group Work

> *Eren: My teacher, when you work individually, other people's ideas are not yours, but it is not like that. Because sometimes we need the opinions of others. for example, when it comes to the things we don't know as a group, I don't know, Selin, for example, but Mert can; For example, Mert can't know, Selin can't, I can. That's why group work is more fun and enjoyable.*

> *Serra: It was difficult at first, but as it progressed, it became fun and instructive. Also, group work is better. The longer the question, the more time it took to spend, and the things inside the question were the same. We saw a difference here when we always go from 5 marbles to 10 marbles. That was nice too. Mathematical modeling activities would not be nice if they were one by one. We did it one by one. I think it would not be very nice if we did everything that you probably wanted to measure, what you did for him, one by one.*

> *Selin: I realized that I have a little more difficulty individually, I was doing the questions better as a group. But I can also say that the questions we solved individually were a bit easier for me. It might be because we are not a group, the "Car" question, the "Sille Park" question was one click easier. Group work helped our self-confidence a little more, that is, it helped our entrepreneurship a little, and it also improved my relationship with my friends. There were sometimes discussions within the group, we had a little difficulty because of it, but everyone defended their ideas and we accepted the good thing.*

It can be understood that some students' belief that group work can solve the activities more efficiently is understood from the statements "group work is more fun and enjoyable", "modeling activities could not be better if they were done individually" and "I had a little more difficulty individually, I was doing better as a group".

Active Mission Request

During the study, the students wanted to take an active role in the preparatory work, modeling practices and group presentations. Students expressed their willingness to take on a task with expressions such as "expressing oneself better", "fulfilling responsibility" and "importance of helping each other". The students' views on this theme are presented with the direct quotations given below.

Expressing Yourself Better

> *Kerim: I am no longer looking for simple ways, but ways that will never happen, but ways that few people will think of.*
>
> *Eren: We used to be a little afraid to try to do it, but now we sometimes argue so we can do it. In the past, our discussion was "you do this, you do that", now our discussion is "I will do this, I will do that" and it is very entertaining.*
>
> *Burak: I think the questions were easier in the first weeks. For example, bazaar was easy, picnic was easy, but we did not understand. In the following weeks, we got used to the group, understood the questions, and made it easier. The olive question was very difficult, but the pontoon question was very difficult, but we expressed our thoughts well, nevertheless we did it with difficulty, everyone in the group was carrying out an opinion.*

As can be seen from the direct quotations above, it is understood that some students are looking for ways to express themselves better by saying "I am looking for ways that few people will think of" and "The pontoon question was very difficult, but we expressed what we had in mind well".

Fulfilling Responsibility

> *Duygu: I had difficulties with some questions, but my friends helped me. In the first weeks, we didn't know how to work with groups, everyone seemed like an individual again. In the following weeks, we became a group with the division of labor, and everyone fulfilled their responsibilities.*
>
> *Cem: For example, the groups found storks differently. Also, we didn't want to present it in the first weeks, everyone was saying that you could present it. But then we were encouraged and wanted to present, which was one of our duties. So you liked our presentation, and we have confidence in ourselves.*
>
> *Eren: At first, we didn't really understand. Selin, Mert, I thought the questions would be normal math questions. We said that it would be multiplication, division, addition, subtraction, but when it was different like this, we got a little excited and scared, but you don't need to be afraid, it has been very good until now when we have done our duties.*

Throughout the study, students were given three basic responsibilities. The first responsibility is to do research in accordance with the context before the activities, the second responsibility is active participation in group work, and the third responsibility is to present the solution to the class. It is understood that the students want to fulfill their responsibilities from the expressions "We became a group with the division of labor in the following weeks, everyone has their responsibility" and "then we were encouraged and wanted to present it".

The Importance of Solidarity

> *Ece: For example, there was a sorting, we were writing the same ones in the same color; There was the question of cities, and the transactions seemed easy, but it was important which operation we would do. The division of labor was necessary because time might not be enough.*
>
> *Duygu: We had difficulties in some situations, but eventually we overcame it by discussing the questions seemed a bit difficult, but later we got used to it. I had problems with the transaction, sometimes I made mistakes while trading; my friends taught me through group work. I don't have many problems anymore, but we need to keep these activities going.*
>
> *Gonca: The activities started to seem simple, so I came up with different ideas. I also had the courage to myself, so I got on better with my friends. The ones that were done alone were difficult, but the ones that were done with the group started to come easier. At the presentations, we did not want to present at first, but then we were very eager to present. We had difficulties, but we solved it with the help of our friends in the group. In the first question, I did not understand exactly what he wanted, which car should we rent, but maybe it would be better if I had friends.*

Undoubtedly, one of the important elements of group work is helping each other. It is seen that the students helped each other with the statements "The division of labor was necessary because time might not be enough" and "My friends taught me through group work". When the answers to all the questions given by the students are examined holistically, summary information about the positive opinions is presented in Table 5.

Table 5. *Frequency table of Students' Positive Opinions*

Opinions	Frequency(f)
Increasing interest in the course	23
Relating mathematics to real life	21
Developing positive attitudes towards mathematics	19
Ensuring permanent learning	14
Expressing yourself better	11
Development of social skills	10
Increasing Success in mathematics lesson	9
The belief that problems can be solved with group work	9
Belief that problems can be solved	5
Fulfilling responsibility	3

The importance of solidarity	1
Total	125

As can be understood from the Table 5. 125 positive opinions under 11 headings were given by the students about the mathematical modeling activities. Students' most common positive opinion on mathematical modeling activities is ‘Increasing Interest in the Course’. This positive opinion was expressed 23 times by the students. The second most frequently repeated positive opinion is ‘Relating Mathematics to Real Life’. It was used 21 times by the students. The least reported positive opinion about mathematical modeling activities is ‘The Importance of Solidarity’. This positive opinion was expressed one time by the students. The second least reported positive opinion is ‘Fulfilling Responsibility’. This positive opinion was expressed three times by the students.

Students' Negative Opinions about Mathematical Modeling Activities

Negative opinions of students about mathematical modeling activities were grouped under three themes: "Unusual question structure", "Negative situations that may affect classroom management" and "Problems posed by the education system". Under these themes, the students expressed different views and the statements of the students regarding these views are given below, through direct quotations:

Unusual Question Structure

All students participating in the study had not encountered mathematical modeling activities before, but students attending Science and Art Centers encountered this type of question structure. Students stated that modeling activities are different from natural learning environments; They were conveyed with expressions such as “the questions being too long”, “not being able to relate the solutions with real life”, “differentiation of the correct answers” and “not understanding the questions”. The students' views on this theme are presented with the direct quotations given below.

The Questions Are Too Long

> *Duygu: Modeling questions are very different, these questions are too long at first, you think you didn't understand towards the end, you read them again. These activities are definitely more fun, but they are a bit long, they need to be read again when we do not understand. It's a bit boring to be long, but when you learn by having fun, you don't understand how time passes.*

> *Cem: We are not used to working with groups, but group work is needed here. Because the questions are both very long and like other questions, how much does it cost, not how long, but questions that we need to interpret a little more. Teacher Harun sometimes makes us solve such long questions, but it wasn't that long.*
>
> *Ece: We thought it would be difficult when the question was too long, as if we were taking the university entrance exam. In fact, when we were talking with friends after you, we were saying to each other if there would be such a question.*
>
> *Selin: At first I couldn't start from somewhere, we didn't know where to start because the questions were too long. Our other friends were not talking until one of us said something and put forward an idea.*

Modeling activities are longer than multiple-choice applications that students solve in natural learning environments. Students expressed this difference with the statements “the activities are definitely more fun but a little longer”, “like other questions, how much does it cost, not how long” and “The question is too long as if we are taking the university exam ”.

Inability to Relate Solutions to Real Life

> *Eren: At first, we had a little difficulty because we didn't know much, we saw it for the first time anyway. We didn't do anything when we couldn't relate the logic to the 4 operations. So we did not know exactly how many storks would come. When I went to Bilsem, we did some of these transactions.*
>
> *Mert: First we read the question, after everyone read it, we took turns saying our thoughts about the question. After that, we were starting to solve the question, but it didn't seem to make sense in some solutions.*
>
> *Duygu: It can be boring when you don't understand. But in general, it was good, I mean, it was easy to understand the questions after understanding, but it was difficult to relate the questions to life.*

Students especially had difficulties in associating activities such as "Giant Foot" and "Uncle Farmer Hüseyin" with real life situations. They stated this difficulty with the statements “So we did not know exactly how many storks would come” and “But it was difficult to relate the questions to life”.

Differentiation of Correct Answers According to People

> *Cem: We need to think a little more about these questions, we need to comment. Because different answers can be correct. We were faced with such a question for the first time in the first weeks, while we were discussing it in the group, we were trying to reach the result by doing the operations with numbers immediately. But as we did the activities, we realized that there was no one correct answer, different answers could have been correct.*
>
> *Selin: Something like this was happening in the classroom, we could create several more questions out of one. But in these activities, we had to focus on one question, that is, differently, if we compare.*

İpek: One of the most challenging situations for us is I say let's do it like this, Kerim says let's do it like that. It was a little difficult for us to come to an agreement, but we were able to come to an agreement when we persuaded. Everyone's answer sounded right, but we knew we had to make one at the presentation.

The correct answers differ due to the nature of mathematical modeling activities. During the application, the students gave different answers to the activities. They explained these different answers with the expressions "there was not one correct answer, different answers could have been correct" and "everybody's answer was right for himself".

Not Understanding Questions

Ece: These activities were not like the other activities we solved in the lesson, they were a bit difficult, but they were fun to solve. We had to think a little bit, so we had to work our heads a little bit. We have to be very careful in the lessons where we solve these activities. There are fine details. Did we miss that point, you can solve the question, but it can be wrong. We need to be careful, we need to be careful in other lessons, but a little more attention is needed in these activities.

İpek: I was having a hard time understanding some problems. That's why, as a group work, I listened to what my friends said, sometimes I could not come up with ideas, sometimes I was producing, there were no other difficulties.

Selin: I told you, sometimes we were even angry with you. These are high school questions, how can we do it, for example, we were solving a question in an hour. We didn't understand it at first, this was the bad part, it took a long time and it was difficult.

Cem: We had a little difficulty at first. How are these questions, how do we do it, we were biased at first. But especially I was having this problem, I don't know where to start. When we first started, we couldn't start from anywhere, but as we progressed, we started to get used to the problems.

The long modeling activities made it difficult for students to understand the question. The students expressed this difficulty with the expressions "we need to be very careful, there are fine details", "I was having trouble understanding some problems" and "these are high school questions, how should we do it".

Adverse Situations Affecting Classroom Management

It was observed that the students participating in the study had problems with classroom management since group work was not carried out in natural learning environments, and these problems were expressed by the students in the interviews with the students. These problems experienced by the students are explained with direct quotations under the headings of "not being used to group work, having problems", "noise in the classroom" and "late answering the questions by the teacher".

Unaccustomed to Group Work, Having Problems

> *Ali: The first weeks were difficult, but once I got used to it, I saw that it was easy. If we teach like this, we will get used to it more, it will be easier. Also, we cannot work with groups, we are not used to group work. And the feeling was always saying something "let it be what I say". I was always used to working individually, we sometimes had problems in group work; Duygu Ece was talking to each other.*

> *Selin: They had a discussion on the pontoon question, and we lost some time there on whether to tie the rope or not, but we do not experience these individually. But still, I prefer group work. Something like this happened as a group, in the group work we did towards the last weeks, Selin and Mert had a fight, which we do not experience individually.*

> *Ece: When you do it in the group, everyone wants their opinion to be valid, but we were discussing, I think it's fun but a little difficult. There were also problems between our group in the first weeks, who did not know what to do. We never do group work in the classroom. In fact, it better be when we play outside, we only become a group, we solve tests in lessons, we do it individually too.*

The students participating in the study had difficulties in getting used to group work in the first weeks. However, they overcame this difficulty, albeit partially, in the following weeks. students; It can be understood from the statements "We sometimes had problems in group work", "Selin and Mert had a fight, we do not experience this individually" and "There were problems between our group in the first weeks".

Having Noise in the Classroom

> *Ece: We are asked questions in the class, we try to solve them, sometimes there are long questions, but we do not need to think much like yours. If we think about these questions, we can think of different solutions. But it is a little difficult to think in a noisy place.*

> *Cem: Like other lessons, we were reading right away, then we were trying to solve it, but we had difficulties. First we thought, then we read. We used different colored pencils to see what we could do. Even when we were doing this, we couldn't do it silently, there was a bit of confusion in the classroom.*

> *Eren: At first, we had difficulties in individual and group work as we did not know modeling, but later on, as we got used to it, those problems began to come very easily. Some of them immediately started talking among themselves without thinking of other ways and there was a lot of noise in the classroom.*

While the activities were being solved in group work, the discussions among the students caused noise. In the interviews, the students reflected these noises with the statements "But it is a little difficult to think in a noisy place", "We can't be quiet, there was some confusion in the classroom" and "He started talking among himself and there was a lot of noise in the classroom".

Late Answering of Asked Questions by the Teacher

> *Selin: At first, we were doing this, we were reading the question, we were underlining it, we were understanding the question. We were having a consultation among ourselves and we were starting to plan, usually we were either graphing or comparing. We were trying to ask you about the points we did not understand, but it was not our turn, we were waiting, time passed and then we forgot our question.*

> *Serra: In the first weeks, there was an adjustment problem, we were all saying something. Everyone was following what they said, not obeying what others said. There were disagreements, sometimes you and Aysun teacher were looking at other groups, it was our turn to be late. Kerim was puffing and puffing…*

During the application, there were cases where the students' questions were answered late due to the answers to the questions of other groups. Some of the students participating in the research were asked to answer the questions late: "We were trying to ask you about the points we did not understand, but it was not our turn" and "it was late. Kerim was puffing and puffing, so…" they stated.

Problems Caused by Traditional Educational Practices

Students, who have not encountered modeling activities before, have problems that may arise from our education system during the application. They stated that "these questions do not appear in the exams" and "the course duration is short".

Not Asking These Questions in Exams

> *Kerim: For example, there was unnecessary information, we couldn't capture that information, we couldn't figure out its logic. In our other mathematics, there was not much unnecessary information. We will enter the scholarship this year, I think they do not have such questions, I would do it, but in those exams there are questions like our other mathematics course.*

> *Selin: Because if I do a question in an hour in the exam, I will get zero for sure. If there is 1 question in the exam and 1 hour is given, then I can do it in an hour, I can do that.*

Some students explained that modeling activities are different from other types of questions by saying "we will enter scholarships this year, I don't think they will have such questions" and "if I do a question in one hour in the exam, I will get zero for sure".

Short Lesson Duration

> *Ali: If we had a little more time, we could have found different answers. When we got used to it, we started to use time well in the last few weeks, and we did a division of labor.*

> *Burak: My friends were very good. We were listening to each other, we weren't listening at first, but later on, we listened to each other in the events that*

followed. There was a lot of time, but it seemed short to us, we learned to use it effectively towards the end.

Cem: We are not used to working with groups, but group work is needed here. Because the questions are both very long and, like other questions, not how much it costs, but how long it is, but the questions that we need to interpret a little more. Your questions should be at least 1 hour, but our lessons would be better if 40 minutes.

Although the students were given 50-60 minutes on average for the solution of the activities, some students said that the time should be longer, "If we had a little more time, we could have found different answers", "the time was too much, but it seemed short to us" and "the lessons would be better if 40 minutes." will happen”. When the answers to all the questions given by the students are examined holistically, summary information about the negative opinions is presented in Table 6.

Table 6. *Frequency table of Students' Negative Opinions*

Opinions	Frequency(f)
The questions are too long	18
Unaccustomed to group work	14
Not asking these questions in exams	12
Inability to relate solutions to real life	11
Short lesson duration	7
Having noise in the classroom	4
Differentiation of correct answers according to people	4
Not understanding questions	3
Late answering of asked questions by the teacher	2
Total	75

According to the Table 6. 75 negative opinions under nine headings were given by the students about the mathematical modeling activities. Students' most common negative opinion on mathematical modeling activities is ‘The Questions Are Too Long’. This negative opinion was expressed eighteen times by the students. The second most frequently repeated negative opinion is ‘Unaccustomed to Group Work’. This negative opinion was expressed fourteen times by the students. The least reported negative opinion about mathematical modeling activities is ‘Late Answering of Asked Questions by the Teacher’. This negative opinion was expressed two times by the students. The second least reported negative opinion is ‘Not Understanding Questions’. This negative opinion was expressed three times by the students

Students' Suggestions for the Application of Mathematical Modeling Activities

While the students were evaluating the modeling process, they were grouped under three themes: "recommendations for the work done before the application", "recommendations during the application" and "recommendations for the presentation" . Under these themes, the students expressed different views and the statements of the students regarding these views are given below, through direct quotations:

Recommendations for Studies Before Implementation

Before the activities, students were asked to do research in accordance with the context of the activity to be held that week as a preparatory work. The opinions of the students regarding these studies were determined as "let's do it together as a group" and "help us solve the activities".

Help us Solve Events

> Eren: We did the kermis problem first, we had a little difficulty with it since it was the first. At first, I was very excited about it. Our previous work was good, it was guiding us and helping us.

> *Selin: At first we were reading, it seemed like it was difficult for us, then as we read the problem one by one, we knew that we had to start from somewhere, we solved the problem in this way. The homework you gave us was very useful for us, sometimes we searched the same site, it was revealed, for example, it was the case with storks.*

> *Ali: We were reading first to solve it, but generally we had difficulties, we were looking for a clue, what should we do to solve this. The most important clue was the work you gave earlier. I did not know exactly where olives grow, migratory birds come according to light pollution, I learned these, it helped me to solve the questions.*

Some students' statements such as "it was guiding us, it was helping us", "the homework you gave us was very useful" and "the most important clue was the work you gave before" reveals that the work done by the students before the activities helped them solve the activities.

Let's Do It Together as a Group

> *Selin: At first, our group friends ask, we all read, then one of us, this is usually me, reads the question aloud, and then we highlight important information while reading aloud. Everyone who had researched Selin from another place and Mert from another had different knowledge. I could hardly explain the golden ratio to my friends. For example, Mert has seen a tractor before, but we don't know exactly how big it is, we can do it better if we know.*

> *İpek: You were asking questions for a solution, and we were answering them. We were also underlining and stating important points, and we were following the*

> *path according to the important information given. We came up with an idea to measure our waist widths in the big foot problem. But everyone was saying something different, it would be better if we looked from the same site.*
>
> *Burak: I say that baklava is not cheap because my mother buys it for a tiny 50 lira. But others say it will be cheap, in fact, if they knew, they could do it right. Also, if everyone's knowledge is different, if we are of the same mind, the solution may be easier.*

Some students stated that the preparatory work done before the activities should be done together by the group members with the expressions "Everyone who has researched from another place, Mert has different knowledge", "It would be better if we looked from the same site" and "The solution would be easier if we were of the same mind".

Implementation Recommendations

Students' suggestions for practice are explained with direct quotations under the headings "We should choose the groups" and "More time should be given".

We Must Choose The Groups

> *İpek: There were disagreements in the groups, but as we progressed, we were able to get along better and generate more ideas. It would have been better if we had the groups, especially if we were chosen able to solve better questions, depending on group work. Sometimes there were disagreements in our group.*
>
> *Serra: I know you created the groups, but there were disagreements in some groups other than ours. Even Senalar had problems while solving, they did not want to present the running race that week. In fact, it would be less of a problem if we created the groups beforehand. Everyone can get along better.*
>
> *Eren: In the first week, I had even more difficulties in the individual work since there was no group, then in the last weeks individual work started to come easy, the easiest and most beautiful was the group work in the last weeks. Working as a group was good, but when you can't get along, he says get what I say and the other says get what I say. But we find the right way, this time too, time is passing, I think time is very important in these events.*

During the study, it was observed that the students worked in harmony. However, this situation, in which some students could not agree within the group, was reflected in the students' opinions with the statements "it would be better if we chose the groups" and "actually, it would be less of a problem if we formed the groups beforehand".

More Time Should Be Given

> *Eren: I think it is a very nice and enjoyable event, even though it was a bit difficult at first. But time was not enough, and we were getting excited and wasting time because we didn't know what to do. Time seems like a lot compared to other questions, but when these questions are different, time sometimes does not reach.*

> *Gonca: First, we were reading the question silently, then everyone was saying their own opinion. But there was usually an argument here. It was a waste of time.*
>
> *Cem: When we didn't understand what we were reading, we would ask our friend, he was telling his opinion, and if what he said was logical, what he said was true. But Duygu was not convinced and we were always arguing. Our procedures were not difficult, but it took time for him to understand the question. The most difficult part was understanding what was asked in the question.*

It is understood that some students, who could not use time well in the first weeks of the study, stated that "we were excited because we did not know what to do, we were wasting time" and "our procedures were not difficult, but it took time to understand the question".

Presentation Suggestions

The students presented the solutions they reached during the activities to their friends. The suggestions of the students for the presentation stage were stated under the headings "We should not be interrupted" and "Everyone should present".

We must not Be Interrupted

> *Serra: We were mostly trying to find the result by sharing each other's ideas and then adding, dividing and multiplying. During the solution, everyone was arguing, but in the end, especially some of our friends were interrupting us. In fact, if he had listened to us, he would have understood what we were saying, but he was hastily interrupting us.*
>
> *Ece: While our group was presenting, we couldn't have a good division of labor. Actually, our presenter was clear, but friends or something intervened, and it wasn't appropriate. But we did it anyway, you said so you wouldn't interrupt your friends, but there were some who didn't comply.*
>
> *Cem: Sometimes there were situations that we did not understand, but we were able to deal with it by talking to our friends. At first, we didn't want to make a presentation, we were hesitant. Just as we are explaining the question, Emre was intervening, it was getting on our nerves, but in the following weeks, everyone was respectful to each other.*

The stage of presenting the solution that the students have reached is an important dimension of the study. However, presentation activities of students in natural learning environments are not used. The students emphasized that their words should not be interrupted during the presentation with the expressions "especially some of our friends were interrupting us", "You said so as not to interrupt your friends, but there were some who did not comply" and "just we are explaining the question, Emre (a student in the class) was interfering".

Everyone Should Submit

> *Eren: our p 's hardly happened in the first week but now we understand problems in recent weeks, we have made many beautiful prepare the presentation of each problem is also very nice. We were wondering how others solved it. It would be nice if everyone presented it, but there might not be enough time for it.*
>
> *Serra: Some of our friends' presentations were the same as ours, while others were very different. The problems were fun, we were arguing amongst ourselves after you left, about who is more logical, etc. But I think it would be nice if there was another lesson for the presentation.*
>
> *Cem: To be honest, we didn't hesitate when playing during regular recess, but we were hesitant in presentations. Afterwards, we did not hesitate, everyone wanted to present. I was wondering how many storks they found, how many trees they planted. It would be nice if they also made a presentation.*

The students stated that the solution reached should be presented by all students with the expressions “We were wondering how others came up with the solution”, “We were arguing about who made more sense” and “It would be better if they also made a presentation”.

Suggestions for Associating with Other Courses

When the students were asked whether mathematical modeling applications could be used in other courses, some students stated that they could be used, while others stated that they could not be used.

It Will Be Difficult to Use In Other Lessons

> *Serra: Maybe in other math classes, but maybe in social or something like this, there will be no group work, maybe there will be no projects. I don't think it's possible in math either, because we're solving tests, it should be one in general.*
>
> *Duygu: Maybe it's group work, but it might not be such a presentation. It's not boring, sometimes I stay between the choices in other math lessons, but the answers should be clear here.*

Students who stated that mathematical modeling applications could not be used in other lessons expressed their opinions with the statements "Maybe there will be no group work in social or something like this" and "There may be group work, but there may not be such a presentation".

Can Be Used In Other Lessons

> *Serra: Now what I'm about to say may not be in math, it may be in fun. Maybe we could do something now, like making a poster and painting it as a decoration. I think modeling activities were fun, whether they were both a little bit, as in the lesson and modeling activity, but it can also help us to understand it differently, we may need to understand it differently. We may also need the way that Aysun teacher told us.*
>
> *Selin: Let the normal classes be like that, but not the exams. In fact, I would like to continue this kind of work in secondary school, it is very nice.*

Kerim: So these are fun, you make them fun, you other math teachers don't make it that fun. It can also be in science, which is a numerical course, we can compare the subjects, and we can study this subject because it is more difficult or that subject is more difficult.

Gonca: Sometimes it was hard, but mostly it was fun. It's like we're bringing together other lessons, that is, we understand Turkish, we do mathematics. These were fun but difficult when I couldn't solve them. We needed the ideas of our friends to solve them.

Some of the students who participated in the study stated that mathematical modeling applications can be used in other lessons as well, "as in the lesson and modeling activity," and "Let the normal lessons be like that, but not the exams". When the answers to all the questions given by the students are examined holistically, summary information about the suggestions for the application of mathematical modeling activities is presented in Table 7.

Table 7 *Frequency Table of Students' Suggestions for the Application of Mathematical Modeling Activities*

Opinions	Frequency(f)
More time should be given	21
Help us solve events	17
We must choose the groups	9
Let's do it together as a group	7
We must not be interrupted	6
Everyone should submit	4
It will be difficult to use in other lessons	4
Can be used ın other lessons	2
Total	70

As can be understood from the Table 7. 70 opinions under eight headings were given by the students about the application of mathematical modeling activities. The most common opinion of applying mathematical modeling activities is ‘More Time Should Be Given’. This opinion was expressed 21 times by the students. The second most common opinion is ‘Help us Solve Events’. This opinion was expressed seventeen times by the students. The third most common opinion is ‘We must choose the groups’. This opinion was expressed nine times by the students.

Conclusion and Discussion

The aim of this study is to reveal how elementary school 4th grade students perceive these activities as a result of mathematical modeling activities and their opinions and evaluations about the application process in the learning environment. The findings show that; Primary school 4th grade students expressed their "positive" and "negative" views on mathematical modeling activities, as well as suggestions for future modeling activities.

Students expressing positive opinions; They stated that mathematical modeling activities affect mathematics learning positively, contribute to their individual development, and group work affects teaching positively. This result of the study can be found in the literature (Tekin-Dede and Bukova-Guzel, 2013; Güder, 2013; Deniz and Akgün, 2014; Tutak and Güder, 2014; Işık and Mercan, 2015; Bilen and Çiltaş 2015; Işık and Mercan 2015; Urhan and Dost, 2016; Pilten, Serin and Işık, 2016; Deniz and Akgün, 2017; Şahin and Eraslan, 2019). In addition, one of the results reached in this study, which is not mentioned in other studies in the literature on the subject, is that the students want to "take an active role" during the applications. Primary school students' willingness to take part in modeling activities can be explained by the fact that they do these activities fondly and willingly. As it can be understood from these views, the students did the modeling activities with love and fun. Students' interest in the lesson has increased and their relationships with their friends have improved. In addition, in the presentations made after the activities, the students had the chance to express themselves better.

Some students, who expressed negative opinions about the application process of mathematical modeling activities, stated that they were not accustomed to the question structure applied in mathematical modeling activities. It was concluded that the students approached the questions with an unusually long structure with prejudice and had difficulty in understanding the long question structure. In addition, the inability to associate the solutions reached with real life and the fact that the correct answers differ according to the group members are among the negative opinions of the students towards the question structure. Another negative view of students towards practice is the problems experienced in classroom management. The fact that the students were not accustomed to group work caused them to have difficulties in practice. Şahin and Eraslan (2019) also stated that the students had difficulty in working in groups in the study in which they took the opinions of the pre-service teachers. The presence of noise in the classroom as a result of the creation of a classroom arrangement different from the natural learning environments during the applications, the late answering of the questions directed to the teacher by the group are among the negative opinions of the students towards the implementation of the modeling activities. The students gave negative opinions about the modeling practices because the lesson time in natural learning environments was limited to 40 minutes and questions were asked in a different structure from the modeling activity in the exams. In the literature, there are studies stating that modeling activities are not used in lessons because it takes a lot of time. (Akgün et al., 2013; Ören Vural et al., 2013; Tutak & Güder, 2014; Pilten, Serin & Işık, 2016; Şahin & Eraslan, 2019).

One of the results of this study is the students' views on the implementation of modeling activities. The students stated that it helped them to solve the activities related to the work done before the application. When the relevant literature is examined, Tekin-Dede and Bukova-Guzel (2013) also concluded that the studies given before the activity affected the modeling process positively. According to the results of this study, it was concluded that it would be more efficient for students to do the work given before the activities together. The students stated that it would be more beneficial for them to choose the groups for the implementation process of the modeling activities and that more time should be given to the implementation period. Similar to this result of the research, it was emphasized in the studies of Güder (2013), Karalı (2013) and Tutak and Güder (2014) that the students were not accustomed to modeling activities and that the time given was insufficient. In the opinions about the student presentations, the students who stated that all the solutions reached by the groups should be presented, stated that their words should not be interrupted during the presentation. It is thought that starting student interactions in the research process before group work will contribute positively to the activities. In terms of associating mathematical modeling activities with other lessons, some of the students stated that mathematical modeling activity could be used in other lessons, while some students stated that it could not be used especially in verbal lessons. From the students' opinions, it was concluded that besides the application of preparatory studies and group activities in other lessons, it would be beneficial to use modeling activities in mathematics lessons.

Suggestions

The results of this study are limited to the opinions of 12 students selected among 64 students attending 4 classes in a state primary school. Considering the possibility of different results, the number of students whose opinions are taken can be increased. Studies based on mathematical modeling activities, which have been done frequently at undergraduate, high school and secondary school levels in recent years, can be applied more intensively to primary school students in order to prepare students for these levels.

It is thought that it would be beneficial for teachers to have experienced these practices in order to effectively implement mathematical modeling activities in the classroom. In this context, seminars and workshops on mathematical modeling should be given to teachers. In these activities, modeling examples that primary school teachers can apply in their classrooms should be created. It is thought that national and international projects to be carried out on the subject will be similarly beneficial for the dissemination of modeling activities in primary schools.

In addition to the modeling activities to be created by the teachers, materials and resources should be provided for the creation of modeling activities suitable for the subjects included in the primary school mathematics curriculum. With this understanding, well-arranged modeling activity examples should be included in mathematics textbooks and teacher guidebooks.

Undoubtedly, it is beneficial to carry out modeling activities with a seating arrangement that is different from the usual classroom seating arrangement. It is recommended to carry out modeling activities in classrooms where group work can be done, a seating arrangement can be provided and there are computers, projections and interactive boards.

The modeling activities to be created should be selected from areas where data can be collected about the problem situation, in accordance with the real-life situation, where they can determine the relations between the variables and the accuracy of the solution reached can be checked. In addition, it is recommended that the activities be created from the contexts of the students' living spaces. The fact that the studies done by the students before the activities and suitable for the context of the modeling activity are carried out separately by each student leads to the prolongation of the discussions during the activities and the inefficient use of time. It is recommended that students do the work given before the activity together in the designed learning environments.

Title in Turkish

Özet:

Matematik öğretiminin temel amaçlarından biri de öğrencilerin gerçek yaşam problemlerini çözmeleri ve gerçek yaşam durumu ile matematiği ilişkilendirmelidir. Matematik öğretimini kolaylaştırmanın bir yolu olarak derslerde gerçek yaşam problemlerinin matematiksel modelleme etkinlikleri ile uygulanmalıdır. Bu çalışmanın amacı 9 hafta süresince matematiksel modelleme etkinliklerini deneyimleyen ilkokul 4. sınıf öğrencilerinin modelleme etkinliklerine yönelik görüşlerini ve ileride yapılacak modelleme etkinliklerine yönelik önerilerini belirlenmektir. Çalışmanın katılımcıları 2019/2020 eğitim öğretim yılında Konya'da bir devlet okulunun 4. sınıfına devam eden 69 öğrenci arasından amaçlı örnekleme yöntemi ile belirlenen 12 öğrencidir. Çalışmanın verileri öğrencilerle gerçekleştirilen yarı yapılandırılmış görüşmelerden elde edilmiş olup içerik analizi ile çözümlenmiş ve yorumlamıştır. Çalışmanın sonucunda öğrenciler; matematiksel modelleme etkinliklerinin derse olan ilgilerini arttırma, matematik derslerinde başarılarını arttırma ve sosyal becerilerinin gelişmesi gibi olumlu görüşlerin yanında soruların uzun olması, grup çalışmalarında yaşanan sorunlar ve sürenin yetmemesi gibi olumsuz görüşler de belirtmiştir.

Anahtar kelimeler: Matematiksel Modelleme, Model Oluşturma Etkinlikleri, İlkokul, Öğrenci görüşleri, Öğretim deneyi,

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About the Author

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Yazar Pamukkale Üniversitesi Sınıf öğretmenliği(2003) bölümünü tamamladıktan sonra Selçuk Üniversitesi eğitim yönetimi ve teftişi alanında yüksek lisans derecesini (2010) almıştır. Doktora eğitimini ise sınıf eğitimi alanında (2021) tamamlamıştır. Araştırma alanları arasında matematik eğitimi, matematiksel modelleme etkinlikleri ile matematik öğretimi, rutin olmayan problem çözme stratejileri, matematiğin oyunlaştırılması ve teknolojik araçların matematik öğretiminde kullanımı bulunmaktadır. Yazar 19 yıllık öğretmenlik deneyimi boyunca farklı öğretim yöntem ve stratejileri kullanarak özellikle Matematik dersini öğrencilerine aktarmaya çalışmıştır. Sınıf öğretmenlerine ilkokullarda Matematiksel Modelleme uygulamaları ile ilgili hizmet içi eğitimin yanında, 225021 ID Numarası "Math Fundamentals" e-twinning projesi kapsamında "Uygulamalı Matematiksel Modelleme" semineri vermiştir. Yazar matematik bağlamı olan farklı sempozyum ve kongrelerde bildiriler sunmuştur.

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Appendices

Appendix 1.

HAVA DURUMU

Yurt dışına turlar düzenleyen bir seyahat şirketi tatile çıkacak müşterilerine gidecekleri yerin seçiminde danışmanlık yapmaktadır. Seyahate çıkacak olan müşteriler ilk olarak o yerin iklimi ile ilgilenip; ne kadar yağmur yağdığına, bir yılda havanın kaç gün güneşli yada kapalı olduğuna ve ne kadar sıcak yada soğuk olduğuna önem vermektedirler. Bu faktörlerin her biri seyahate çıkacaklar için farklı öneme sahiptir.

İki müşteri şirkete mail yollayarak tatil için istedikleri şehrin özelliklerini belirtmişler ve en uygun şehirleri tavsiye etmelerini istemişlerdir. Seyahat şirketi müşterilerine gitmeleri için dokuz şehir belirlemiş ve bu şehirlerle ilgili iklimiyle ilgili bazı bilgiler toplamıştır. Bu bilgiler ve müşterilerin mailleri aşağıda verilmiştir.

Sayın Seyahat Şirketi Yetkilisi; *Eşim ve ben geçen ay emekli olduk. Sıcak ve güneşli bir şehirde tatil yapmayı planlıyoruz. Çok soğuk bir şehir olmasın ama yağmurun yağmasını önemsemiyoruz. Eşim ve benim için uygun şehirler hangileridir önerilerinizi bekliyoruz.* *Meral Fatih BULUT…*	*Sayın Seyahat Şirketi Yetkilisi;* *Bir markette yönetici olarak çalışmaktayım. Bu yaz tatil için gideceğim yerde açık havada yapılabilecek her türlü sporu özellikle doğa yürüyüşünü denemek istiyorum. Çok sıcak olmayan ve aynı zamanda havası iyi olan bir şehirde tatil yapmak istiyorum. Hangi şehirleri önerirsiniz? Selamlar…* *Ahmet DEMİR…*

Sizin Göreviniz:

1- Tatile çıkacak kişilerin isteklerine göre dokuz şehri karşılaştırmak için model (derecelendirme sistemi) geliştiriniz. Geliştireceğiniz bu model sadece bu dokuz şehir için değil başka şehirleri karşılaştırmak için de kullanılmalıdır.
2- Her iki tatilci için şirkete tavsiye mektubu yazınız. Bu mektupta önerdiğiniz şehirleri "en uygun şehirler", "uygun şehirler" ve "uygun olmayan şehirler" olarak ayırmalısınız. Bu sayede tatilciler hangi şehirleri dikkate alması gerektiğini ve hangilerini dikkate almamaları gerektiğini bileceklerdir.
3- Mektuplarınızda puanlama sisteminizin nasıl çalıştığını ve neden oluşturduğunuz sistemin iyi sistem olduğunu seyahat şirketine açıklamalısınız.

ŞEHİRLER	GÜNEŞLİ GÜN SAYISI	15˚C'NİN ALTINDAKİ GÜN SAYISI	30˚C'NİN ÜSTÜNDEKİ GÜN SAYISI	YILLIK ORTALAMA YAĞIŞ(MM/YIL)

LONDRA	85	12	15	1220
MADRİD	195	40	169	274
BERLİN	36	184	6	516
ATİNA	71	0	185	2222
PRAG	45	55	30	661
MİLANO	85	0	328	1534
ROMA	178	4	237	386
VİYANA	84	157	36	633
PARİS	114	10	58	863

Appendix 2.

KİM KOŞSUN

Anadolu Atletizm Kulübü bu yıl düzenlenecek olan 4000 m. koşu yarışmasına sporcu seçecektir. Bunu için öğrencilerin performanslarını belirlemeye yönelik bir çalışma yapılmıştır. Takımın antrenörü Sercan Hoca 4 gün süresince biri öğleden önce diğeri öğleden sonra olmak üzere 8 kez öğrencilerin 4000 metreyi kaç dakikada koştuklarını ölçmüştür. Sercan Hoca ve takımın diğer antrenörü Atakan Hoca 4000 metre yarışına hangi sporcunun katılması gerektiği konusunda anlaşamamışlardır.

Aşağıda sporcuların 4000 metreyi kaç dakika ve saniyede koştuklarını belirten tablo bulunmaktadır.

Sizce hangi sporcu yarışmaya katılmalıdır? Kulübümüz için en uygun sporcuyu nasıl seçtiğinizi Atakan ve Sercan Hoca'ya bir mektup yazarak açıklayınız.

	BADE	TALYA	DAMLA	AYBÜKE
1.TUR	34' 25 "	36' 50 "	37' 40"	34' 40"
2.TUR	37' 45 "	34' 40 "	36' 40"	38' 50"
3.TUR	36' 20 "	36' 10 "	37' 50"	34' 40"
4.TUR	35' 55 "	35' 15 "	34' 50"	39' 40"
5.TUR	36' 40 "	35' 10 "	34' 50"	34' 20"
6.TUR	35' 55 "	36' 30"	36' 30"	35' 50"
7.TUR	37' 55 "	34' 50"	37' 50"	38' 20"
8.TUR	38' 50 "	36' 40"	35' 40"	34' 40"

Appendix 3.

ÇİFTÇİ HÜSEYİN AMCA

Çiftçi Hüseyin Amca'nın zeytin fidanı dikmeyi düşündüğü bir tarlası vardır. Eni boyunun iki katı uzunluğunda olan bu tarlanın alanı 5000 metrekaredir. Hüseyin Amca tarlasına en fazla sayıda zeytin dikmek istemektedir. Ancak zeytin bitkisinin gelişimi ve verimi göz önünde bulundurulduğunda zeytinleri belli aralarla dikmesi gerekmektedir.

İlçe Tarım Müdürlüğüne giden Hüseyin Amca şu bilgileri almıştır.

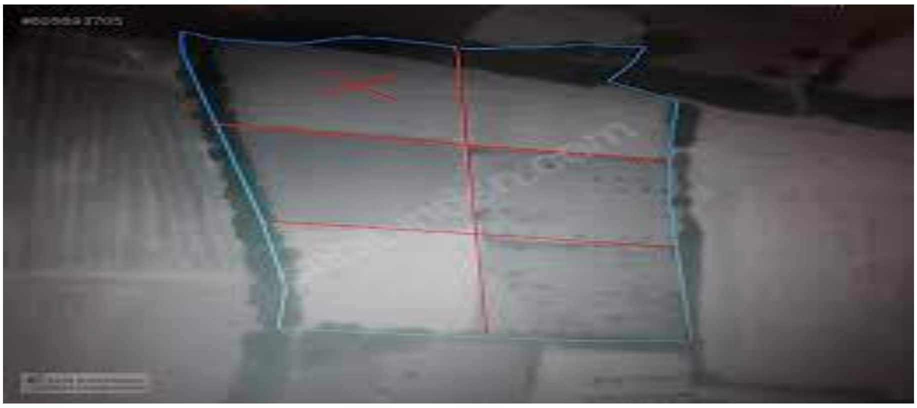
- *Yüksek verim elde etmek için zeytin fidanları birbirlerine mesafeli dikilmelidir.*

-*Yetişkin bir zeytinin kökleri kazık şeklinde değil saçak şeklinde ilerlemektedir. Bu kökler 2-5 metreye kadar ilerlemektedir.*

-*Ağaçların taç (yaprağın gövde uzunlukları) uzunlukları saçak kök kadar olmaktadır.*

-*Verimli ağaçlarımız olsun istiyorsak; tarlamız yılda iki kez traktörle sürülmelidir.*

Bu bilgiler doğrultusunda Hüseyin Amca'ya tarlasına kaç zeytin fidanı dikmesi gerektiği ve bu fidanların kaç metre arayla dikilmesinin uygun olacağı konusunda tarlanın kuş bakışı görünümünü çizerek yardımcı olunuz. Yapacağınız yardımı Hüseyin Amca'ya bir mektup yazarak açıklayınız.



Appendix 4. Başarı Değerlendirme Formu

1- Aşağıdaki sayıların rakamla yazılışlarını ve okunuşlarını yazınız

a- Kırk sekiz bin yüz dokuz:..................

b- 8008:..

2-"1078" sayısını çözümleyiniz, en yakın onluğa ve yüzlüğe yuvarlayınız

Çözümleme:...

En yakın onluk:.....................

En yakın yüzlük:....................

3- "780201" sayısının onlar basamağı ile on binler basamağı yer değiştirdiğinde yeni sayı kaç olur?

Yeni sayı: ....

4-Aşağıdaki işlemleri yapınız. Verilmeyenleri bulunuz.

25467	59634	9A3B
+74895	+ ☐	+C8D6
	76013	14302

5-Ardışık beş doğal sayıdan ortanca olan 47 olduğuna göre bu sayıların toplamı kaçtır?

6- 242 eksiği 505 olan sayının 57 fazlası kaçtır?

7-Bade ile kardeşi Cemre arasında 5 yaş fark vardır. Bade 2009 doğumlu olduğuna göre cumhuriyetimizin 100. yılında ikisinin yaşları toplamı kaç olur?

8- 2965 ceviz, 5784 ceviz, sepet ve toplamak ifadelerini kullanarak bir problem yazınız.

Problemim:……

9- Aşağıdaki işlemleri yapınız.

960284	65420	7A43B
-782317	- ☐	-39CD8
	8347	

10-"78638 – 54947> ...." ifadesinde noktalı yere yazılabilecek en büyük doğal sayı kaçtır?

11-Tüm rakamları aynı olan dört basamaklı en büyük çift sayıya hangi sayıyı eklersek toplam 15723 sayısı olur?

12-Aşağıdaki işlemleri yapınız?

54	769	825× ☐ =13925
×38	× 37	47×65×29=65×29× ☐

Ek 4. (Devam) Başarı Değerlendirme Formu

13- "24,35,5 arkadaş, kilo ve şeker" ifadelerini kullanarak çarpma işlemi gerektiren bir problem oluşturunuz.

Problemim:……..?

14- Fazıl SAY'ın Konya'da vereceği konsere, 37 koltuk bulunan 24 sıra ve 48 koltuk bulunan 127 sıra bulunmaktadır. 6400 bilet satıldığına göre konseri kaç kişi ayakta izleyecektir?

15-Aşağıdaki bölme işlemlerini yapınız. Verilmeyenleri bulunuz.

84 | 4 378 | 16

6A6÷38=17 4128÷ ☐ =96

16- Aşağıdaki işlemlerdeki bölümün kaç basamaklı olduğunu işlem yapmadan tahmin ediniz.

75 | 6 87 | 9 576 | 6

...basamaklı … basamaklı …basamaklı

17- Ali, 597 sayfa olan kitabı dokuz günde okumaktadır. İlk gün 64 sayfa, ikinci gün 78 sayfa okumuştur. Kalan günlerde eşit sayıda sayfa okudu. Ali kalan günlerde kaçar sayfa kitap okumuştur?

18-"450, 18, öğrenci ve otobüs" ifadelerini kullanarak bölme işlemi gerektiren bir problem kurunuz.

Problemim:…………………………………………………………………………………… …………………………………..?

13 ve 18. Sorular 10 puan diğer sorular 5puandır… BAŞARILAR ☺ ☺

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Araştırma Makalesi / Research Article

Trends in Postgraduate Thesis Studies on Pedagogical Content Knowledge in Mathematics Education in Turkey: A Systematic Review

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Abstract – The aim of this study is to systematically review and assess the subliminal theses in Turkey which were subject to pedagogical know-how from the mathematical studies published in the national thesis center between 2016 and 2021. To examine prostheses according to certain criteria, qualitative research methods were based on a systematic examination method. In order to obtain the data of the study, 25 postgraduate theses about pedagogic content knowledge in math education were included in the analysis based on the PRISMA information flow, at the National Thesis Data Center of the Presidency of the Council of Higher Education. The theses chosen in line with the aim of the study were examined in the framework of the study evaluation form. As a result, it has been found that the knowledge of program, content knowledge, knowledge of understanding student, and knowledge of instructional strategies - subcomponents of pedagogical content knowledge as revealed by theorists who reviewed pedagogic content knowledge - is not scrutinized in detail by the graduate thesis studies.

Key words: pedagogical content knowledge, mathematics education, postgraduate thesis, systematic review.

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Introduction

The quality of teaching requires that the teacher have knowledge of the students and their culture, their political and social environment (Ball and McDiarmid, 1990). In order for the teacher to teach the students the field in a useful and effective way, it is believed that the teacher must have a deep knowledge about the subject first. Shulman (1986) suggests that teachers should be able to transmit their knowledge in a way that makes it easier for students to understand. Since the mid-1980s, studies on teacher training have emphasized the importance of including studies on teachers' knowledge, beliefs and competencies rather than on their behavior (Işıksal Bostan and Osmanoğlu, 2016). Shulman (1986) described the effects of subject knowledge on teaching for teachers as a missing paradigm in education research. He created the pedagogical content knowledge concept (PAB) for the first time by mentioning the importance of how teachers transform their content knowledge to make it easier for students to understand and revolutionized the work in teacher training.

Shulman (1987) defined PAB as a special combination of content knowledge and pedagogy knowledge, and conceptualized the PAB that connects content and pedagogic knowledge in two categories. Grossman (1990) added the PAB as follows: has examined four components: knowledge of understanding student, knowledge of teaching strategies, knowledge of teaching objectives and curriculum knowledge. Marks (1990) PAB he has defined knowledge on the subject he points to interact with each other as students' understanding of the subject knowledge on teaching the media and knowledge of teaching processes.

Numerous studies were carried out after 1990 to conceptualize the PAB (Ball, Thames and Phelps, 2008; Cochran, DeRuiter and King, 1993; Gess-Newsome, 1999; Hill, Ball and Schilling, 2008; Hill, Rowan and Ball, 2005; Magnusson, Krajcik and Borko, 1999; Park and Oliver, 2008). These theorists considered the definition of Shulman (1986) as Grossman (1990) and Marks (1990) and tried to conceptualize the PAB through various components or approaches.

Gess-Newsome (1999) conceptualized the PAB differently and based the information structure that teachers should have on two main structures as an integrating model and transformative model. The integrative model covers the intersection of pedagogical and contextual information gathered by the teacher during teaching, while the transformative model reveals a new information that encompasses the synthesis of these three information.

Blömeke et al. (2015) consider that information a teacher should have: that the background knowledge, pedagogy knowledge, PAB, and described these types of knowledge as the cognitive competences that a teacher has. Baki (2018) has identified five key components for PAB modeling for math education. In this model, while using the expression Teaching Math Knowledge for teaching, Blömeke et al. (2015) defined a Teacher Adequacy Model.

The work on PAB conceptualizing mathematical education was done by Hill, Ball et al. (Ball, Thames, and Phelps, 2008; Hill, Ball and Schilling, 2008; Hill, Rowan and Ball, 2005). Ball et al. (2008) have developed a new comprehensive model based on Shulman's PAB model based on the results of experimental studies in mathematical education. They identified two basic components for the PAB model. In this model, which addresses the subject content knowledge and PAB concepts in a holistic way, we have defined the teaching knowledge of mathematics for teaching. This model, which is explained as a mathematical knowledge model for teaching, is described under the headings content knowledge and PAB to teach mathematics. Rowland et al. (2009) demonstrated a quadruple information model that assessed teachers' mathematical domains and mathematical pedagogic content knowledge information together. According to this understanding, the four dimensions of knowledge a teacher should have are: basic information has been defined as transformation information, contact information and unexpected events information.

The dynamic, complex and holistic structure of the PAB makes it difficult for researchers to reach consensus on the issue (Şimşek and Boz, 2016). However, it is a factor that is primarily emphasized in the importance of teacher knowledge, teaching and learning (Fennema and Franke, 1992; Hill et al., 2008). The PAB directly impacts student learning by introducing demonstrations, representations, examples, simulations, and explanations used to ensure understanding of the concepts taught in the field. At this point, the PAB's influence in math education and learning emerges unambiguously. PAB teaching is important in planning, organizing, and developing a positive attitude towards teaching (Kaya, 2010). It is the teacher competence that concretely demonstrates the knowledge, skills, attitudes and values a teacher must have.

The studies on mathematics education in Turkey, compiled in the PAB mostly involve prospective teachers and qualitative data (Depaepe et al., 2013; Şimşek & Boz, 2016; Sayın et al. , 2021). Examination of the PAB components used in the studies revealed that the most commonly used PAB components are the understanding of students and education strategies (Depaepe et al., 2013; Şimşek and Boz, 2016) and the subject area has been defined as (Sayın et al., 2021). The PAB in the work of Depaepe et al. (2013) included: it has been examined from six different perspectives, namely, the nature of the PAB of teachers, the relationship between the PAB and its content knowledge, the relationship between PAB and teaching practice, the relationship between PAB and student learning, between PAB and personal characteristics, and the development of the PAB of teachers. The studies reviewed revealed that teachers had gaps in pedagogic content knowledge, a strong relationship between the PAB and content knowledge, the necessity of the PAB for effective teaching, the positive impact of teaching experience on the PAB and the PAB had an effect on student learning.

The studies on PAB reveal that the PAB and its components were revealed differently by various researchers. For example, in some models, the relevant field or program knowledge was not specified as a sub-component of PAB (Grossmnan, 1990; Shulman, 1987), in some models it is emphasized as a subcomponent. Examinations of these components showed that teaching a field discussed topics such as the aims of teaching, understanding students, program, teaching strategies and presentations, evaluation, subject area, context, pedagogy (Park and Oliver, 2008). It was determined that in some studies, the teacher solely bases the knowledge of understanding student and knowledge of instructional strategies in recognizing the students' errors/misconceptions and identifying their causes (Akkaş, 2014; Şahin et al. 2014; Gökkurt et al., 2015; Aksu and Konyalıoğlu, 2014) The studies on the PAB in the literature consist primarily of the understanding of students, knowledge of instructional strategies, subject area knowledge and program knowledge of the subcomponents of the PAB (Altaylı et al., 2014; Gökbulut, 2010: Şahin, 2016). The PAB students' understanding, knowledge of instructional strategies, knowledge of the subject content, and program knowledge were evaluated in this study.

Mathematical discipline is very important in the educational process. However, since mathematics is inherently abstract and concepts are acquired through abstractions, it is of great importance to nurture competent and sufficient teachers for the knowledge of pedagogical fields in the teaching of mathematics. Review of theories on pedagogical content knowledge in the context of the studies carried out, discuss the results, and reveal the situation with a holistic understanding will provide guidance to PAB studies in the field of mathematics education. In this context, it is thought that the purpose of the study would be to bring together the studies conducted and provide an overview of the PAB studies within the scope of mathematical education and provide information about the research conducted in the subject in Turkey.

The objective of this study is to systematically review and assess the postgraduate theses involving the PAB in mathematics education studies published in Turkey's national thesis center between 2016 and 2021. Problem of the research in line with the stated purpose: "What are the trends and results of postgraduate theses that examine the PAB in mathematics education in Turkey?" records).

1. What are the methodologies for the post-graduate theses on mathematics education (PAB) held in Turkey between 2016-2021?
   - how was the distribution of the research by years (2016-2021)?
   - which workgroup was discussed?
   - which research method (quantitative-qualitative-composite -other) was used?
   - what data collection tools have been used?
   - which data analysis methods are used?
2. How were the components of the PAB (program knowledge, content knowledge, knowledge of understanding student, and knowledge of instructional strategies) covered in related post grade thesis and what were the associated learning outcomes?

emi

Method

Model

This study is based on a systematic review method, which is a qualitative research method, to examine under certain criteria the undergraduate theses involving the PAB in the mathematical education studies published in the national thesis center between 2016 and 2021. Systematic review: there are three basic tools—identifying relevant studies, critically evaluating them, and consistently synthesizing findings (Gough, Oliver, & Thomas, 2012).

Study Group

The criterion sampling was used in determining the study group. The criteria for the work to be included in the analysis are:

- The activity related to "Pedagogical content knowledge" in the mathematics training
- Publication of the study at the National Thesis Data Center of the Presidency of the Council of Higher Education
- Published and available access to the study between 2016-2021.
- The study is written in Turkish and English

In the study, theses examining the PAB in the field of mathematical education were discussed. In the Detailed Search section of the National Thesis Data Center of the Presidency of the Council of Higher Education to obtain data about the study, a scan was conducted between 2016 and 2021 using the key concept of "pedagogical content knowledge" within the subject field of "Education and Training." In the review, 168 postgraduate thesis studies were completed, including 17 graduate thesis studies related to mathematics. In the Detailed Search section, a search was conducted between 2016 and 2021 using the keyword "Education and

Training" in the subject field and "pedagogical content knowledge" in the summary section. The review included 161 postgraduate thesis studies, including 16 postgraduate thesis studies, which differs from the original search results. In the advanced screening section, the search word(s) section was searched using the keywords "pedagogic domain information" and "math" and consequently there were 74 post-graduate thesis studies. Abstracts unrelated to the subject were examined and included in 26 post-language thesis working groups. In 26 postgraduate theses, 11 doctoral and 15 master's dissertations were completed. The selection of the workgroup is based on the PRISMA information flow in figure 1. PRISMA is a "preferred reporting clause for systematic reviews and meta-analyzes" and provides a clear presentation on how systematic review is done based on complete and transparent reporting. (Liberati et al., 2009)

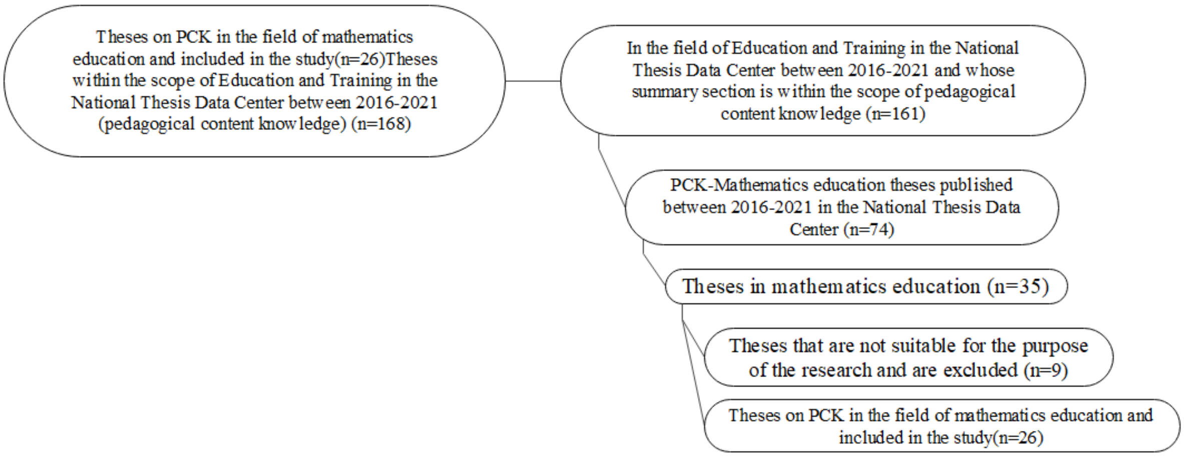

Figure 1. Flow chart for determining the study group

Analysis of Data

The theses selected for the purpose of the study were reviewed within the framework of the evaluation form prepared by the researchers. In this context, the title (reference, authors, year, etc.) of the postgraduate theses, their purpose, methodological aspects (approaches, working group, data collection tools, data analysis method) and results (learning outcomes regarding the sub-dimensions of the PAB model used) were analyzed. Following the research objective, the literature was reviewed, and the most examined sub-components of the PAB models based on graduate theses were identified as "program knowledge", "content knowledge", "knowledge of understanding student" and "knowledge of instructional strategies ". In this context, the postgraduate these included in the study were examined by content analysis in terms of process and results within the framework of four sub-components determined. Content analysis is to combine similar data into specific concepts and themes and to interpret these in a way that the reader can understand (Marshall & Rossman, 2006; Yıldırım ve Şimşek, 2013). The agreed PAB components were selected as themes, and the data were individually coded by the researchers under four main themes. Miles and Huberman's (1994) [Reliability: The coherence among the researchers was calculated as 87 percent by applying the formulas of the Consensus / (Opinion Union + Difference of Opinion).

The findings of the studies, and the obtained findings on the theses reviewed within the framework of the four PAB components determined, were presented in descriptive form with frequency and percentage values.

All these selected were scanned simultaneously by the researchers within the determined criteria. As the systematic analysis used in the study was a study based on the literature review, the data obtained was examined by three experts in an effort to achieve internal validity by varying researchers. The manner in which the data was obtained, the keywords used, the selection procedure of the postgraduate theses to be included in the analysis, and many details of the theses examined are clearly indicated. To compare the results with data within the PAB, we aimed to establish external security by taking the opinions of two mathematics education experts.

Findings

Findings related to the 1st sub problem

In this study where the trends and results of the postgraduate thesis studies covering the PAB in mathematics education in Turkey are studied, the first study examines the methodologies of postgraduate theses in order to reveal the trends. Data on the distribution of 26 post-graduate theses (11 doctoral dissertations, 15 postgraduate theses) by year (2016-2021) are presented in Chart 1.

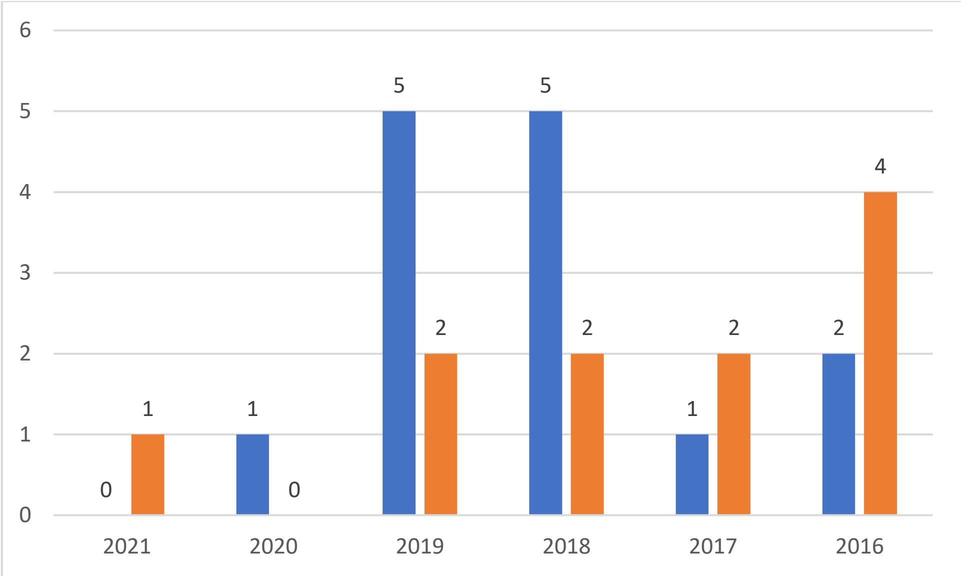

Chart 1. Distribution of graduate theses by years

When the chart is examined, among the selected theses, 1 doctoral thesis in 2021, 1 master's thesis in 2020, 7 theses in 2019 (2 doctorate, 5 master's), 7 theses in 2018 (2 doctorate, 5 master's), 3 theses in 2017 (2 doctorate, 1 master's), and 6 theses (4 doctorate, 2 master's) were published in 2016. Distribution of the analyzed theses regarding their working groups is presented in chart 2.

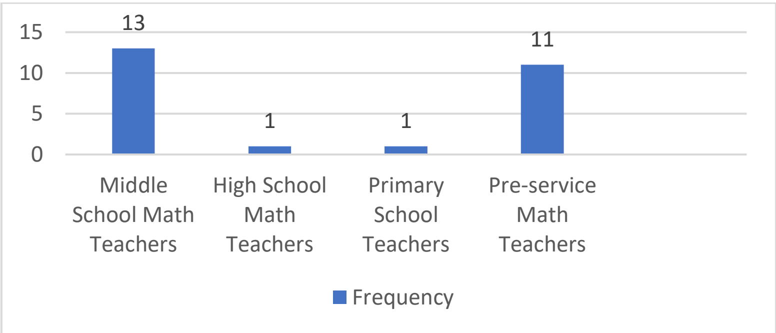

Chart 2. Study group frequencies of graduate theses

The study revealed that 13 theses were performed with the middle school mathematics teachers, one thesis with high school mathematics teachers and one thesis with classroom teachers and 11 dissertations with the prospective math teachers. It was observed that pedagogic content knowledge of the most secondary school math teachers were studied. Chart 3 shows the distribution of the research methods by the postgraduate theses reviewed.

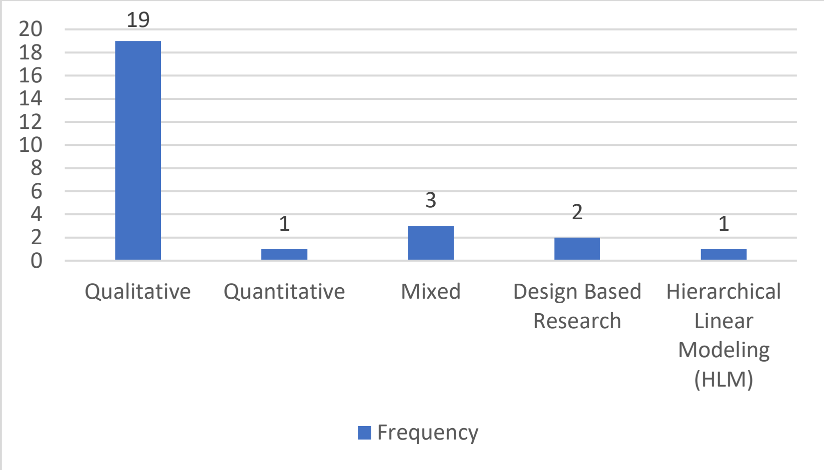

Chart 3. Distribution of research methods in graduate theses

Chart 3 reveals that 73 percent of the theses were based on qualitative research method, 3.8 percent on quantitative research method and hierarchical linear model, 11.52 percent on mixed research method, and 7.8 percent on design-based research method. Studies on PAB in math education revealed that the most qualitative research methods were used. Illustration 4 shows the breakdown of the data collection tools used in the postgraduate theses examined.

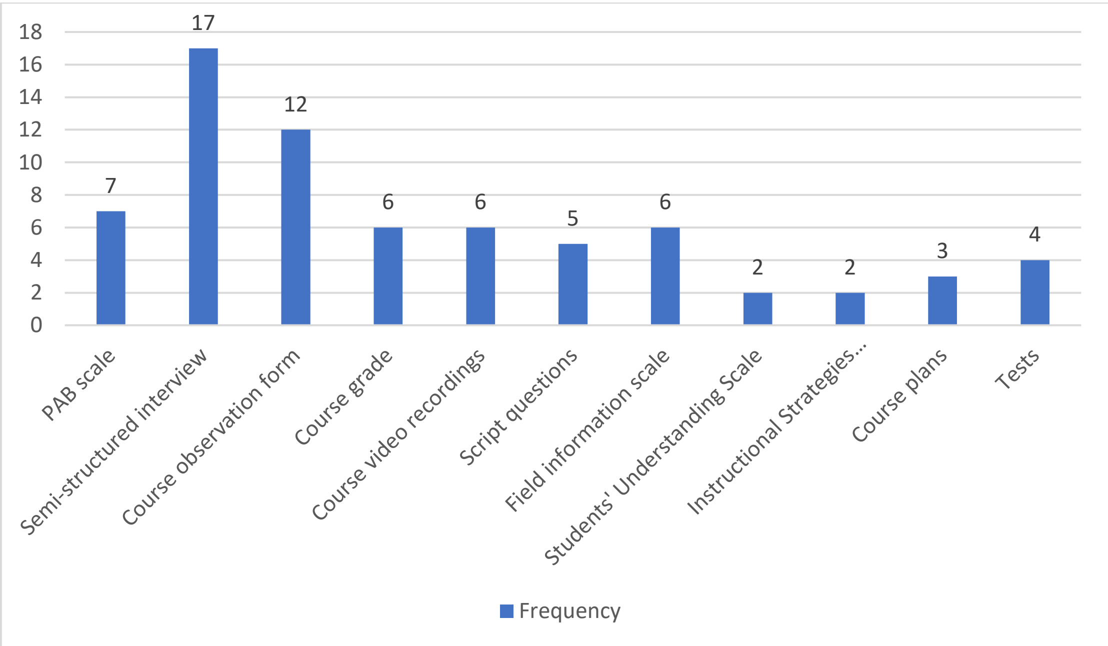

Chart 4. Distribution of the data collection tools used in graduate theses

Chart 4 reveals that 65 percent of the theses use semi-structured interviews, 46 percent use lesson notes, course video recordings, and content knowledge scale. In addition, the theses used course video recordings, PAB scales, course notes, scenario questions, domain knowledge scale, tests (Pedagogical Math Knowledge Test, PAB Self-Sufficiency Test, Faculty Cluster

Test, Success Test). A breakdown of the data analysis methods used in the postgraduate theses examined is presented in Chart 5.

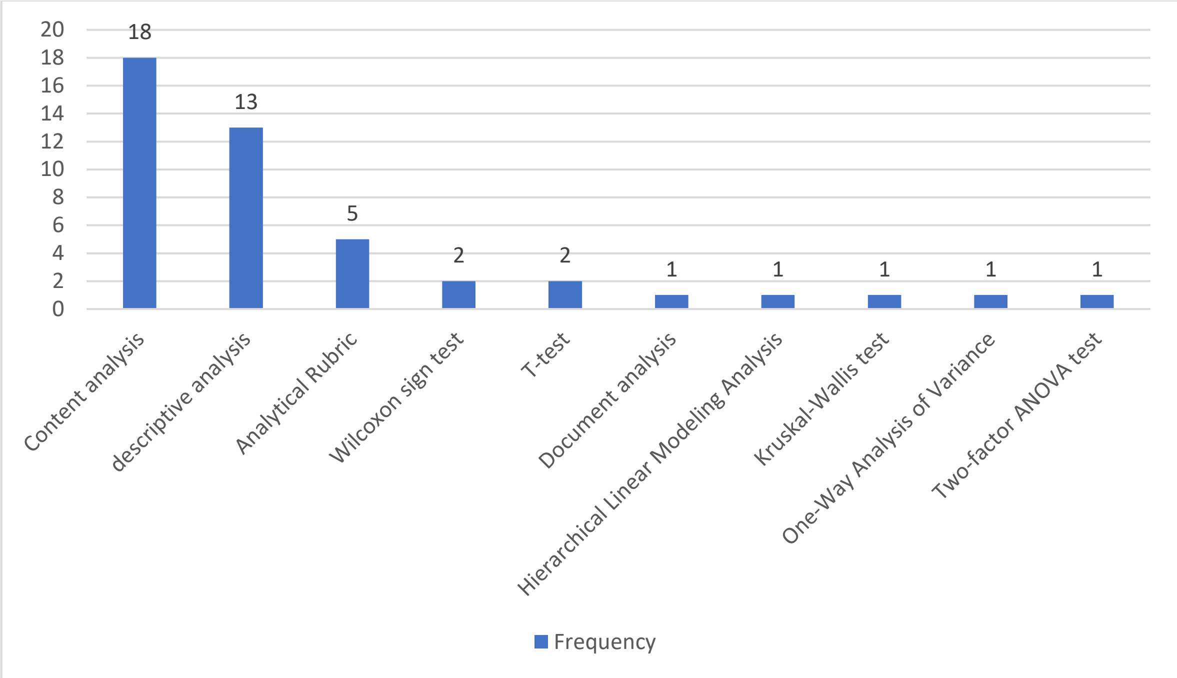

Chart 5. Distribution of data analysis methods used in graduate theses

The Chart revealed that 69.2 percent of post-graduate theses utilized content analysis while 50 percent used descriptive analysis methods. Since the studies are mostly based on the qualitative research pattern, the most preferred methods are content analysis and descriptive analysis. Analytical rubrics are used for the analysis of various scenario questions in some scales. T-test and Wilcoxon marker test in 8% of the analysis of quantitative data, Kruskal-Wallis test in 4%, one-way variance analysis and two-factor ANOVA test; Four percent used document and hierarchical linear modeling analysis.

Findings for the 2^nd^ sub-problem

In the postgraduate theses related to the PAB carried out in the field of mathematics education in Turkey between 2016-2021: How the components of the PAB (program knowledge, content knowledge, knowledge of understanding student, and knowledge of instructional strategies) were addressed and the related learning outcomes were individually examined under the identified components. The findings obtained in this context are explained based on the definitions of the theorists.

Program knowledge

In the evaluation of program knowledge, which is one of the components of the PAB in the postgraduate theses examined in the research, it was found that 19% (5 theses) of the theses included the dimension of program knowledge in their studies. When we examined which PAB model was the basis for the theses involved in the Program Knowledge dimension study, we found that 20% of the theses were based on the PAB model defined by Shulman (1987), 20% on Baki (2012), 20% on Blomeke and their friends (2015) and 40% on the PAB model defined by Ball and friends (2008). Examination of the said PAB models reveals different descriptions that theorists have come up with regarding program knowledge. According to Shulman (1987), program knowledge content knowledge and related education program is described as information to use teaching materials (textbooks, technological tools, concrete teaching materials), knowledge of horizontal programs (information on intra- and inter-disciplinary relationships), knowledge of vertical programs (information on subject ranking and materials covering this sequence). Ball et al. (2008) described program knowledge to be given by a teacher in what order, as knowledge of how topics relate to each other and with other disciplines, while Baki (2012) described the place of the subject in the curriculum and its relevance to other achievements or topics as knowledge-making knowledge. Blömeke et al. (2015) defined program knowledge as a curriculum related to content knowledge, acquisition, use of teaching materials, design of learning environment, teaching methods, and techniques knowledge. Table 1 presents the PAB models used in the dissertation studies for the program knowledge component and the scope of the program knowledge examined in this context.

Table 1. PAB models and scope of program knowledge

PAB model based on	Thesis	Scope of program knowledge reviewed
Shulman (1987) Ball et al. (2008) Baki (2018) Blömeke et al. (2015)	Şimşek (2016) (Functional subject) Keleş (2019) (learning area to process data) Girit (2016) (algebra learning area) Güler (2019) Kutlu (2018)	Knowledge of benefits in a curriculum for a given content
Shulman (1987) Ball et al. (2008)	Şimşek (2016) Keleş (2019)	Information on the distribution of subjects by class level
Shulman (1987) Ball et al. (2008) Baki (2018) Blömeke et al. (2015)	Şimşek (2016) Keleş (2019) Girit (2016) Güler (2019) Kutlu (2018)	Information about pre-requisite relationships and which order they should be taught
Shulman (1987)	Şimşek (2016)	Knowledge of the skills program aims for
Baki (2018) Blömeke et al. (2015)	Güler (2019) Kutlu (2018)	How to prepare the content of a lesson for basic knowledge (operational/conceptual) and skills (reasoning, association, communication...)

Table 1 shows that all of the studies that examined program knowledge included "knowledge of achievements in education program for content" and "information on pre-condition relations and which order education should be realized," 2 studies "distribution of subjects by class level, 1 knowledge of skills targeted by the program of study, 1 understanding of basic knowledge (operational/conceptual) in 2 studies and knowledge on how to prepare content of a course in relation to skills (reasoning, association, communication...) within the scope of program knowledge. On the basis of the PAB model developed by Ball et al. (2008), it was found that program knowledge, the sub-component of the model, was not discussed in the studies (Amaç, 2018; Çıkrıkçı, 2015; Çopur-Gençtürk, 2012; Doğruel, 2019; Hacıömeroğlu, 2013: Özdoğan, 2018; Yazıcı, 2017; Yılmaz, 2016).

Ball et al. (2008)'s program knowledge, which is the PAB component of the "Mathematical Knowledge for Education" model of "Mathematical Knowledge"; the knowledge about the educational program on content knowledge was considered as informational, intra-disciplinary, interdisciplinary relationship knowledge and subject ranking. however, in the studies under consideration, it was analyzed that the outcomes of the content-related education program generally addressed variables such as class-based distribution of subjects, pre-condition relations and order of teaching need to be carried out (Esen, 2013; Lannin, et al., 2013; Stacey and Chick 2004).

Ball et al. (2008) have found that studies based on the PAB model look at some of the questions added to scales at the dimension of program knowledge. It has been noted that, within the context of program knowledge, studies do not examine educational programs and skills or information variables for using teaching materials (computer-aided materials, concrete materials, etc.). In studies based on the PAB model developed by Shulman (1986), it is found that the program's sub-component is: the knowledge of program is also seen in the Kula (2011) study, which looks at the dimensions of knowledge, intra-disciplinary and interdisciplinary relationship and subject ranking information to use teaching materials (textbooks, technological tools, concrete teaching materials).

The results obtained for studies on program knowledge in postgraduate theses are presented in Figure 2.

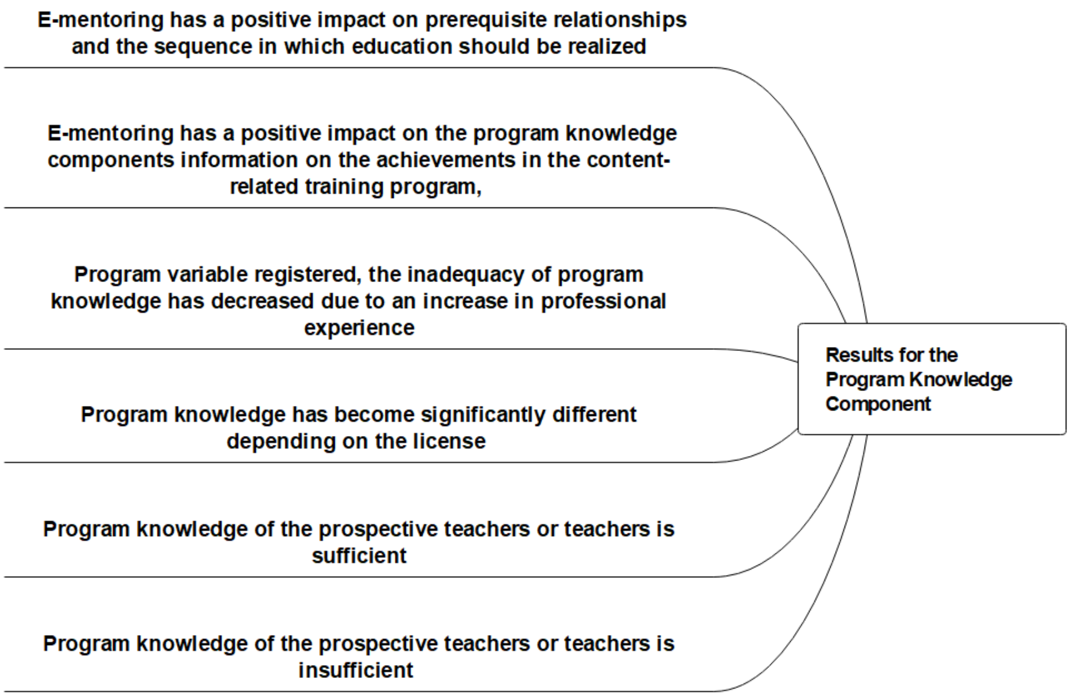

Figure 2. Results for program knowledge component

In reviewing the results regarding program knowledge, we found that program knowledge of the prospective teachers or teachers is insufficient or insufficient, program knowledge has become significantly different depending on the license program variable registered, the inadequacy of program knowledge has decreased due to an increase in professional experience, and e-mentoring has a positive impact on the program knowledge components (information on the achievements in the content-related training program, prerequisite relationships and the sequence in which education should be realized).

According to reviews on program knowledge in the studies, we see that program knowledge is examined in terms of information related to the algebra, function, and data processing learning areas. It was determined that program knowledge was not considered in terms of the knowledge of gains in the educational program for a given content, knowledge of the distribution by class levels of the subjects, prerequisite relationships and what order teachings should be taught, knowledge of the skills the training program is intended for, knowledge of basic knowledge (operational/conceptual) and knowledge on how to prepare content of a course for the skills (reasoning, association, communication...), knowledge of the use of teaching material in the underlying PAB models, which is analyzed.

Content knowledge

In the evaluation of content knowledge, which is one of the components of the PAB in the postgraduate theses examined in the research, it was found that 60 percent (15 theses) of the theses included the content knowledge size in their studies. In the study of the PAB model on which the studies including the content knowledge dimension are based, it has been determined that 53.4 percent of the theses are based on the content knowledge component defined by Shulman (1987), 6.6 percent by Lesseig (2016) and 40 percent by Ball et al. (2008). Examination of the said PAB models reveals that the definitions suggested by theorists differ for the content knowledge. According to Shulman (1987), the field knowledge is the type of information that covers the concepts, processes, proofs and problem-solving skills that teachers will teach. According to Lesseig (2016), "the subject area for content teaching is two parts. General content knowledge for content teaching; information held by the person using math in relation to the content; specific domain knowledge for content teaching; it is described as unique knowledge solely that teachers have in teaching content. Ball et al. (2008) categorizes content knowledge into general domain, custom field, and horizontal space. General content knowledge is mathematical knowledge that can be used in areas other than education, with knowledge of mathematical concepts, identifying correct and incorrect answers. Specific field knowledge is the mathematical knowledge that mathematical teachers have, unlike experts in other fields; horizontal content knowledge is described as the knowledge of mathematical concepts in relation to the other concepts in the educational program.

Table 2 presents the PAB models used in the dissertation studies that examine the content knowledge component and the scope of the content knowledge examined in this direction.

Table 2. PAB models and content knowledge

PAB model based on		**Scope of content knowledge**
Shulman (1987)	Uçar (2019) (non-routine problems); Yurtyapan (2018) (triangles and rectangles); Duran (2018) (derivatives and applications): Aliustaoglu (2018) (linear equation and slope); Özdemir Baki (2017) (area measurement sub-learning area); Sahin (2016) (algebra learning area)	Information on what to teach (content knowledge)
Ball et al. (2008)	Keleş (2019); Doğruel (2019) (rate and ratio); Girit (2016); Gürel (2016) (central trend and distribution measurements); Bulut (2021) (ratıonal numbers) Yazıcı (2017) (basic concepts in clusters); Yılmaz (2016) (addition and removal operations in fractions)	
	Uçar (2019): Yurtyapan (2018)	Information on the methods used for problem solving

Shulman (1987) Ball et al. (2008)	Uçar (2019); Tosuncu (2019) (numbers, geometry, data processing and measurement learning area); Yurtyapan (2018); Duran (2018), Aliustaoğlu (2018), Özdemir Baki (2017): Sahin (2016) (Keleş, 2019; Girit (2016); Gürel (2016); Doğruel (2019): Yazıcı (2017) Yılmaz (2016);Bulut (2021)	Information for concepts relevant to the subject to be taught (e.g. Definitions, treatments)
Shulman (1987) Ball et al. (2008)	Duran (2018) Tosuncu (2019); Duran (2018), Özdemir Baki (2017) Keleş (2019); Girit (2016); Gürel (2016); Yılmaz (2016); Bulut (2021)	Uses of the subject to be taught Links between the concepts and concepts contained in the subject
Shulman (1987) Ball et al. (2008)	Tosuncu (2019); Duran (2018), Özdemir Baki (2017): Sahin (2016); Aliustaoğlu (2018) Doğruel (2019): Yazıcı (2017)	Skills to problem and solve the issue
Shulman (1987)	Duran (2017) (multiplication and division by fractions)	Information for model usage (field model, length model, cluster model)
Lesseig (2016)	Cihan (2019) (methods of proof)	Knowledge of the method of proof and know how to apply it Main proof diagrams they have

Upon examination of the data from Table 2, PAB model, most frequently deployed by Shulman (1987) and Ball et al. (2008) is preferred for work addressing content knowledge. Non-routine problems, triangles and quadrants, derivatives and applications; linear equation and slope; area measurement sub-learning area, algebra learning area, rate and proportion central tendency and dissemination metrics have been determined that in the clusters, fundamental concepts, such as collecting and subtracting with fractions, content knowledge is examined. Information on the subject to be taught in 12 studies upon examination of the table data related to the area of knowledge covered in the theses reviewed; 2 problems resolution method information of the study; 13 studies' information on the subject to be taught; 1 information on the use area of the study, 7 information on the studies' concepts and connections; Seven studies had examined problem-building and problem-solving skills, while one study had examined information about model use. In addition, one study examined the information on proofing methods that Lesseig (2016) preferred the PAB model and the information on implementation, within the scope of the content knowledge on the main proof diagrams they have.

The content knowledge, which is the sub-component of the PAB model, is expressed as the information required or used in the teaching of lessons in the math program based on class levels, as well as the concepts, processes, proofs of the field, their underlying mathematical meanings, mathematical representations, and information on how mathematics develops and changes as a discipline (Ball et al., 2008; Lesseig, 2016; Shulman, 1986). In studies based on the PAB model developed by Shulman (1986), content knowledge, which is the sub-component of the model, can be found: it was determined that the mathematical program is studied in the dimensions of establishing the relationship between the information required or used in lesson teaching according to class levels, as well as the underlying mathematical meanings and processes, and the concepts and concepts related to the field (Aliustaoğlu, 2018; Duran, 2018; Özdemir-Baki, 2018; Şahin, 2016; Uçar, 2019; Tosuncu, 2019;Yurtyapan, 2018). The content knowledge along with these dimensions; It has been examined by Uçar (2019) and Yurtyapan (2018) according to the methodology used in the subject: the studies Duran (2018), Aliustaoğlu (2018), Özdemir Baki (2017) Şahin (2016) showed the ability to provide a problem with respect to the subject. Duran (2017) reviewed the content knowledge for model use by factoring multipliers and dividing operations. Ball et al. (2008) looked at the subject area knowledge holistically in the "Mathematical Knowledge for Teaching" model. Subject content knowledge: general content knowledge is defined in three categories: specific content knowledge and horizontal content knowledge. In the postgraduate studies reviewed, only the information related to the Keleş (2019) study was examined in an integrated manner. Within the scope of general field and specific content knowledge, we have reviewed the works of Doğruel (2019), Girit (2016) and Yazıcı (2017); Gürel (2016) study demonstrated that she was only exploring general content knowledge, and Yılmaz (2016) examined only specific content knowledge. Figure 3 provides the results of the content knowledge research for the postgraduate theses examined.

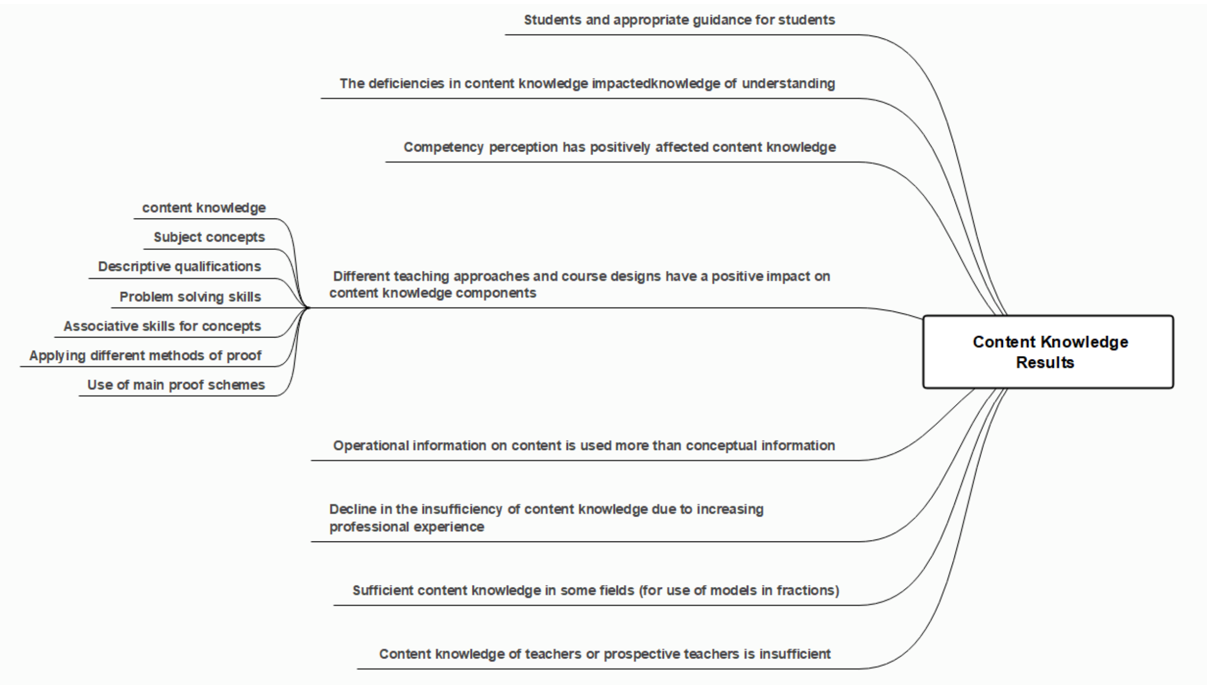

Figure 3. Results for the content knowledge

Upon examination of the results, we found that the content knowledge of teachers or prospective teachers is insufficient for the majority of the studies that address content knowledge. Moreover, there is a decline in the insufficiency of content knowledge due to increasing professional experience (Uçar, 2019; Yurtyapan 2018: Duran, 2018. Doğruel, 2019), that operational information on content is used more than conceptual information (Duran, 2018), and different teaching approaches and course designs have a positive impact on domain information components (content knowledge, subject concepts, descriptive qualifications, problem solving skills, associative skills for concepts, applying different methods of proof, use of main proof schemes) (Aliustaoğlu, 2018; Özdemir and Baki, 2017: Cihan, 2019), has sufficient content knowledge in some fields (for use of models in fractions) (Duran, 2017), and competency perception has positively affected content knowledge (Tosuncu, 2019). The deficiencies in content knowledge impacted knowledge of understanding students and appropriate guidance for students (Girit, 2016; Gürel, 2016).

Knowledge of understanding student

PAB models described by 4.2% as Grossman (1990), Magnusson et al. (1999), Lesseig (2016), Baki (2018), Rowland et al. (2009), Blömeke et al. (2015), where 24 studies examined the understanding of students who are PAB components in the post-graduate theses study, of this 41.5 % is Shulman (1987); 8.3 % in Covariates (2008); 25% were based on the PAB model defined by Ball et al. (2008). The evaluation of the said PAB models show that the explanations about understanding the student differ. Shulman (1987): Understanding students a teacher's awareness of the preliminary knowledge, misconceptions, what are the points they experience difficulty, their motivation, attitudes and level of interest in a course from students of different ages and experiences, Ball et al. (2008) defined a particular mathematical concept or process-oriented information as knowledge of more what students think or do. Grossman (1990) believes that knowledge of understanding student is the teacher's knowledge of pupils' understanding, concepts and misconceptions in a given subject area; the ability to connect initial knowledge and new information according to Magnusson et al. (1999), and how to identify students' misconceptions, errors, and student challenges. According to Kovarik (2008), understanding information is grouped into two sub-dimensions. Preliminary information of the students; it includes mathematical background in the subject students will learn according to their development levels, as well as misconceptions, the ability to guess the questions in their minds, and the information to identify where they have difficulties and their reasons. Lesseig

(2016) has been described as clear knowledge for the student's schemes (characteristics of extrinsic, experimental and deductive schemes, students' tendency to rely on authority or experimental examples, descriptions and expressions appropriate to students, representations within students' conceptual access, discussion patterns appropriate to the students' level, and the relation between the mathematical and everyday use of terms). Rowland et al. (2009) reported that there is information within the medium of knowledge of incidents with unexpected learners in a lesson about events with unexpected understanding of the student, that it is information about unanticipated learning opportunities, opportunities as they arise during learning, deviations from schedule or plan as needed, which is almost impossible to plan in class. Blömeke et al. (2015) knowledge of understanding student; learning disabilities, difficulties of learning and knowledge of students' prior knowledge. When the definitions in the underlying PAB models are examined, students' understanding of the student can be defined as preliminary information, misconceptions, what are the difficulties, their motivations, attitudes and awareness of their level of interest, understanding the student's conceptual or operational knowledge, and relational thinking. Table 3 lists the PAB models used in the dissertation studies that examine the knowledge of understanding student component and the scope of the content knowledge examined in this direction.

Table 3. PAB models and knowledge of understanding student

PAB model based on	**Investigations**	**Knowledge of understanding student**
Shulman (1987) Ball etc. (2008) Baki (2018) Blömeke, etc. (2015) Kovarik (2008)	Uçar (2019); Aliustaoğlu (2018) Keleş (2019); Girit (2016) Güler (2019) Kutlu (2018) Duran (2017)	Knowing the mindset-strategies of students
Shulman (1987) Grossman (1990) Magnusson Tax Office 1999 Kovarik (2008) Baki (2018) Blömeke, etc. (2015) Rowland Tax Office (2009) Ball etc. (2008)	Uçar (2019); Uz (2019), Aliustaoğlu (2018), Amaç (2018) (algebraic use of letters in algebra); Şimşek (2016); Özdemir Baki (2017): Sahin (2016); Tosuncu (2019) Can (2019) (fractional transactions) Sert Çelik (2018) (equality and equation) Duran (2017) Güler (2019) Kutlu (2018) Bilik (2016) (triangle) Keleş (2019); Gürel (2016); Doğruel (2019): Yazıcı (2017); Özdoğan (2018) (concept of function)	Identifying learning challenges
Shulman (1987)	Uçar (2019); Uz (2019), Duran (2018), Aliustaoğlu (2018), Amaç (2018) Şimşek (2016); Özdemir Baki (2017): Sahin (2016); Tosuncu (2019)	

Grossman (1990) Kovarik (2008) Magnusson Tax Office 1999 Rowland Tax Office (2009) Ball etc. (2008) Baki (2018) Blömeke, etc. (2015)	Can (2019) Orman (2020) (chamomile numbers); Duran (2017) Sert Çelik (2018) Bilik (2016) Girit (2016); Gürel (2016); Doğruel (2019): Yazıcı (2017); Özdoğan (2018) Güler (2019) Kutlu (2018)	Identifying misconceptions, errors and causes Knowledge of questions to be asked of the student to understand his/her mistake
Shulman (1987) Ball et al. (2008) Kovarik (2008) Magnusson Tax Office 1999 Baki (2018) Blömeke, etc. (2015)	Yurtyapan (2018); Duran (2018), Özdemir Baki (2017): Sahin (2016) Keleş (2019); Gürel (2016); Doğruel (2019) Orman (2020); Duran (2017) (multiplication and division by fractions) Sert çelik (2018) Güler (2019) Kutlu (2018)	Identifying preliminary information of students
Shulman (1987)	Aliustaoğlu (2018), Sahin (2016)	Attracting student attention, Motivating students
Ball v.d (2008)	Girit (2016)	Identifying students' mathematical needs
Baki (2018) Blömeke, etc. (2015)	Güler (2019) Kutlu (2018)	Analyzing students' mathematical solutions and discussions
Baki (2018) Blömeke, etc. (2015)	Güler (2019) Kutlu (2018)	Determining the relevance of the activities to the student level
Lesseig (2016)	Cihan (2019)	Information for identifying evidentiary schemes Information on identifying student difficulties for proving and explaining the reasons

Looking at the table data regarding the factors that are examined within the scope of understanding the students who prefer the PAB, we can see that 7 studies are interested in students' thinking, 20 studies' learning difficulties, 21 studies identify students' misconceptions and errors, 11 studies identify the reasons for conceptual misconceptions and mistakes, 12 studies identify students' preliminary information, 4 the questions to be directed to the student to understand the student's mistake, 2 to get the students attention of1 studies, determine the students' needs 2 study analyzed the students' mathematical solutions and discussions, 2 determined the suitability of activities to student level, 1 determined the proofing schemes of the study, 1 assessed the knowledge of understanding student components regarding the information to identify the student's difficulties in proving the study and explaining the reasons. It was determined that the most common research was conducted on identifying learning difficulties and misconceptions, errors and causes.

Within the studies based on the PAB model developed by Shulman (1986), understanding students as sub-components of the model could include: Students were examined to determine learning difficulties, errors, misconceptions and reasons for content (Uçar, 2019; Oz, 2019; Tosuncu, 2019; Yurtyapan 2018: Duran, 2018. Aliustaoğlu, 2018: Amaç 2018. Şimşek, 2016; Özdemir Baki, 2017: Sahin, 2016). Understanding the learner with these dimensions; student preliminary information has been reviewed in the studies conducted by Yurtyapan (2018), Duran (2018), Özdemir Baki (2017), Şahin (2016). The study was conducted by Aliustaoğlu (2018) with the aim of getting students' attention and motivating them.

Figure 4 presents the results of the studies on knowledge of understanding student about the student in the evaluation postgraduate theses.

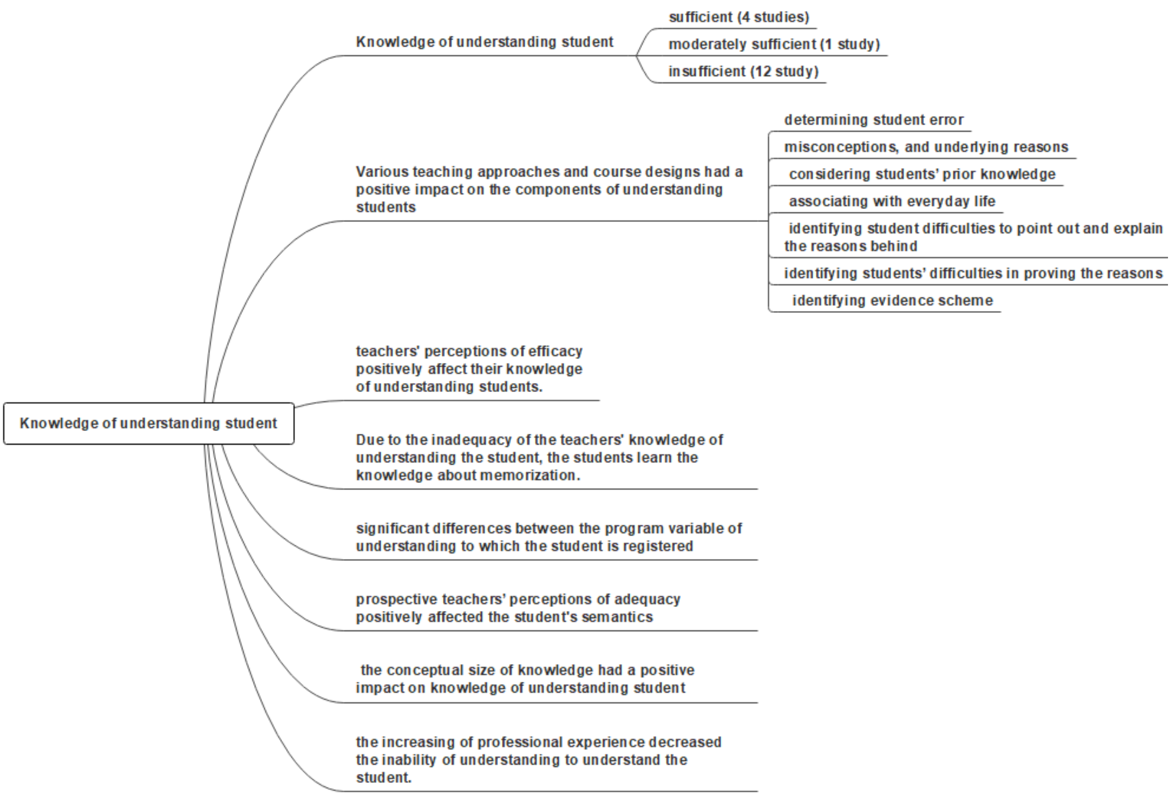

Figure 4. Results from knowledge of understanding student

When the results of the assessment of student comprehension in the studies that selected the PAB were examined, it was observed that prospective teachers or teachers had insufficient understanding of the student in 12 studies, moderate level in one study and adequate level in four studies. Knowledge of understanding student on memorizing students due to insufficient teacher understanding knowledge (Yurtyapan, 2018), various teaching approaches and course designs had a positive impact on the components of understanding students (determining student error, misconceptions, and underlying reasons, considering students' prior knowledge, associating with everyday life, identifying student difficulties to point out and explain the reasons behind, identifying students' difficulties in proving the reasons, and identifying evidence schemes) (Aliustaoğlu, 2018; Özdemir and Baki, 2017: Cihan, 2019; Güler, 2019); significant differences between the program variable of understanding to which the student is registered (Şimşek, 2016) have been reached. (Tosuncu, 2019), prospective teachers' perceptions of adequacy positively affected the student's semantics (Tosuncu, 2019), the conceptual size of knowledge had a positive impact on understanding the student (Can, 2019), and the increasing of professional experience decreased the inability of understanding to understand the student.

Knowledge of instructional strategies

4.8% of the theses examined and examined the knowledge of instructional strategies of the PAB components in postgraduate theses, as defined by Lesseig (2016), Baki (2018), Rowland et al. (2009), Blömeke et al. (2015); of this 42.8% is Shulman (1987); Kovarik of 9.5% (2008); 28.5% were based on the PAB model defined by Ball et al. (2008). Following are the explanations for knowledge of teaching strategies when the related PAB models are examined. Knowledge of teaching strategies according to Shulman (1987); is described as the teacher's teaching method and know-how to communicate space to the students, eliminate students' misconceptions and increase students' success. Knowledge of strategies, methods, techniques, decision-making and implementation skills that can be used to teach an subject based on Ball et al. (2008), knowledge of teaching strategies by Kovarik (2008); knowledge of mathematical representations describes in three sections: representations (Charts, tables, and formulas), examples (real-world problems), and analogies (teaching the students about abstract concepts of math education with the concepts or facts known to students). Baki (2018) knowledge of teaching strategies explaining the strategy, method, and technique used by teachers during teaching. Lesseig (2016), on the other hand, the relationship between demonstration schemes and education (answering questions, responding to student opinions, using examples, and teaching methods that raise or reduce authoritarian or experimental demonstration schemes), questioning strategies (to justify beyond transactions and encourage thinking in the general case), use of significant examples or contrasting examples (broadening thought, bridging or scaffolding, focusing on key ideas in evidence), and knowledge of proof links (How to connect visual, symbolic or oral evidence to the agreed characteristic or contextual definitions? how to generate a generic argument from the diagram?). In general, knowledge of instructional strategies can be defined as teaching methods and technical knowledge that aim to convey the knowledge of these fields to students, eliminate misconceptions, and increase students' success. Table 4 presents the PAB models used in the dissertation studies that examine the Instructional strategies knowledge and the scope in this direction.

Table 4. PAB models and Scope of the Coverage of the Learning Strategies Reviewed

PAB model based on	Thesis	Instructional strategies knowlegde components
Shulman (1987)	Uçar (2019); Tosuncu (2019); Yurtyapan (2018); Duran (2018) Aliustaoğlu (2018);Goal (2018) Şimşek (2016); Özdemir and Baki (2017): Sahin (2016); Tosuncu (2019)	Educational, technical, and strategy information to be used in correcting mistakes made by the student and misconceptions.
Ball v. d. (2008)	Keleş (2019); Girit (2016); Gürel, 2016	
Baki (2018)	Güler (2019)	
Blömeke v. d. (2015)	Kutlu (2018)	
Rowland v. d. (2009)	Bilik (2016)	
Shulman (1987)	Yurtyapan (2018);Duran (2018); Aliustaoğlu (2018), Şimşek (2016) Özdemir - Baki (2017): Sahin (2016)	Teaching method, strategy, and know-how to be used by the teacher during the teaching process
Ball et al. (2008)	Keleş (2019); Girit (2016); Gürel (2016); Doğruel (2019): Yazıcı (2017); Yılmaz (2016); Bulut (2021)	
Baki (2018)	Güler (2019)	
Blömeke et al. (2015)	Kutlu (2018)	
Shulman (1987)	Aliustaoğlu (2018), Şimşek (2016); Özdemir and Baki (2017): Sahin (2016)	Designing suitable learning environments to effectively teach concepts
Kovarik (2008) Baki (2018) Blömeke v. d. (2015) Ball v. d. (2008)	Orman (2020) Güler (2019) Kutlu (2018) Keleş (2019); Girit (2016); Gürel, 2016	Knowledge of mathematical representations, examples, and analogies
Kovarik (2008)	Duran (2019)	Use of modeling for information on mathematical representations (area model, length model, cluster model)
Ball v. d. (2008)	Yazıcı (2017); Yılmaz (2016)	Subject-specific lesson planning information
	Yazıcı (2017)	Ability to use technological materials and programs
	Yılmaz (2016)	Teaching multi-representation use
Lesseig (2016)	Cihan (2019)	Identifying teaching strategies to overcome student challenges
Baki (2018) Blömeke v. d. (2015)	Güler (2019) Kutlu (2018)	Knowledge of relationship between mathematics and the real world

Examination of the Table 4' data on the scope of Instructional strategies knowledge in studies based on PAB models reveals that 16 studies will use the teaching methods, techniques and strategies to eliminate errors and misconceptions made by the students; 14 teaching methods, strategies and technical information to be used in the teaching process; 4 information on designing suitable learning environments to effectively teach concepts of the study; six studies' knowledge of mathematical representations, examples and analogies; 1 model of the study for mathematical representations knowledge of the study (field model, length model, cluster model); 2 use of subject-specific course materials and use of teaching programs 1, it was determined that 1 study examines the information about determining knowledge of instructional strategies to overcome student challenges, and 2 study examines the information about teaching strategies to create relationships between mathematics and the real world. Within the studies based on the PAB model developed by Shulman (1986), education strategies subcomponent of the model is included in the following: Students were identified with the objective of eliminating misconceptions and mistakes related to content, preparing suitable learning environments for teaching concepts, the teaching method, strategy and technical information used by the teachers, and how to apply them in the teaching process (Uçar, 2019; Yurtyapan 2018: Amaç 2018. Duran, 2018. Aliustaoğlu, 2018: Şimşek, 2016; Özdemir Baki, 2017: Sahin, 2016). The educational knowledge by Ball et al. (2008) which is the PAB component of the "Mathematical Knowledge for Education" model; a subject has been examined as the type of knowledge covering strategies, methods and techniques to be used in teaching and applying them (Doğruel, 2019; Keleş, 2019;Yazıcı, 2017; Girit, 2016; Gürel, 2016). The results obtained for studies on instructional strategies knowledge in graduate theses are presented in Figure 5.

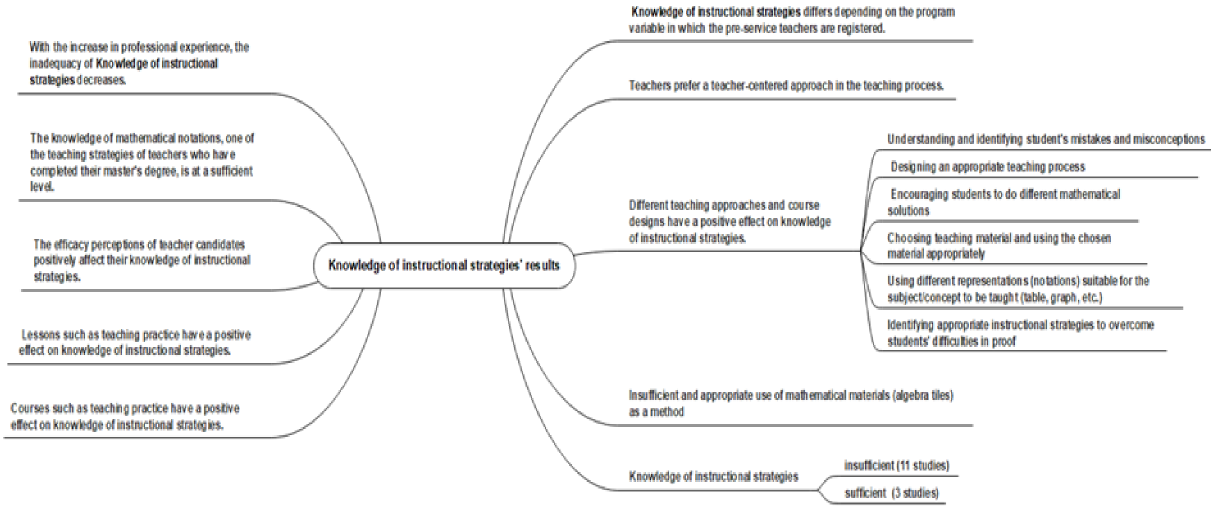

Figure 5. Results with respect to the knowledge of instructional strategies

In examining the data on the outcomes of the knowledge of instructional strategies components in which the knowledge of instructional strategies of the participants who prefer the PAB are examined, it is seen that the knowledge of instructional strategies of prospective teachers or teachers are insufficient in 11 studies and that they are sufficient in three studies, that they follow a teacher-centered approach during the education process, that different teaching approaches and course designs have a positive impact on the teaching information components of the education strategies (e.g., table and charts), that they identify teaching strategies to overcome the difficulties of students that aim to perceive errors of concept, design and prove a suitable teaching process, use the appropriate representations(), encouraging students to develop different mathematical solutions), results have been obtained regarding the significant difference between the program variable in which the Instructional strategies knowledge is recorded in the scale development study, where prospective teachers can vary positively with the class level, and the effects of this variation on the teaching profession have, perceptions of competence of teachers have a positive effect on the education of the teaching profession, the mathematical representations of teachers who have obtained their master's degree are of sufficient knowledge, the knowledge of field within the scope of mathematical representations is used, and the insufficiency in knowledge of instructional strategies is reduced with increasing professional experience.

Conclusion, Discussions and Recommendations

This study investigated methodological aspects of postgraduate theses (distribution by year, workgroup, research methods, data collection tools and data analysis methods), how to handle the components of the PAB (program knowledge, content knowledge, understandings to students and teaching strategies) and what the related outcomes were.

An examination of the results of the PAB achieved by the mathematical teachers and teacher candidates revealed that the PAB had been largely inadequate for teachers and teacher candidates. With the increase of professional experience, the inadequacy of the PAB was reduced, different teaching approaches and course designs had a positive impact on the development of the PAB. It was concluded that the training courses offered for the teaching profession positively improved the PAB, and that teachers' perceptions of competence had an impact on the level of PAB. It also found that teachers use operational information about the content more than conceptual information. Teachers tend to provide teacher-centered teaching due to PAB deficiencies.

The program knowledge component in postgraduate theses, based on PAB models; the achievements in the context-oriented training program were expressed as the distribution of subjects by class level, pre-condition relations, the order in which the teaching was to be realized, the selection and use of teaching materials in accordance with the objective of the subject and establishment of in disciplinary and interdisciplinary conceptual relations. It was determined that 20 percent of the examined graduate these included the program knowledge component in their research. In the dimension of program knowledge, it was found that the outcomes of the context-oriented training program, the distribution of the subjects by class level, prerequisite relationships and the order in which they should be taught were examined. However, within the scope of program knowledge, it has been noted that the educational program and skills, information variables for using teaching materials (computer-aided materials, concrete materials, etc.), and the dimensions of interdisciplinary relationship information have not been reviewed.

It was determined that 60 percent of the theses examined within the scope of the content knowledge, which is the sub-component of the PAB model, included the content knowledge size in their research. In this dimension, we examined information on subjects to be taught, methods used for problem solving, information on concepts, information on the field of use and connections between concepts, knowledge of problem solving and problem-solving skills, knowledge of model usage and methods of proof, knowledge of the main proof charts they have.

Understanding undergraduate thesis based on PAB models; students' ability to sense preliminary information is expressed as being able to recognize problems, errors, misconceptions and reasons for learning. Lesseig (2016) discusses its understanding of the student for demonstration in its model. It was determined that 96 percent of the theses examined included the student in his studies on the cognitive knowledge component.

It was noted that the dissertation addressed the understanding component of students with the extent to which they identify learning difficulties, errors, misconceptions, and reasons, but not with respect to the interest, attitudes, and motivations of students in the course (Uçar, 2019; Uz, 2019; Tosuncu, 2019. Yurtyapan 2018: Duran, 2018. Amaç 2018. Şimşek, 2016; Özdemir Baki, 2017: Sahin, 2016). Studies on understanding the student have revealed that teachers' competence in identifying students' preliminary information about a certain subject, students' perceptions, motivations and attitudes are not reviewed, and students' misconceptions and difficulties are limited to knowledge.

The background information on teaching strategies, which is the sub-component of the PAB model, is expressed as the teacher’s general knowledge to transfer to students, dispensing students’ misconceptions and teaching methods, strategical and technical knowledge used by teachers. It was noted that 84% of the examined graduate these included the teaching strategies knowledge component in their research. It was concluded that some studies examined the knowledge of knowledge of instructional strategies solely in terms of eliminating the misconceptions of students on the subject but did not examine the concepts effectively as part of the design of suitable learning environments, and the use of appropriate methods and strategies. The studies that examine the knowledge of knowledge of instructional strategies in terms of the teaching methods, strategies, and technical information used by teachers did not involve the use of technology. Yazıcı (2017) study alone showed that the knowledge of teaching strategies was examined in terms of their ability to use technological materials or programs.

In the examination of results regarding program knowledge, it was concluded that program knowledge of the prospective teachers or teachers is insufficient or adequate, program knowledge has become significantly different depending on the license program variable registered, the inadequacy of program knowledge has decreased due to the increase in professional experience, e-mentoring has a positive effect on the program knowledge components (information on the achievements in the content-related training program, prerequisite relations and the sequence in which education should be realized).

Looking at the scope of program knowledge reviewed in the studies, we see that program knowledge is examined in terms of information related to the algebra, function, and data processing learning areas. It was determined that program knowledge was not considered in terms of the knowledge of gains in the educational program for a given content, knowledge of the distribution by class levels of the subjects, prerequisite relationships and what order teachings should be taught, knowledge of the skills the training program is intended for, knowledge of basic knowledge (operational/conceptual) and information on how to prepare content of a course for the skills (reasoning, association, communication...), knowledge of the use of teaching material in the underlying PAB models, which is analyzed.

When the results obtained from these studies, we concluded that in the vast majority of the studies in content knowledge, prospective teachers or teachers have insufficient content knowledge, there is a decrease in the insufficiency of content knowledge due to increasing professional experience, the processing knowledge of content is more used than conceptual information, different teaching approaches and course designs have a positive impact on the components of content knowledge, there are sufficient content knowledge in some areas (areas for model use in fractions), competency perception affects space information positively, and the inadequate information perception of students and suitable guidance can be given to students. Upon examining the results regarding knowledge of understanding student, we found that the understanding of students by teachers or prospective teachers is largely insufficient, that inadequate teacher understanding information is learned about students due to inadequate teacher understanding skills, that different teaching approaches and course designs have a positive impact on the components of understanding the student, that there are significant differences about the program variable of which the understanding of the student is recorded, that the perceptions of teacher competencies and conceptual knowledge have a positive effect on the knowledge of understanding student, and that due to improved professional experience, there is a lack of understanding of students.

Upon examining the data on the results of the components of the knowledge of instructional strategies, we found that the instructor or teacher candidates' instructional strategies had a largely insufficient understanding, that different teaching approaches and course designs had a positive impact on the components of the education strategies, that the education strategies had a significant effect on the program variable of record, that the education strategies of prospective teachers varied correctly with the class level, and that this variation had an effect on the application classes for the teaching profession, that the competency perceptions of teachers had a positive effect on the teaching strategies and that the insufficiency in teaching strategies had decreased.

The PAB is an important element for teaching profession qualifications. However, there are differences in theories. The reason for this is considered to be related to the language, association and representation characteristics of the discipline. While content information is a prerequisite for linking special content with other content subjects and concepts, understanding the student and learning strategies may differ depending on the content chosen. Therefore, the PAB must be examined and evaluated as a whole. Whereas, in other studies conducted in literature and in the postgraduate theses examined, it is observed that the PAB components sometimes focus only on the knowledge of the field or student mistakes. Similarly, to the results shown, in most of the systematic analysis or meta-synthesis studies in the literature there seems to be a PAB concept following Shulman (Depaepe et al., 2013). Similar to the results obtained after examining the studies conducted in the literature, teachers' PAB is not sufficiently adequate (Dönmez, 2009; Sezer, 2012; Eroğlu, and Tanışlı, 201; Çıkrıkçı, 2015; Köklü, 2008; Şimşek and Boz (2016) it has been established that prospective teachers or teachers have difficulty in identifying and eliminating misconceptions (Şahin, Erdem, Başıbüyük, Gökkurt, & Soylu, 2014; Güler, 2014: Şimşek ve Boz, 2016). It was determined that the range of approaches proposed by the teachers regarding the teaching of the subject was far from conceptual perception and that plain narration and rote learning were preferred. Furthermore, PAB has been observed to develop gradually during university education (Bulut, 2021). In-service training has also been found to be significantly contributing to PAB's development (Altaylı, Konyalıoğlu, Cihan, Hızarcı, and Kaplan, 2014). The most striking result in the field is that the content knowledge of teachers and teacher candidates is not sufficient for math teaching. Because the concept of mathematics is operational, the explanations of teachings remain at the operational level. It has also been established that a linear relation between PAB and professional experience was created due to lack of content knowledge making it difficult for students to understand and analyze their mistakes (Cankoy, 2010; Köklü, 2008).

Mathematical understanding is known to be problematic, and many abstract concepts include forms of representation. It is important that the PAB of teachers and teacher candidates develop and improve in math teaching. There is a need for teachers who are skilled in recognizing and resolving the errors or mistakes of their students and who can use a variety of teaching strategies. Furthermore, it is necessary to create a relation between the concepts and achievements in the training program and to pay attention to these elements in the design of the teaching process. Studies show that PAB gradually develops within the licensing process and within the teaching profession. Accordingly, teachers who develop themselves while being aware of the PAB must keep up with new education strategies and methods as well as innovative technologies.

Methodological tendencies, study areas, and results of postgraduate theses on the PAB in mathematics education were established with a current perspective. In this respect, it is recommended that comprehensive studies that identify and develop the PAB are conducted. Course content can be developed to improve the PAB as part of the updated new higher education program. Self-regulatory skills of teachers and teacher candidates may be improved. Mathematical educators can work together to develop a common language and educational policy around the PAB. This way, the results obtained can be evaluated and integrated into the teaching process.

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Türkiye'de Matematik Eğitiminde Pedagojik Alan Bilgisi Konulu Yüksek Lisans Tez Çalışmalarında Eğilimler: Sistematik Bir İnceleme

Özet:

Bu çalışmanın amacı Türkiye'de 2016-2021 yılları arasında ulusal tez merkezinde yayınlanmış matematik eğitimi araştırmalarında pedagojik alan bilgisini konu alan lisansüstü tezlerin sistematik olarak incelenerek değerlendirilmesidir. Bu doğrultuda lisansüstü tezleri belirli kriterlere uygun şekilde incelemek amacıyla nitel araştırma yöntemlerinden sistematik inceleme yöntemi temel alınmıştır. Çalışmanın verilerini elde etmek için Yüksek Öğretim Kurumu Başkanlığı ulusal tez veri merkezinde yer alan matematik eğitiminde pedagojik alan bilgisi konulu 26 lisansüstü tez PRISMA bilgi akışı referans alınarak analize dahil edilmiştir. Çalışmanın amacı doğrultusunda seçilen tezler oluşturulan çalışma değerlendirme formu çerçevesinde incelenmiştir. Bu kapsamda lisansüstü tezlerin künyesi (referans, yazarlar, yıl vb.), amacı, metodolojik unsurları (örneklem sayısı ve grubu, modeli, veri toplama araçları, veri analiz yöntemi) ve sonuçlar (kullanılan pedagojik alan bilgisi modeli alt boyutlarına ilişkin öğrenme çıktıları) analiz edilmiştir. Pedagojik alan bilgisini inceleyen teorisyenlerin ortaya koyduğu pedagojik alan bilgisi alt bileşenleri olan program bilgisi, alan bilgisi, öğrencileri anlama bilgisi ve öğretim stratejiler bilgisinin lisansüstü tez çalışmalarında nasıl ele alındığı ve sonuçları ayrıntılı olarak değerlendirilmiştir.

Anahtar kelimeler: pedagojik alan bilgisi, matematik eğitimi, lisans üstü tezler, sistematik inceleme

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Araştırma Makalesi/Research Article

On Eigenvalue And Eigenvector Perceptions of Undergraduate Pre-service Mathematics Teachers

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Abstract – Analysis of systems that can be expressed in matrices is very important in the field of application. Whether such a system works properly is determined by eigenvalues of the matrix representing the system. Eigenvalue and eigenvector concepts are taught within the scope of linear algebra course at undergraduate level. In this study, perceptions of undergraduate students who took the linear algebra course about the eigenvalue-eigenvector concepts are investigated. The research is conducted with the participation of 95 students from the Faculty of Education, Department of Mathematics Education. A scale measuring the students' approached about eigen theory was developed. For the reliability of the scale, Kuder-Richardson 20 (KR-20) reliability analysis was done and 0,72 was obtained. To see the relationship between learning outcomes and academic achievement is used the chi-square test and descriptive analysis are made in the study. Problems arising in the perception of eigenvalue and solutions are presented.

Key words: linear algebra, matrices, eigenvalue, eigenvector.

Introduction

Undergraduate level students first encounter the eigenvalue and eigenvector concepts in linear algebra classes. Linear algebra is one of the first undergraduate courses that students have to understand theoretically systematically, so it is cognitively a frustrating experience for both students and supervisors (Stewart & Thomas, 2011; Thomas & Stewart, 2011). The students taking the course face two basic difficulties, cognitive and conceptual difficulties related to the nature of linear algebra (Dorier & Sierpinska, 2001). Stewart and Thomas (2011) stated that this course was very intense for students and they found it difficult to relate the definitions and theoretical results to their previous knowledge. According to these authors, students' difficulties are due to the fact that many teachers and texts do not introduce the geometry of eigenvalues and eigenvectors. When geometric representation is left aside and the relation of algebraic and geometric representation is not discussed, many students learn these concepts mechanically, while when both representations are considered in the teaching of these concepts, students understand these concepts better (Salgado, 2015). International studies show that linear algebra students often develop an analytical-arithmetic thinking style that leads to the development of procedural knowledge (Gol Tabaghi & Sinclair, 2013). The fact that mathematical equations are not considered geometrically in two and three-dimensional spaces prevents students from developing their synthetic-geometric thinking skills. The relationship between mathematical operations and their geometrical equivalents should be given in the field of algebra in the most intense and clearest way. The resulted difficulties in the linear algebra course are described in Dorier et al. (2000): "The main criticisms made by the students toward linear algebra concern the use of formalism, the overwhelming amount of new definitions and the lack of connection with what they already know in mathematics". Although students can perform operations that require calculation in linear algebra, they have difficulty understanding concepts and establishing relationships between concepts (Dorier, 1998; Harel, 1989). Thomas and Stewart (2011) show that students generally do not understand the meaning of eigenvalue and eigenvector definitions and try to algebraically change symbols without understanding these concepts. Moreover, they observed that many students did not know the geometric image of the eigenvalue and eigenvector. In linear algebra courses, students have difficulties in topics such as linear dependence and independence, row and column spaces, and vector spaces of matrices, geometric interpretation of the action of linear transformations (Carlson, 1993). Larson et al. (2007); Larson, Zandieh and Rasmussen (2008) studied the way students approached a basic equation about eigen theory. The results of these few studies have inspired us to identify the problems.

The learning outcomes of the linear algebra course on eigenvalue and eigenvector can be expressed in the following 7 items:

i. Defines the concepts of eigenvalue and eigenvector

ii. Expresses the explicit state of the system of equations given in implicit form

iii. Uses the theoretical knowledge about the existence of the solution of the system

iv. Graphically interprets a system of linear equations *v.* Algebraically calculates the eigenvalues of the two-dimensional square matrix

vi. Knows that if the eigenvalues of a square matrix of dimension 2 are complex, the eigenvectors corresponding to these eigenvalues cannot be represented in the Euclidean plane

vii. Knows that a square matrix has at least one eigenvalue

In order for the eigenvalues and eigenvectors corresponding to a square matrix to be well understood and correctly determined, these outcomes must be realized. In this study, the perceptions of the undergraduate students are investigated by asking in line with the outcomes in these 7 items. For this purpose, the students are asked questions about the evaluation of the above items. The answers given to these questions are evaluated.

Let us express some remarkable points about the mathematical definitions of eigenvalue and eigenvector concepts. In linear algebra and engineering mathematics books, the concepts of eigenvalue and eigenvector are usually started like the question "When does the image of an x vector become a scalar multiple of itself under linear transformation given by a square matrix?". In basic linear algebra and engineering mathematics books, the term scalar is used in the introduction to the subject, and it is clarified in the next chapters that the term scalar corresponds to what kind of numbers (e.g. Meyer 2000; Burden, 1993; Bretscher, 2013; Szabo, 2000; Kreyszig, 2006). Since the subject is not usually mentioned in courses, or because the students' attention is not drawn, our study shows that the students think that this scalar is a real number. However, with the scalar concept, a number associated with the field of the linear vector space in which the problem is addressed should be understood, which is a real or complex number. In practice, vector spaces that are usually built on a complex number field are studied. Let the n-dimensional real Euclidean space be $\mathbb{R}^n$ and the complex Euclidean space $\mathbb{C}^n$.

In the linear algebra book of Szabo (2000), the definition of eigenvalue and eigenvector is defined as follows:

> A real number λ is an eigenvalue of a real $n \times n$ matrix A if there exists a nonzero column vector $x \in \mathbb{R}^n$ for which $Ax = \lambda x$. A nonzero column vector $x \in \mathbb{R}^n$ is an eigenvector of a real $n \times n$ matrix if there exists a real number λ for which $Ax = \lambda x$ (p. 375).

According to this definition, for example consider the matrix $A = \begin{bmatrix} 3 & -5 \\ 1 & -1 \end{bmatrix}$ For all nonzero $x \in \mathbb{R}^2$ and $\lambda \in \mathbb{R}$, the equation $Ax = \lambda x$ does not satisfy. However, this matrix A can also be the coefficient matrix of a linear differential equation with a constant coefficient, and the λ scalar and the vector x must be obtained for the solution of this differential equation. It is not possible to find eigenvalues and eigenvectors of the system without working with complex numbers ($\mathbb{C}$) to obtain the solution of that system. In fact, each real number is also a complex number, and the basics of the complex number system are taught to students in both high school mathematics and general mathematics (calculus). When the eigenvalue and eigenvector definitions are given with mathematical notation as follows, it is easily understood which structure of the scalar and vector in $Ax = \lambda x$ equation:

Definition: Let A be a given $n \times n$ dimensional matrix. If a $\lambda \in \mathbb{C}$ and nonzero vector $x \in \mathbb{C}^n$ satisfy the equation $Ax = \lambda x$, then λ is called an eigenvalue of A and x is called an eigenvector of A corresponding to λ. (Horn & Johnson, 2013).

Using the definition, the eigenvalues and corresponding eigenvectors of matrix A given above are obtained as follows: $\lambda_1 = 1 + i$, $x_1 = (5,2 - i)^T$ ve $\lambda_2 = 1 - i$, $x_2 = (5,2 + i)^T$. (Here, "i" and "T" stand for the complex unit element and the transpose of the matrix respectively).

Linear transformations can be expressed in matrices, and in low dimensional examples, the image of the vector x can be displayed on the graph under this linear transformation. If the eigenvalues of the matrix are real, the transformed image of the eigenvector corresponding to that eigenvalue can be illustrated. For this, by using virtual manipulatives (e.g. GeoGebra application), the student can see any vector of this vector under transformation by selecting any vector that the student wants on the plane with the help of dynamic vectors. In this way, the student sees the definition of eigenvalue and eigenvector better by seeing the appearance of an eigenvector under transformation (Gol Tabaghi & Sinclair, 2013; Gueudet-Chartier, 2004). If the matrix has complex eigenvalues, it is not possible to show this situation to students with the help of graphics. For this reason, the definition of eigenvalue and eigenvector have been mentioned above can be given to students by carefully emphasizing the set of complex numbers. In addition to, real examples and matrix samples with complex eigenvalues can be solved in the course.

The research questions guiding this study are the following:

Q1: How do you express the definition of eigenvalue and eigenvector of a real matrix?

Q2: Let $B = \lambda I - A$ be an $n \times n$ matrix. Consider homogeneous equation $Bx = 0$. What is the dimension of the matrices x and 0?

$$B_{n\times n} x_{\square\times\square} = 0_{\square\times\square}$$

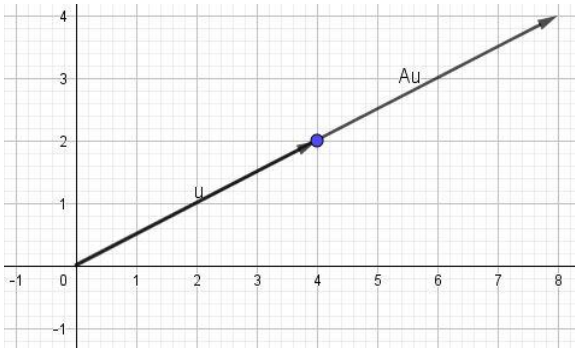
Q3: Let B be an n-dimensional square matrix and x be an n-dimensional vector. Does the equation $Bx = 0$ always have a solution?

☐ Yes ☐ No

Q4: Let B be an n-dimensional square matrix and x be an n-dimensional vector. What property is sufficient for matrix B to have an infinite number of solutions for the $Bx = 0$ system of equations?

☐ B is a square matrix ☐ $\text{trace}(B) = 0$
☐ $\det B = 0$ ☐ B is a diagonal matrix

Q5: For the matrix $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$ and $u \in \mathbb{R}^2$, the relationship between the vector u and the vector Au is given in the following figure.



Which of the following is exactly true according to this figure?

☐ A does not have an eigenvector ☐ Au is an eigenvector
☐ u is an eigenvector ☐ A does not have an eigenvalue

Q6: Which of the following is an eigenvalue of matrix $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$?

☐ 2 ☐ -2 ☐ 3 ☐ -1

Q7: Consider the matrix $A = \begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}$. For every vector u in the xy-plane, if the vector Au is not a multiple of the vector u, can you say "The matrix A does not have an eigenvector"?

☐ Yes ☐ No

Q8: Which of the following is true about the eigenvalues of an $n \times n$ dimensional matrix A?

☐ Infinite number ☐ At least one ☐ It may not have any eigenvalues

Method

The research design is quantitative, it is based on a descriptive analysis method since it puts forward the existing situation. Knowing linear algebra demands that the student starts thinking about the objects and operations of algebra not in terms of relations between particular matrices, vectors and operators but in terms of whole structures of such things: vector spaces over fields (real or complex), algebras, classes of linear operators, which can be transformed, represented in different ways, considered as isomorphic or not, etc. (Hillel & Sierpinska, 1994). Since the aim of this study is to measure the level of knowledge about the concepts of eigenvalue and eigenvector, a scale that includes the basic concepts mentioned above and related to these concepts is proposed. The suggestions in Gol Tabaghi and Sinclair (2013) were also used in the preparation of the scale.

The study group consists of 95 randomly selected third and fourth year students from Mehmet Akif Ersoy University Mathematics Education Department students who were taken a linear algebra course in Turkey in the 2021-2022 academic year.

In the study, a test consisting of 8 open-ended and multiple choice questions has been applied to determine the status of students who took the linear algebra course in which the concept of eigenvalue had been given before, in perceiving these concepts. In the preparation of the test, linear algebra textbooks and three field experts teaching linear algebra at these universities have been considered. In the light of the comments received, three of the 10 questions in the draft have been omitted and one question has been added. The test includes the definitions of eigenvalue and eigenvector concepts and questions about the calculation of eigenvalues and eigenvectors of a matrix.

The questions Q3 and Q4, which measure the existence of a solution based on theoretical knowledge, are questions that corresponds to the same learning outcome. Except for these two questions, each question correspond to the different sub-outcomes respectively. The question Q1 evaluates first outcome, Q2 second outcome, Q3 and Q4 third outcome, Q5 fourth outcome, Q6 fifth outcome, Q7 sixth outcome and Q8 seventh outcome.

A general evaluation of the students' answers has been made as "correct, incorrect, none". The student's answers have been examined in detail. It is determined that difficulty indexes of the questions are between 0.17 and 0.76. For the reliability of the scale, Kuder-Richardson 20 (KR-20) reliability analysis was done. The KR-20 reliability coefficient of the test is calculated as 0.72. According to Fraenkel and Wallen (2008), the obtained measurements are reliable. Also, data processing was carried out by using the chi-square test to determine the relationship between learning outcomes and student academic achievement with alpha degrees = 0.05 through the SPSS program. In the line with the results obtained from the analysis, inferences have been made on the perceptions of the students about eigenvalue-eigenvector and the reasons for the problems experienced.

Findings and Discussions

The percentage of correct answers given to the questions prepared by considering the solution of the equation system, the existence of the solution of the system, the root number of the polynomial, the basic theorems and the concepts have been mentioned above, are evaluated.

In question Q1 the students are asked to describe the definition of eigenvalue and vector of a real matrix. The set of complex numbers has not been mentioned in most of the students' answers. Our study shows that students think that the scalar number is a real number. However, the concept of scalar number represents the number belonging to the field of linear vector space, and this can be a real or complex number depending on the problem being studied. Student T has described it as (Figure 1):

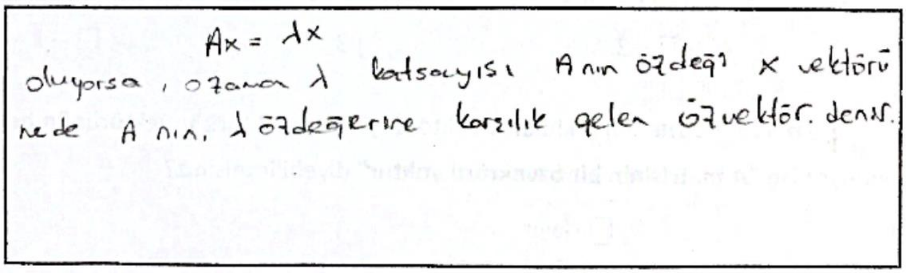


Figure 1 The Answer of the Student T to Question 1.

As it is seen, the student T has only written the equation and stated the scalar number λ is an eigenvalue and the vector x is an eigenvector. In the definition has not been given information from which space to take of x and λ. Student B has replied as (Figure 2):

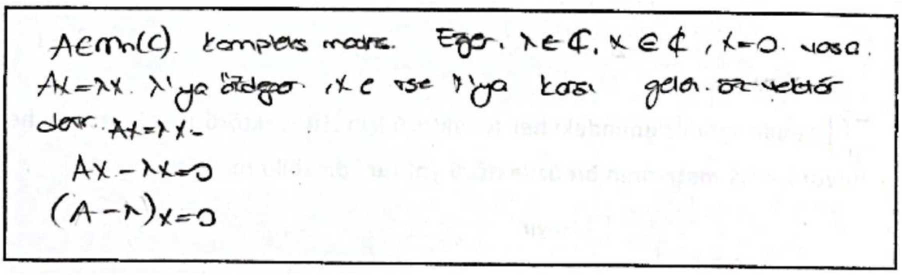


Figure 2 The Answer of the Student B to Question 1.

In this answer, λ have been taken from $\mathbb{C}$ but the vector x has not been taken from $\mathbb{C}^n$.

It is seen that students answer the questions Q3, Q4 and Q6 correctly in the range of 40 to 65 percent. They are relatively successful in analytical arithmetic operations. It is observed that they are less successful in problems that require "analytical-structural thinking" (Sinclair & Gol Tabaghi, 2010) skills, namely in Q1, Q2 and Q8.

Question Q8 is quite confusing for students. In fact, they have to state that a matrix has at least one eigenvalue. 24 percent of the answers to question Q8 are "It may not have any eigenvalues". It can be considered that a student who gives this answer comes to the conclusion that "if the polynomial has no real root, it has no root". Students who cannot give the correct answer correspond to a rate of 61 percent. On the other hand, the definition of eigenvalue is requested in Q1. It is seen that complex numbers are not expressed in 66 percent of the answers given for this definition. Based on this, it can be said that most students ignore the fact that polymomas can have a complex root.

In Q5, the real eigenvalue of a matrix and its corresponding eigenvector are given graphically. This question has two purposes, to see if students could carry out the state given by graph (visually) and whether they could write an equation system form. From the answers given to Q5, it is seen that 51 percent of the students are able to connect arithmetic calculation and visual representation. Only a few of the 48 students who correctly answer this question interprete the figure directly. Student A is able to link the arithmetic calculation and visual representation (see Figure 3).

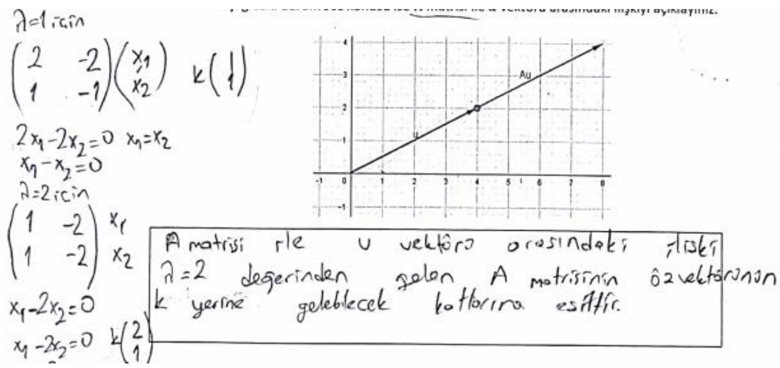


Figure 3 The Answer of the Student A to Question 5.

The most blatant feature of the students' practical thinking is their tendency to base their understanding of an abstract concept on 'prototypical examples' rather than on its definition. For example, linear transformations were understood as 'rotations, dilations, shears and combinations of these'. This way of understanding made it very difficult for them to see how a linear transformation could be determined by its value on a basis, and consequently, their notion of the matrix of a linear transformation remained at the level of procedure only (Dorier, 2002).

In the question Q7, the fact that a situation like Q5 does not occur is discussed. The eigenvalues of the matrix given in this question are complex. It can be evaluated that the graph given in Q5 is effective in the correctness of 56% of the answers given by the students. Accordingly, it can be thought that the visual handling of the concepts contributes positively to learning and reaching conclusions.

It is remarkable that students generally have answered question Q7 correctly, but many students have answered question Q6 wrong. Student H has not answered the question correctly; but has tried to link the information to the discriminant concept such that if the discriminant is less than zero, it has no eigenvalue (Figure 4).

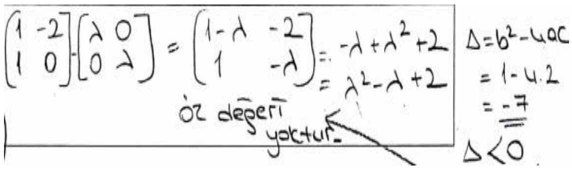


Figure 4 The Answer of the Student H to Question 6.

In the definition, the eigenvalue must be taken as a complex number. Since students generally think that the root of a polynomial is only a real number, these eigenvalues of a matrix with complex eigenvalues are ignored. In fact, if the eigenvalues of a matrix are desired to be two-dimensional and only complex eigenvalues, the answer is “this matrix has no eigenvalue”.

According to the data obtained as a result of the analysis of the answers given to the questions with the Chi-square test, there is a significant relationship between the questions Q1 and Q4; Q1 and Q8; Q2 and Q3; Q5 and Q7; Q7 and Q8.

The questions aiming to evaluate seven learning outcomes are evaluated. The following Table 1 reflects students’ percentage of correct answers and their standard deviations. Since Q3 and Q4 target the same learning outcome, the percentage of correct answers to Q5 and Q7 is calculated taking an average of correct answers of these questions.

Table 1 *Descriptive Statistics of Students (n=95)*

Learning Outcomes	**Questions**	**Percentage of Correct Answers**	**Standard Deviation**
Defines the concepts of eigenvalue and eigenvector	Q1	34%	0.724
Expresses the explicit state of the system of equations given in implicit form	Q2	17%	0.729
Uses the theoretical knowledge about the existence of the solution of the system	Q3 and Q4	64%	0.495

Graphically interprets a system of linear equations	Q5	51%	0.614
Algebraically calculates the eigenvalues of the two-dimensional square matrix	Q6	41%	0.597
Knows that if the eigenvalues of a square matrix of dimension 2 are complex, the eigenvectors corresponding to these eigenvalues cannot be represented in the Euclidean plane	Q7	56%	0.581
Knows that a square matrix has at least one eigenvalue	Q8	39%	0.509

As can be seen from Table 1, the percentage of correct answers to the question about the existence of at least one eigenvalue of a square matrix is 39%. In addition, the percentage of students who fully expressed the definition of eigenvalue is obtained as 34%. These two values are the lower percentages in the table.

The percentage of correct answers to the question about implicit representation of linear equations via matrix form which is one of the basic subjects of Linear Algebra course is 17. One reason why the ratio is low can be interpreted as the lack of understanding of the concept of dimension. On the other hand, the percentage of correct answers given to the questions about theoretical knowledge of linear equations system, namely basic theorems and concepts of Linear Algebra course and the visual representation of equations system are 64 and 51 respectively. The image of an eigenvector corresponding to a real eigenvalue under a linear transformation is different from the images of other vectors. The relatively high percentage of this problem, such as 51, may be a result of the graph given. The effect of graphic can also be seen in the percentage of correct answers given to the question about the eigenvector of the matrix with the complex eigenvalue, which is 56 percent. In general, students investigate the solution of the problem based on the formula, and if the result of the formula cannot make sense, they choose the negative one among the answers (Figure 4). This is why the percentage of correct answers given to this question is 39 percent. Their success is low as can be seen from the percentage of correct answers to the questions in the analytical structure. This result is consistent with the conclusion Sinclair and Gol Tabaghi (2010) has stated

> Analytic-arithmetic thinking involves describing a proper set-up to carry out computations and specifies an object by a formula. Synthetic-geometric thinking involves using geometric descriptions to visualize mathematical objects in two and three-

dimensional space. Analytic-structural thinking involves thinking about an object in terms of its properties (p. 149).

When dealing with the solution of a mathematical problem, students avoid evaluating the problem geometrically (Gueudet-Chartier, 2004). In one of the questions given to the students, there is a vector in the plane and the image of that vector under a linear transformation. An eigenvalue of the matrix corresponding to the linear transformation and an eigenvector corresponding to this eigenvalue is expected to be easily and accurately expressed by the students. Whereas the students have preferred to convert the given graph to an algebraic equation so they could not reach a solution. In calculus or geometry lessons, students examine many different examples such as straight-line graphs, quadratic equations, function graphs, and solutions of linear equation systems, etc. In this question, it was seen that these students had difficulties with representing a vector by point or arrow (Gueudet-Chartier, 2004; Brousseau, 1998). Gol (2012) concluded that use of the software stimulated both dynamic imagery and different communication strategies. The experience allowed the students to see vectors' direction and position on the plane; to analyze the behavior of vector x and its transformation under the matrix A.

Whether the answers to the questions are dependent on each other is measured by the Pearson's Chi-Square test. The results of the Pearson's Chi-Square test:

Table 2 *Answers to Q1 and Q4*

		Q4			
		empty	*wrong*	*correct*	*Total*
Q1	*empty*	2	5	12	19
	wrong	0	15	29	44
	correct	0	1	31	32
	Total	2	21	72	95

For Q1 and Q4, p value is 0.001 so there is a meaningful relationship between the answers to these questions. The equation for eigenvalue-eigenvector is a homogeneous equation system. The students who are aware of this relationship gave generally correct answers.

Table 3 *Answers to Q1 and Q8*

		Q8			
		empty	*wrong*	*correct*	*Total*
Q1	*empty*	1	14	4	19
	wrong	0	29	15	44
	correct	0	14	18	32
	Total	1	57	37	95

The *p* value is obtained as 0.032. Students who have paid attention to complex numbers into account in the definition have answered the question Q8 correctly.

Table 4 *Answers to Q2 and Q3*

		Q3			
		empty	*wrong*	*correct*	*Total*
Q2	*empty*	23	17	40	23
	wrong	20	19	39	20
	correct	2	14	16	2
	Total	45	50	95	45

For Q2 and Q3, the *p* value is obtained as 0.008. The number of wrong answers to both questions is high. This result shows that the students can not make sense of the equations given in matrix form.

Table 5 *Answers to Q5 and Q7*

		Q7			
		empty	*wrong*	*correct*	*Total*
Q5	*empty*	2	1	3	6
	wrong	0	24	17	41
	correct	2	13	33	48
	Total	4	38	53	95

The p value is 0.000 for these questions Q5 and Q7. The general tendency of the correct and wrong answer is the same. The number of correct answers to Q7 is very high. Here, it is seen that the graph given in Q5 has a positive effect on the interpretation of the question. The visual handling of the concepts contributes positively to learning and reaching conclusions.

Table 6 *Answers to Q7 and Q8*

		Q8			
		empty	*wrong*	*correct*	*Total*
Q7	*empty*	1	1	2	4
	wrong	0	29	9	38
	correct	0	27	26	53
	Total	1	57	37	95

For Q7 and Q8, *p*=0.000. In Table 6, it is seen that students who have given the wrong answer to Q7 generally have given the wrong answer to Q8. More than half of the students do not think that it can be a complex eigenvalue-eigenvector of a matrix.

Conclusions and Suggestions

In the study, it is seen that the majority of the students can not define eigenvalue and eigenvectors. It is determined that 17 percent of the students are able to express the explicit state of the equation system given in an implicit form and the image of a vector under linear transformation, 64 percent of the students are able to use the theoretical knowledge about the existence of the solution of the system, 39 percent of students knew that a square matrix has at least one eigenvalue. In addition, it is seen that more than half of the students are able to interpret the linear system of equations graphically, and with the help of this graphic. They have determined that the eigenvectors of a square matrix with complex eigenvalues could not be represented in the plane.

The system of equations in the eigenvalue definition is a homogeneous system of equations. The problem of finding an eigenvalue and its corresponding eigenvector requires the existence of an infinite solution of the corresponding homogeneous system. It is seen that the students who know the definition of eigenvalue and eigenvector also know that the determinant of the coefficient matrix must be zero. When the answers to the first 6 questions are examined, it is seen that the students have basic knowledge about obtaining the characteristic polynomial of a matrix. However, it is determined that 61 percent of the students have not known how many roots a polynomial should have. As a result, there are deficiencies and mistakes in the questions about the existence of the complex root. In order to solve these problems, it has been taught that it would be useful to mention the complex situation, to give explanatory examples, and to interpret the solutions geometrically while discussing the concept of eigenvalue in the lectures. In addition, it emerges from the results that sufficient attention should be paid to the examples of matrices with complex eigenvalues in the books. The fact that students generally deal with real numbers and consider the set of complex numbers at a secondary or even superficial level may cause complex roots to be ignored in root calculations.

Among the prepared questions, there are two questions about calculating the matrix eigenvalue. The first one has real eigenvalues and the second one has complex eigenvalues. The matrix $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$ have real eigenvalues and the equation $Au = \lambda u$ for this matrix is provided for example $\lambda = 1$ and $u = [2,1]$ vector. Thus, students can see the equation $Au = \lambda u$ in $\mathbb{R}^2$ plane. However, in another question, no matter which $u \in \mathbb{R}^2$ $(u \neq 0)$ vector is taken for the matrix $A = \begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}$, the equation $Au = \lambda u$ is not provided. In this case, students can naturally say that this matrix has no eigenvalue. From the evaluation results, it appears that while dealing with the concept of eigenvalue in the lectures, the complex situation should be mentioned, illustrative examples and geometric interpretation of the solutions, as well as more attention should be paid to such examples in the books. The fact that the complex numbers are not addressed sufficiently in the course causes the most basic structure in eigenvalue and eigenvector definitions to be neglected. Students working mostly on real numbers and leaving the complex number system at the second level or even the basic level may cause the roots to be ignored during calculation of the polynomial root.

The concreteness that seems to lack in linear algebra could be more efficiently provided by the use of drawings, especially drawings illustrating concepts and properties in abstract vector spaces (Gueudet-Chartier, 2004). In the case of eigenvectors, when students can compare eigenvectors related to different eigenvalues or when they can determine linear independence of different vectors associated to an eigenvalue, they show evidence of having constructed a process (Salgado, 2015). Larson et al. (2008) mentioned the difficulty involved in transforming $Av = \lambda v$ into $(A - \lambda I)v = 0$ and later into $|A - \lambda I| = 0$. Our results are consistent with results presented by the studies carried out by Salgado (2015), Stewart and Thomas (2011), Gol (2012), Gueudet-Chartier (2004), and Larson et al. (2008). In the research conducted by Gol Tabaghi and Sinclair (2013) it was determined that students had difficulties in making the formal definition of the concepts of eigenvalue and eigenvector, understanding the behavior of the eigenvector, and establishing the connection between these concepts with the concept of linear transformation, as well. Perhaps before the eigenvalue and eigenvector concepts are given, even before the solution of linear systems, the fundamental theorem of algebra about how many roots of an n^{th} order polynomial should be given. As Tabaghi and Sinclair (2013) concluded, the use of software contributes to a deeper understanding of abstract concepts. Approximate roots of high-order polynomials as well as second-order samples should be sampled with appropriate mathematical software (for example Maple, Matlab, GeoGebra, Sketchpad), and then matrices should be addressed.

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Lisans Matematik Eğitimi Öğrencilerinin Özdeğer ve Özvektör Algıları Üzerine

Özet:

Matrislerle ifade edilebilen sistemlerin analizi, uygulama alanında oldukça önemlidir. Böyle bir sistemin düzgün çalışıp çalışmadığı sistemi temsil eden matrisin özdeğerlerine göre belirlenmektedir. Lisans seviyesinde özdeğer ve özvektör kavramlarının öğretimi doğrusal cebir dersi kapsamında yapılmaktadır. Bu çalışmada doğrusal cebir dersini almış lisans öğrencilerinin özdeğer ve özvektör kavramları hakkındaki algıları araştırılmıştır. Araştırma Eğitim Fakültesi Matematik Eğitimi bölümü öğrencilerinden 95 öğrencinin katılımıyla gerçekleştirilmiştir. Öğrencilerin özdeğer teorisine yaklaşımlarını ölçen bir ölçek geliştirilmiştir. Ölçeğin güvenilirliği için Kuder-Richardson 20 (KR-20) güvenirlik analizi yapılmıştır ve 0,72 olarak bulunmuştur. Çalışmada öğrenme çıktıları ile akademik başarı arasındaki ilişkiyi görmek için ki-kare testi kullanılmış ve betimsel analiz yapılmıştır. Özdeğer ve özvektör kavramlarının algılanmasında ortaya çıkan problemler ve çözüm önerileri sunulmaktadır.

Anahtar kelimeler: lineer cebir, matrisler, özdeğer, özvektör.

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Araştırma Makalesi / Research Article

Design Study To Develop The Proof Skills Of Mathematics Pre-Service Teachers

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Abstract – Mathematical proof, in addition to its duties such as verifying and explaining a claim, helps to understand mathematics and constitute new information, thereby increasing its importance in mathematics education. Proving skills need to be developed to learn mathematics and find effective solutions in real-life problem situations. Given the importance of proof in mathematics education, it is critical to develop appropriate teaching practices to increase pre-service teachers' knowledge of proof, proving, and understanding levels, particularly in mathematics teacher training programs, to provide effective and permanent mathematics learning for future students. The effect of designed proof tasks on the development of pre-service mathematics teachers' proving skills was investigated in this study. This research, the design-based research, was carried out with the participation of three volunteer mathematics teacher candidates studying at a state university in Ankara. The data obtained from the proof tasks and individual interviews during four weeks period. Proof tasks designed according to the findings obtained from the data analyzed with the qualitative analysis method contributed to the development of the participants' proving skills.

Key words: mathematical proof, proving skills, proof task, pre-service mathematics teacher.

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Introduction

Mathematical proof is an important mathematical argument, a connected sequence of assertions against a mathematical claim, that has set of accepted statements as true and does not require justification, and employs known and valid reasoning forms, as well as forms of expression that are appropriate in communication (Stylianides, 2007). The development of proof skills is not only important in mathematics, but also among the primary objectives of the mathematics teaching curriculum (NCTM, 2000). The most important reason for this is that reasoning and proof help students relate their previous knowledge to new ones, make inferences and make sense of their new knowledge (Brodahl et al., 2020). Thus, learning and teaching proof is critical in mathematics education (Hanna & de Villiers, 2008, 2012; Yan, Mason & Hanna, 2017) in terms of providing learning and making sense of mathematics, as well as assisting in the discovery of effective solutions in real-life problem situations (Mariotti, 2006).

In the process of mathematical proof, there are processes such as verifying a proposition, explaining why it is true, systematizing the results obtained (de Villiers, 1990; Selden & Selden, 2003), discovering new results and hypotheses through inferences, and expressing the results using mathematical language and notation. (Harel and Sowder, 2007; Ko and Knuth, 2009). This complex nature of the proof process (Moore, 1994; Dreyfus, 1999) causes students, pre-service teachers and even teachers to experience difficulties in this process. (Knapp, 2005). In the conducted studies, it was revealed that difficulties such as not knowing where to start proving and how to continue (Healy and Hoyles, 2007), not using the existing preliminary information strategically (Weber, 2001), lack of knowledge of definition (Dane, 2008; Polat and Akgün, 2016; Cihan, 2019), inability to use mathematical language and notation (Moore, 1994; Knapp, 2005), inability to determine the appropriate proof method (Baki and Kutluca, 2009), inability to understand the nature of proof (Çontay and Duatepe-Paksu, 2019), lack of mathematical concept readiness and lack of proof image (Pala, Aksoy and Narlı, 2021) occur. It is thought that the teaching methods, techniques, and tools used are effective in understanding the proof conceptually (Weber, 2004), gaining the ability to prove (Harel, Selden, & Selden, 2006) and overcoming difficulties experienced with proof (Hanna and de Villiers, 2008; Skott, Larsen and Østergaard, 2020). The difficulties that arise in the studies show that there are problems, deficiencies, or mistakes in teaching proof (Cihan, 2019; Zeybek Şimşek, 2020).

Related Literature

Although the difficulties experienced in the proving process are revealed in the studies, there are very few studies to find solutions to these difficulties and improve the current situation (Selden, Selden and Benkhalti, 2017; Yan, Mason and Hanna; 2017). Since the concept of proof is an important part of mathematics education, it is emphasized that appropriate teaching practices should be designed, effective application guidelines should be developed, and such researches should be increased to enhance pre-service teachers' knowledge of proof, ability to make proof, and concept comprehension levels, especially in mathematics teachers training departments (Yan, Mason and Hanna; 2017; Zeybek Şimşek, 2020). In this direction, it is important for pre-service teachers to deal with proof (Harel, Selden, & Selden, 2006), and to examine and develop the proving processes for the development of proof skills (Sarı Uzun, 2020). In the studies conducted in this field, proof schemes (Balacheff, 1988; Harel & Sowder, 1998; Weber, 2004), ways of thinking in the proof process (Raman, 2003), categories of explanations presented as proof (Miyazaki, 2000) have been constructed, and methods, strategies, and techniques (Dean, 1996; Schabel, 2001; Selden and Selden, 1995) for teaching proof have been presented. Many studies have been conducted in the recent past to use the methods, techniques, and strategies obtained as a result of these studies (Arslantaş İlter, 2020; Selden, Selden, and Benkhalti, 2017; Yan, Mason, and Hanna, 2017;), but no study has been found that provides a guide for the teaching of proof and the development of the proving process.

Proof tasks were developed and implemented in this study in response to the difficulties experienced by pre-service mathematics teachers during the proving process, such as not knowing where to begin the proof, not being able to continue the proof, using mathematical language and symbols, a lack of knowledge, and not being able to distinguish between hypotheses and judgments. These proof tasks are designed as a mathematical task (Greenberg, 1993; Weber, 2005) that covers the stages of applying inference rules and completing the proof until the desired result is obtained by providing some preliminary information (e.g., inferences, axioms, definitions).

Research Problem

This research is a part of a PhD dissertation that aims to improve pre-service mathematics teachers' proving skills. In the scope of this study, the contribution of designed proof tasks to the development of proof skills in pre-service mathematics teachers was investigated. For these purposes, the research problem and its subproblems are as follows:

- How did the designed proof tasks affect pre-service mathematics teachers' proving skills?
    - How did the 1st designed proof task affect pre-service mathematics teachers' proving skills?
    - How did the 2nd designed proof task affect pre-service mathematics teachers' proving skills?
    - How did the 3rd designed proof task affect pre-service mathematics teachers' proving skills?
- What are the opinions of pre-service mathematics teachers on design?

Method

This section covers the research method, stages, study group, data collection tools and data analysis.

Research Design

In this research, proof tasks including theorems involving basic mathematics subjects and designed as a guide in the proof process were applied to pre-service mathematics teacher in this research. The study focuses to contribute to the development of prospective teachers' proof skills with enhancing the design of these tasks by applying and evaluating it. In this sense, the model of the research was determined as design-based research (DBR).

As a constructivist approach that emphasizes learning by systematically designing teaching strategies and tools, design-based research (DBR) contributes to the creation, development, acceptance, and continuity of knowledge in learning environments (Brown, 1992; Collins, 1992). In this research, the ADDIE model (Branch, 2009), which includes the stages of analysis, design, development, implementation, and evaluation, was used as the DBR model.

The use of a design-based research approach (Doorman, Bakker, Drijvers & Wıjaya, 2016), which provides a systematic approach to the proof process and allows for the generalization of findings on specific contexts, was thought to be beneficial in emphasizing learning, creating, and developing knowledge.

Mathematical tasks are defined as tasks that provide a student with the opportunity to learn new mathematical content such as concepts and procedures or to develop mathematical processes such as analytical skills, creativity, and cognitive skills (Stylianides, 2016). According to the literature, mathematical tasks improve the quality of mathematics education and are an effective learning and teaching tool for learning mathematical concepts (NCTM, 2000; Stylianides and Ball, 2008; Stylianides, 2016). Therefore, in this study, it was considered that proof tasks are an appropriate tool for developing the proof skills of pre-service mathematics teachers.

Stages of Research

The analysis part of the DBR model, in which applications are made and evaluated to determine the current situation for pre-service mathematics teachers, the concept of proof and being able to make proof, and the difficulties and deficiencies experienced in the context of proof as a result of the analysis of the findings, are not presented within the scope of this research. However, it is explained in detail in the PhD dissertation.

The course content and proof tasks were prepared during the design and development phase as a result of the researcher's scan of the field studies (Almeida, 2000; Atwood, 2001; Sarı, 2011; Selden, Selden and Benkhalti, 2017; Selden, Selden and McKee, 2008; Stylianides and Stylianides, 2009), the examination of the relevant books (Hammack, 2013; Houston, 2009; Vellenman, 2006), and the expert opinions, in order to eliminate the deficiencies identified as a result of the applications, to assist in overcoming the difficulties encountered, and to prepare for the proof process.

Only a part of the implementation phase is included in this research. In this part, the explanation of the proving process with basic proof methods and the implementation of proof tasks were carried out in this part over a four-week period, with the researchers and participants meeting every week after the completion of the preparation for proof part, to achieve the following goals:

- To know and to apply the direct proof method.
- To know and to apply the method of proof by contradiction.
- To apply proof methods to fundamental subjects.

During the evaluation phase, the data obtained from the proof task practices in this study were evaluated, and interviews were conducted with three volunteers at the end of each proof task, using a semi-structured individual interview form developed by reviewing the relevant literature, considering the parts in the designs, and experts' opinions. In these interviews, they were asked to explain the proof process, which includes stages such as understanding the theorem, determining the appropriate proof method, determining the desired result, and the proof task inference process. Beside from that, participants were asked about the difficulties they encountered during the proof process, type of information they required, and their opinions on proof tasks. Following the evaluation of the findings, changes were made to the proof task design and implemented again with a different theorem.

Participants

In this research, volunteered three pre-service mathematics teachers who are at peace with symbolic logic, who can express themselves, who are willing to learning and improvement, were selected as the participants of the study, by taking the opinions of the instructors who gave proof-related courses at a state university in Ankara. During the research, pseudonyms as S1, S2 and S3 were defined for the participants in accordance with ethical rules.

Data collection

Data were collected through proof tasks and semi-structured individual interviews in the context of the courses mentioned above.

Proof tasks

Theorems determined by examining the related books (Houston, 2009; Hammack, 2013; Vellenman, 2006) and taking expert opinions are included in the proof tasks prepared to examine and improve the proving processes of prospective mathematics teachers. Three faculty members who are experts in the field evaluated the theorems for suitability and mathematical language, and their final form was determined based on their feedback. The use of different theorems in each design is critical for the development and objective evaluation of the proving process participants. The following are the theorems that were used for each design:

1. "Let A, B, C be sets and $A \setminus B \subseteq C$. Then $A \setminus C \subseteq B$."
2. "Let $f: X \to Y$ be a function and $A \subset X$. If f is one-to-one, then $f(X \setminus A) \subset Y \setminus f(A)$."
3. "*Let* $\lim_{x\to a} f(x) = A$ *and* $\lim_{x\to a} g(x) = B$. *Then* $\lim_{x\to a}[f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = A + B$ *dir.*"

Semi-Structured Individual Interviews

The purpose of semi-structured interviews is to thoroughly examine pre-service mathematics teachers' proof processes. Interviews were conducted three times, at the end of the first, second, and third proof tasks, using the semi-structured individual interview form, and audio recorded with the participants' permission. The interview form, which was prepared considering the literature review and the design components in mind, took its final form after three faculty members who were experts in the field weighed in on its suitability for the purpose, and the following questions were included in the form:

1. *Can you explain the proof task process step by step?*
2. *What kind of external sources did you use when you had difficulties while working on proof tasks? And how has the guidance you received from these outside sources helped you?*
3. *What kind of information would you need that you thought you would have done better if "....." was included in the proof tasks?*

In addition to these questions, during the last interview, the participants were asked some questions such as "*How would you describe your progress in proving in general through this study?*" and "*Did the tasks change your point of view towards proof? Please explain.*" to get the opinions of the participants.

Data Analysis

The research data obtained through proof tasks and recordings of interviews conducted with participants. Content analysis was used as a data analysis method. The data were analyzed by the arrangement of the data, coding of the data and the creation of themes, and the data were interpreted by associating them with each other.

The codes obtained during the analysis process were determined by the researcher's and consultant's shared opinions, and themes and categories were created.

Analysis of Proof Tasks

In the proof tasks, it has been requested that the participants should identify the propositions in the given theorem and express them as conditional propositions in the form of $p \Rightarrow q$, determine the hypothesis and conclusion, write the first sentence together with the reason as the introductory sentence of the proof process, make inferences from the first sentence, write the last sentence and the reason, and complete the proof. These steps determined for the process are presented in the proof task design. The following are the first proof task responses:

In the part of determining the components of the theorem, the expression of the theorem as $p \Rightarrow q$ is “A, B, C is a set and $A \setminus B \subseteq C$ is $A \setminus C \subseteq B$”, hypothesis (p) : "$A \setminus B \subseteq C$", conclusion (q) : "$A \setminus C \subseteq B$".

Let the first sentence, which is the introductory sentence to the proof process, be the hypothesis, that is, the assumption: "$A \setminus B \subseteq C$".

It is expected that the inferences that can be made in the proof process will be made by using the properties of the sets according to the selected proof method. For example, let "$a \in A \setminus B$" for direct proof. Therefore, it can be started as "$a \in A$" and "$a \notin B$". By using the inclusion property in sets, the desired result is obtained. Let's show that there is no "$A \setminus C \subseteq B$" according to the method of proof by contradiction. Let's take an element a, with "$a \in A \setminus C$" and "$a \notin B$". Hence "$a \in A, a \notin C$" and "$a \notin B$". From here,"$a \in A \setminus B$" and "$a \notin C$" are obtained. So to be "$a \in A \setminus B \subseteq C$" it becomes "$a \in C$" and contradicts "$a \notin C$". By making inferences in this way, a contradiction is obtained, and the desired result is achieved.

The last sentence of the proof process, namely the desired result, is "Thus $A \setminus C \subseteq B$".

Changes were made to the design of the second proof task in response to the evaluation of the first proof task and the data obtained from the individual interviews. Due to the difficulties encountered after writing the first sentence to start the proof, steps were added to the inference section for the step of writing the second sentence and for making inferences step by step while making inferences. As in the first proof task, the evaluation was done as true, partly true, and false for each step.

The second proof task responses are given below.

Answers for determining the components of the theorem; restate the theorem: "If f is one-to-one, then $f(X \setminus A) \subset Y \setminus f(A)$ for $\forall A \subseteq X$." Hypothesis (p): “f is a one-to-one function.” Conclusion (q): "$f(X \setminus A) \subset Y \setminus f(A)$”.

The introductory sentence of the proof process (assumption): "Let f be a one-to-one function."

The second sentence of the proof process (according to the proof by contradiction method): “Let us assume that there is no $f(X \setminus A) \subset Y \setminus f(A)$.”

The implications that can be made in the proving process are: “There is $\exists y \in X$ such that $y \in f(X \setminus A)$ and $y \notin Y \setminus f(A)$ (from the definition of inclusion). There is $\exists x \in X \setminus A$ such that $y = f(x)$ (from the image set definition). $y \in f(A)$ (from the difference set definition). There is $\exists a \in A$ such that $y = f(a)$ (from the image set definition). From here, if $f(x) = y = f(a), x = a$ and $x \in A$ contradiction is obtained."

The last sentence of the proof process: "Then $f(X \setminus A) \subset Y \setminus f(A)$".

In line with the evaluation of the second task and the data obtained from the interviews, it was determined that the participants had difficulties while making inference due to the lack of information about the proposition/theorem given. Accordingly, in addition to the second design, it was considered appropriate to provide the information that may be needed while making the proof, and an information box was added in the third design. Evaluation was made in the same way as the first and second designs.

The third proof task responses are given below.

"Let $\lim_{x\to a} f(x) = A$ and $\lim_{x\to a} g(x) = B$. Then

$\lim_{x\to a}[f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = A + B$"

Determining the components of the theorem; restate the theorem: "If $\lim_{x\to a} f(x) = A$ and $\lim_{x\to a} g(x) = B$ then $\lim_{x\to a}[f(x) + g(x)] = A + B$"

Hypothesis (p):$\lim_{x\to a} f(x) = A$ and $\lim_{x\to a} g(x) = B$"

Conclusion (q): "$\lim_{x\to a}[f(x) + g(x)] = A + B$"

The introductory sentence to the proof process (assumption):

" Let $\lim_{x\to a} f(x) = A \text{ and } \lim_{x\to a} g(x) = B$."

Second sentence of the proof process (according to the direct proof method): Give the number ε>0 (from the limit definition) to show that $\lim_{x\to a}[f(x) + g(x)] = A + B$.

Inferences that can be made in the proving process: $if \lim_{x\to a} f(x) = A \; then \; \exists\delta\_1 > 0 \; so \; if \; 0 < |x - a| < \delta\_1 \; then \; |f(x) - A| < \varepsilon / L \; and \; if \lim_{x\to a} g(x) = B \; then \; \exists\delta\_2 > 0 \; so \; if \; 0 < |x - a| < \delta\_2 then \; |f(x) - B| < \varepsilon / L$ (from the limit definition).

$\delta\_i = min\{\delta\_1, \delta\_2\}, if \; 0 < |x - a| < \delta \quad then \; |(f(x) + g(x)) - (A + B)| = |(f(x) - A) + (g(x) - B)| \leq |f(x) - A| + |g(x) - B| < \varepsilon/L + \varepsilon/L = \varepsilon$ (triangle inequality and the definition of limit)"

The last sentence of the proof process (desired conclusion):

"Therefore $\lim_{x \to a}[f(x) + g(x)] = A + B$."

In order to ensure the internal validity of the research and to evaluate the stages in the proof process more objectively, different theorems were used in each design.

Analysis of Semi-Structured Individual Interviews

First and foremost, the interview recordings were converted and edited into written text. The audio recordings were listened to several times by the researchers while they were transcribed in order to ensure the reliability of the research. The researchers identified the steps in the proof tasks as themes and collected data around these themes. As a result, more detailed information about the participants' thoughts was obtained at each stage of the evidence process.

In the final interviews, participants were asked to evaluate the working process and explain how it helped them, and the data gathered was also analyzed.

Findings

The findings and comments obtained as a result of the analysis of the data collected from three pre-service teachers who participated in the four-week classroom studies that included the evidence tasks in the research and where individual interviews were conducted are presented in this section.

Findings of the First Design

In the first proof-of-design event, the participants were asked "*Let A, B, C be sets, and $A \setminus B \subseteq C$. Then the theorem "$A \setminus C \subseteq B$" is given.*" The findings obtained from the answers of the participants are given below.

Determining the Components of the Theorem

Participants were asked to identify and express the hypothesis and clause components of a given theorem in the form of $p \Rightarrow q$ conditional propositions. The following is the response of participant S1.

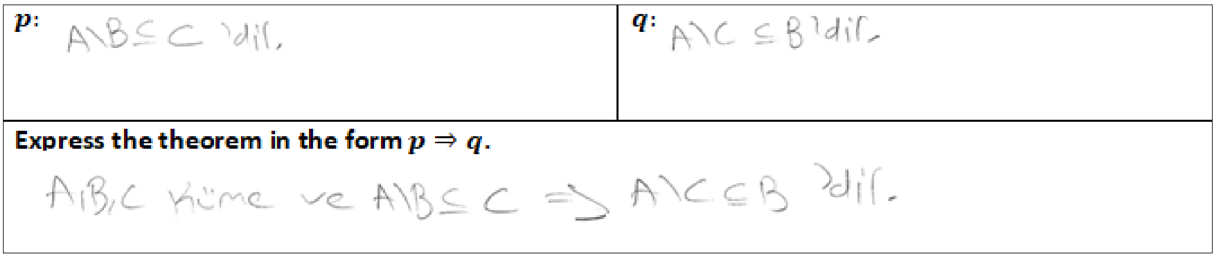
p: A,B,C küme ve $A \setminus B \subseteq C$	q: $A \setminus C \subseteq B$'dir.
Express the theorem in the form $p \Rightarrow q$. A,B,C küme ve $A \setminus B \subseteq C \Rightarrow A \setminus C \subseteq B$'dir.	

Figure 1: S1's response to determining the components of the first proof task theorem

While the participant correctly determined the hypothesis, s/he made a false inference by accepting the sets mentioned in the theorem statement as an inference, despite working in the set theory universe. Furthermore, s/he stated in this section that the proving process described in the lecture came to mind and that when s/he saw the theorem, s/he determined how to determine the propositions p and q in his/her mind as follows.

> S1: *"As soon as I saw the theorem since it is a familiarity after you explained it, I first thought about how to separate "p" from "q". Since the theorem consists of two different sentences, when I saw the 'Let $A \setminus B \subseteq C$' part, I directly sensed that it was "p" and the other was "q"."*

The participant named S2 correctly determined the conclusion and hypothesis components of the theorem, as seen in Figure 2.



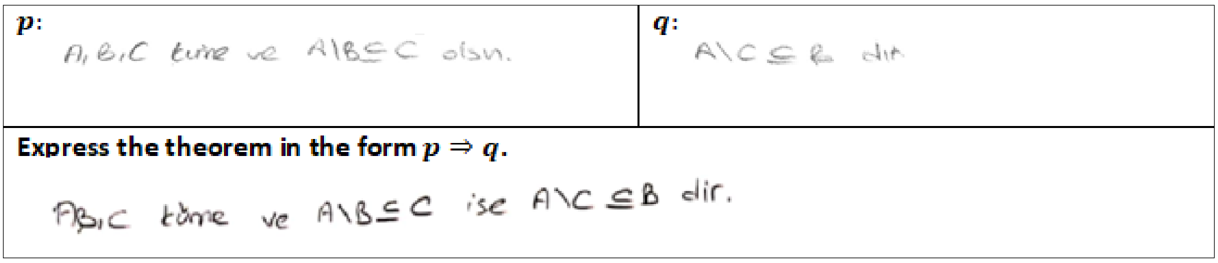
p: A\B⊆C 'dir.	q: A\C ⊆B 'dir.
Express the theorem in the form $p \Rightarrow q$. A,B,C küme ve A\B⊆C ⟹ A\C⊆B 'dir.	

Figure 2: S2's response to determining the components of the first evidence task theorem

As shown in Figure 2, participant S2 correctly determined propositions p and q and correctly restated the theorem. According to the participant, s/he first attempted to understand the theorem and use the conjunction "*if*".

> *S2: "First of all, I tried to understand the theorem. It came to me as a special expression. There are no mathematical conjunctions in this theorem, so it is not a mathematical statement. I even considered it when I asked in the first question for it to be written as " $if\ p, then\ q$." With the conjunction "if...,then..." I stated the theorem."*

Although the participant named S3 determined the hypothesis correctly, she made a mistake by accepting the sets in the theorem statement as assumptions.



p: A, B, C küme ve A\B⊆C olsun.	***q***: A\C ⊆ B dir.
Express the theorem in the form $p \Rightarrow q$. A, B, C küme ve A\B⊆C ise A\C⊆B dir.	

Figure 3: S3's response to determining the components of the first proof task theorem

In this part, the participant explained that s/he tried to understand the theorem by drawing a figure before starting the proof as follows:

S3: *"I read the theorem first, then immediately thought of drawing a figure to visualize it." I gave it a few tries to see what would happen if the shapes were subsets of each other and what would happen if they intersected."*

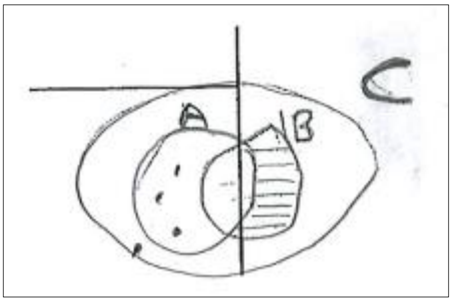


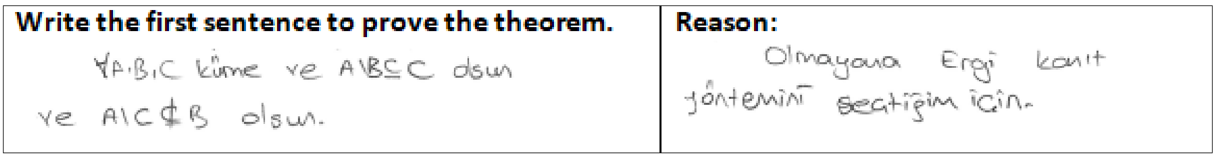
Figure 4: The drawing S3 drew for the theorem.

To understand the theorem, participants were observed using strategies such as looking at keywords such as "*if* ", "*then*", "*in that case*" and drawing a figure.

Introduction Sentence of the Proof Process

Participants were asked to write the first sentence of the proof process according to the proof method they chose.

When the participants' answers were examined, it was revealed that they chose the theorem's assumption as the first sentence of the proving process. As shown in Figure 5, the participant S1 chose the method of proof by contradiction and provided a correct answer based on the method.



Write the first sentence to prove the theorem.	**Reason:**
∀A,B,C küme ve A\B⊆C olsun ve A\C⊈B olsun.	Olmayana Ergi kanıt yöntemini seçtiğim için.

Figure 5: First sentence response of S1's first proof task

The participant stated that s/he first turned to direct evidence but decided to continue with the method of proof by contradiction and expressed his/her first sentence with the reason as “I thought I should accept that p is true and arrive at q, but after separating p and q, I thought I should use the method of proof by contradiction and wrote my first sentence accordingly.”.

The first sentence response of the participant named S2 is as in Figure 6.

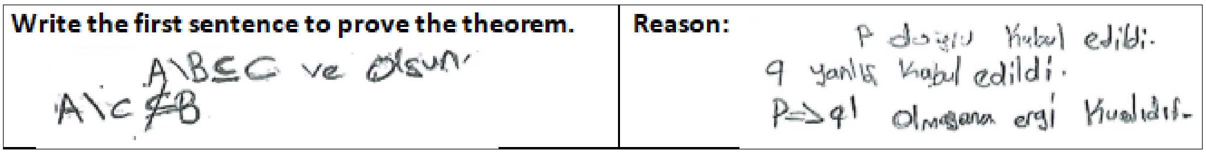


Figure 6: First sentence response of S2's first proof task

When the participant first looked at the theorem, s/he thought that s/he would use method of proof by contradiction as the solution method but stated that s/he tried the direct proof method but could not progress and explained that s/he decided to continue with the method of proof by contradiction as “When I first looked at the theorem, I had guessed that it would be solved with the method of proof by contradiction. But still, since I didn't try with direct proof first, I decided to do it with the method of proof by contradiction. After writing it as $p \Rightarrow q$, I decided to show that $q' \Rightarrow p'$ is true to show that this is true.” and it was seen that his/her answer was appropriate.

The answer of the participant named S3 is given below.

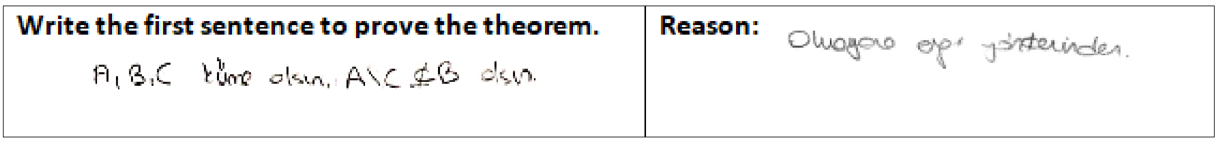


Figure 7: First sentence response of S3's first proof task

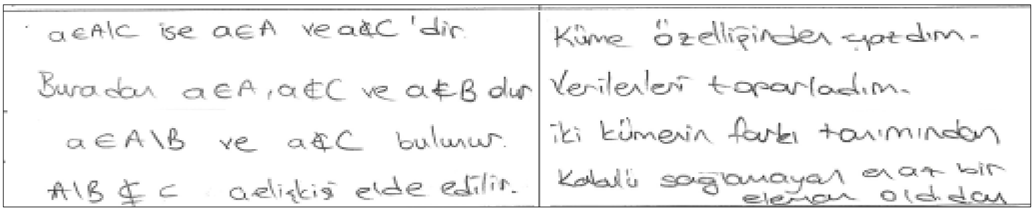
S3 also stated that, like S2, s/he attempted the direct proof method first; however, s/he was unable to prove, and thus chose the method of proof by contradiction. Figure 7 shows his/her response to the first sentence. The participant's explanation for this section is as follows:

S3: *“When I read the theorem, I wanted to try the direct proof method first. I saw that I was unable to settle a matter. I thought I'd try it the method of proof by* *contradiction. So, I took the proposition* $p \Rightarrow q$ *as* $q' \Rightarrow p'$ *and wrote my first sentence."*

The introductory sentence to the proof process, known as the assumption, is determined by the proof method used. Knowing the chosen proof method is critical for taking the first step in the proof. Looking at the answers, it was discovered that the chosen proof method was written as the reason for the introductory sentence rather than "assumption".

Inferences Made in the Proof Process

In this part, the participants are expected to make inferences from the introductory sentence they wrote and explain their reasons. The answer of the participant named S1 is given below.



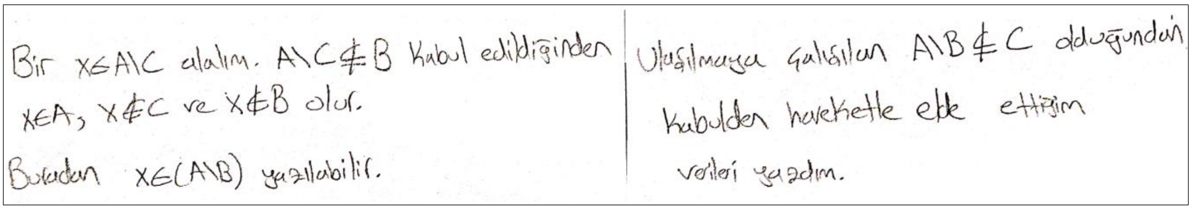
$a \in A \setminus C$ ise $a \in A$ ve $a \notin C$ 'dir.	Küme özelliğinden yaptım.
Buradan $a \in A$, $a \notin C$ ve $a \notin B$ dur	Verilenleri toparladım.
$a \in A \setminus B$ ve $a \notin C$ bulunur.	İki kümenin farkı tanımından
$A \setminus B \not\subseteq C$ çelişkisi elde edilir.	Kabulü sağlamayan en az bir eleman olduğundan

Figure 8: S1's inference response to the first proof task

As in Figure 8, it was seen that the participant made inferences using the properties of sets and difference definition. This part was explained as:

S1: "*I found that if I use the difference set property, if it is an element of set A, it will not be an element of set C. Then I gathered the givens and found at least one element that did not meet the requirement."*

The inferences of the participant named S2 during the proving process are given in Figure 9.

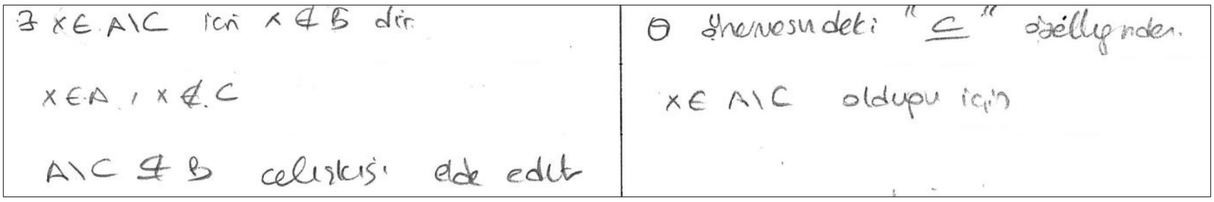


Bir $x \in A \setminus C$ alalım. $A \setminus C \not\subseteq B$ kabul edildiğinden $x \in A$, $x \notin C$ ve $x \notin B$ olur. Buradan $x \in (A \setminus B)$ yazılabilir.	Ulaşılmaya çalışılan $A \setminus B \not\subseteq C$ olduğundan kabulden hareketle elde ettiğim verileri yazdım.

Figure 9: S2's inference response to the first proof task

The answer of the participant named S2 that s/he started the inference process by taking x element and made inferences given in Figure 9. S/he made a statement about this part as "*I thought if I didn't get the x element, I wouldn't be able to make any inferences. So, I started by getting an element."*

S3's response, in which s/he obtained a contradiction by making inferences, is as follows:



$\exists x \in A \setminus C$ için $x \notin B$ dir $x \in A$, $x \notin C$ $A \setminus C \nsubseteq B$ çelişkisi elde edilir	θ teoremindeki "$\subseteq$" özelliğinden. $x \in A \setminus C$ olduğu için

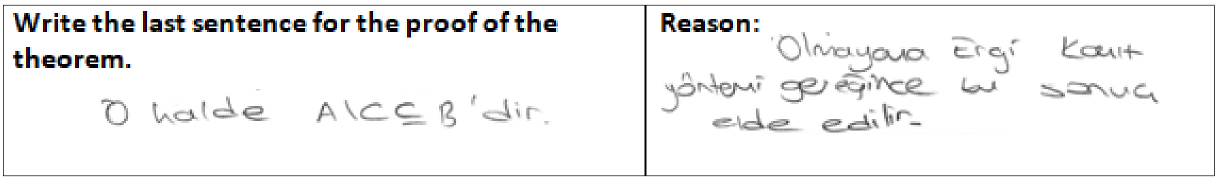
Figure 10: S3's inference response to the first proof task

The participant obtained a contradiction by writing his/her reasons and making inferences and described this process as "*I thought about what I could achieve using the properties of the set and wrote them down.*"

The participants made inferences based on the properties and meanings of the sets, just as they did in their answers and explanations.

Last sentence of the Proof Process

The last sentence of the theorem regarding the proving process was asked to each participant. Figure 11 shows, for example, the last sentence of participant S1 and the reason for the proof process. By writing the desired outcome, the participant provided the correct answer.



Write the last sentence for the proof of the theorem.	**Reason:**
O halde $A \setminus C \subseteq B$'dir.	Olmayana Ergi Kanıt yöntemi gereğince bu sonuç elde edilir.

Figure 11: S1's last sentence response to the first evidence task

The participant stated how s/he wrote this result as “This result is obtained according to the method of proof by contradiction”.

The participant named S2 stated that when he separated the theorem as p and q in his/her last sentence, s/he thought that the result he wanted to reach was q and gave the correct answer as in Figure 12.

Write the last sentence for the proof of the theorem.	Reason:
O halde $A \setminus B \subseteq C \rightarrow A \setminus C \subseteq B$ sağlanır.	

Figure 12: S2's last sentence response to the first evidence task

The participant stated that in order to reach this result, he established the following relationship between the data in the theorem:

> S2: *"When I separated the theorem as p and q, I realized that q is the result I wanted to reach. I interrelated the data and came to this conclusion."*

As shown in Figure 13, the participant named S3, like the other participants, correctly answered his/her last sentence as the result she aimed to achieve after selecting the method.

Write the last sentence for the proof of the theorem.	Reason:
$A \setminus B \nsubseteq C$ elde edilir. O halde $A \setminus B \subseteq C$ ise $A \setminus C \subseteq B$ dir.	

Figure 13: S3's last sentence response to the first evidence task

During the interview, the participant explained how she determined this result by saying, *"At first, I determined the proof method and according to it, I determined what I wanted to achieve in the last sentence while writing the first sentence."*

When the participants' opinions on the proof process were examined, it was determined that the steps provided in the task were useful, but they struggled with the inference process right after the first sentence. Participants S2 and S3 stated that restating the theorem in design is an important step in understanding the theorem, and that the given titles help with the proof process.

Findings of the Second Design

According to the results of the first proof-of-design task application and the individual interviews, the participants stated that they had difficulty making inferences and writing their reasons after the first sentence and were hesitant how to proceed.

> *S1: "After I wrote the first sentence, I had a hard time making inferences right away. Once I started, I was able to continue, but of course it took a while."*

Changes were made to the second design in comparison to the first design for this purpose. Following the introductory sentence to the proof, it was requested to write the second sentence using what was given in the theorem to support the inference process and provide a connection. Following that, it was asked to make gradual inferences from the first two sentences and prove them. The following steps are designed to write the final sentence and the entire proof.

In the proof task prepared in accordance with the second design, "*Let* $f: X \to Y$ *be a function and* $A \subset X$*. If* f *is one-to-one, then* $f(X \setminus A) \subset Y \setminus f(A)$" is given. The findings from this task are given below.

Determining the Components of the Theorem

Participants were asked to identify and express the hypothesis and conclusion components of a given theorem in the form of $p \Rightarrow q$ conditional propositions. Figure 14 illustrates the response of participant S1.

p:	q:
$f:X\to Y$ bir fonk., $A \subset X$ ve f birebir.	$f(X\setminus A) \subseteq Y\setminus f(A)$
Express the theorem in the form $p \Rightarrow q$.	
$f:X\to Y$ bir fonk, $A\subset X$ ve f birebir $\Rightarrow f(X\setminus A) \subset Y\setminus f(A)$ olur	

Figure 14: S1's response to determining the components of the second proof-of-design efficiency theorem

When the response of the participant was examined, it was seen that although she wrote the proposition q correctly, she wrote the statement in the first line extra in the proposition p, and she used the conjunction "and" incorrectly while restating the theorem as $p \Rightarrow q$.

S1: *"I had some trouble understanding the theorem. I had a hard time where to put the expression* $A \subset X$ *when making the distinction between* p *and* q*. Then I thought it should be* p*, but I felt like I had to prove it too."*

As stated by the participant named S1, the inclusion of the expression $A \subset X$ in the theorem caused difficulties in determining the components of the theorem.

In Figure 15, the answer of the participant named S2 regarding the determination of the components of the theorem is given.

p: f:X→Y birebir fonksiyon ve f(X−A)⊂Y−f(A)	***q***: A⊆X
Express the theorem in the form $p \Rightarrow q$. f:X→Y birebir fonksiyon ve f(X−A)⊆Y−f(A) ⇒ A⊂X dir.	

Figure 15: S2's response to determining the components of the second proof-of-design efficiency theorem

When the participant's response is examined, it is clear that he made a mistake in restating the theorem and distinguishing between p and q. This section is explained further below.

S2: *"I tried to express the theorem with an if, but I was stuck in a dilemma. There are not two expressions, there are three here. I didn't know which of these to include in p and which in q."*

Similar to the answer of participant S1, participant S3 gave the answer in Figure 16 that while the statement "f is one-to-one" was sufficient in the proposition p, which is the components of the theorem, she also wrote the statement "Let $f: X \to Y$ be a function and $A \subset X$".

p: f: X→Y bir fonk ve A⊂X ol. f birebir dir.	***q***: f(X\A)⊂Y\f(A) dır.
Express the theorem in the form $p \Rightarrow q$.	

Figure 16: S3's response to determining the components of the second proof-of-design efficiency theorem

When the participant's response was examined, she made the proposition q correctly, but did not write the part of the theorem that should be expressed as $p \Rightarrow q$. In this part, the participant expressed what she did to understand the theorem as follows:

S3: *"I understood the theorem by drawing a figure. Visually it was better. I thought p as a one-to-one function, and that A is a subset of X, and I took the rest as q."*

Participants S1 and S2 mentioned difficulty distinguishing between p and q. It has been observed that the theorem is composed of three propositions, which makes it difficult to make this distinction based on two propositions. The participant named S3 stated that she distinguished between p and q through the use of visuals.

Introduction Sentence of the Proof Process

Participants were asked to write the first sentence of the proof process based on the method of proof they chosen.

It was discovered that the participants preferred the direct proof method in their responses, and they attempted to write the first sentence of the proving process as a result. Figure 17 shows the answer of participant S1.

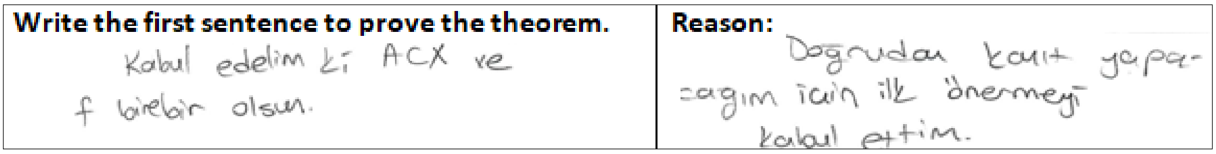


Figure 17: S1's first sentence response to the second proof-of-design task

In his/her first sentence, the participant used the conjunction "and" to combine the expressions. The answer in this section is simply to accept that "f is a one-to-one function." As a result, it has been rated as partially correct. S/he explained the first sentence she determined according to the direct proof method as *"I thought to do it according to the direct proof method, so I thought it was appropriate to accept p as true."*.

S2 also made a mistake in the first sentence of the proof process as a result of his/her error in determining the components of the theorem. Figure 19 shows the participant's response.

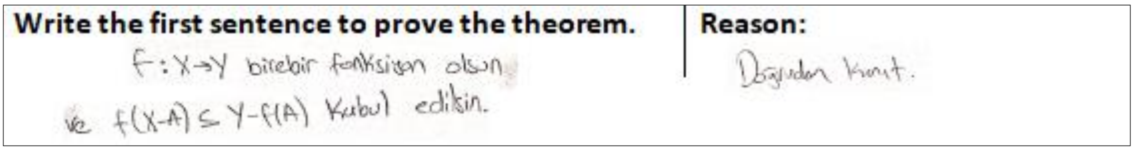
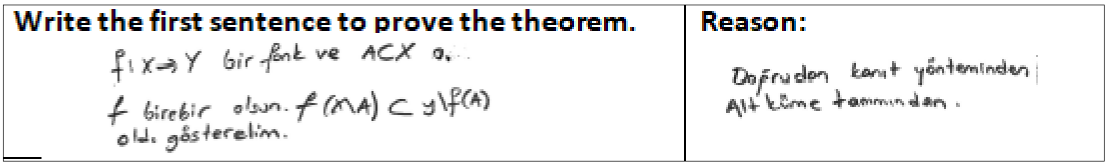


Figure 18: S2's first sentence response to the second proof-of-design task

As can be seen in the participant's response, the mistake s/he made in the distinction between p and q was reflected in the first sentence and s/he was aware of this situation and said, *"I accepted p as correct because I thought I had to prove it directly. But I am not sure that p is correct."*

Participants S1 and S2 stated that they used the direct proof method and that the first sentence should be accepted as true as a consequence. However, it was revealed that neither of the participant wrote a completely correct assumption.

It was discovered that, similar to S1, participant S3 added the expression "$A \subset X$ " to the assumption that s/he had determined as the first sentence of the proof process. Figure 19 depicts the participant's response.



Write the first sentence to prove the theorem.	Reason:
f: X→Y bir fonk ve A⊂X o... f birebir olsun. f(X\A) ⊂ Y\f(A) old. gösterelim.	Doğrudan kanıt yönteminden; Alt küme tanımından.

Figure 19: S3's first sentence response to the second proof-of-design task

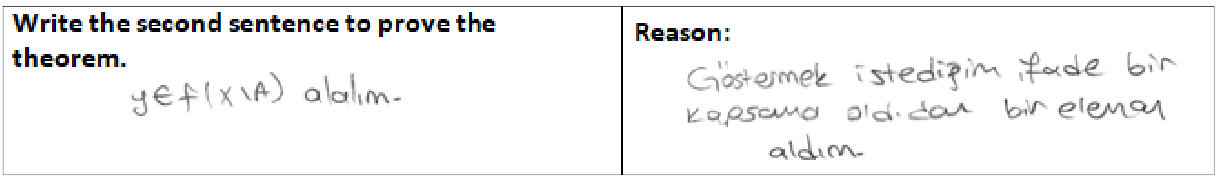
When the participant's response was examined, it was revealed that there were statements that were partially correct but indicated the desired result to be hypothesized. According to the participant, s/he first determined what s/he wanted to show at the end and then expressed what he needed to start the proof as the first sentence.

S3: "*First I determined what I wanted to show. To show this, I thought I had to use f is one-to-one and A is a subset of X, or I wouldn't be able to start the proof.*"

When the answers of the participants were examined, it was discovered that there were no participants who could write the proof completely and correctly along with the reason, even if they assumed correctly.

Writing the Second Sentence

According to the data obtained after the first design was implemented, the part of writing the second sentence was added after the introduction sentence, that is, the first sentence, to assist the inference process. Figure 20 shows the response of participant S1 to this section.



Write the second sentence to prove the theorem.	Reason:
y∈f(X\A) alalım.	Göstermek istediğim ifade bir kapsama old. dan bir eleman aldım.

Figure 20: S1's second sentence response to the second proof-of-design task

It was seen that the participant formed two sentences correctly after the assumption s/he wrote in the first sentence for the introduction to the proof.

The participant named S2 formed the second sentence incorrectly because of the mistakes he made in the first parts. The participant's response is given in Figure 21.

Write the second sentence to prove the theorem.	Reason:
Bir $x \in X-A$ ele alalım, $x \in X-A \Rightarrow f(x) \in f(X-A)$ olur	Verileri kullanarak gerekli ilişkilendirmeyi yapmak

Figure 21: S2's second sentence response to the second proof-of-design task

In the second sentence response, the participant stated that s/he correlated the data for the outcome s/he desired.

The answer of the participant named S3 is given in Figure 22.

Write the second sentence to prove the theorem.	Reason:
En az bir x elemanı vardır ki $x \in X-A$ ve $y = f(x)$ dir.	f fonksiyon olduğundan görüntü kümesi tanımından

Figure 22: S3's second sentence response to the second proof-of-design task

When the participant's response was examined, it was discovered that s/he used the image set definition to write the second sentence; however, it was determined that the expression "$x \in X \setminus A$" was written correctly while the expression "$y = f(x)$" was incorrect. His/her answer was rated as partially correct, as the expression "$y \in f(X \setminus A)$" was expected to be written instead. The explanation of the participant regarding this part is given below.

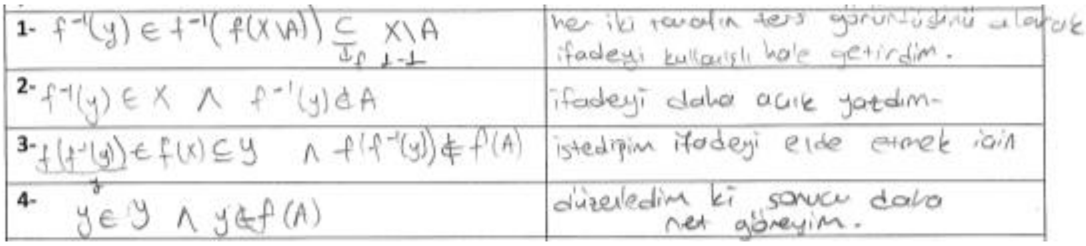
S3: *"After writing the first sentence, I wrote the second sentence by looking at the givens in the theorem."*

As seen in Figures 21 and 22, participants S2 and S3 stated that they wrote their second sentences according to the information given in the theorem. It was observed that the participants had difficulties and made mistakes while writing the second sentence, as in the first sentence, due to the mistakes they made in the part of determining the components of the theorem.

Inferences Made in the Proving Process

In the inference part, it has been rearranged to ensure that inferences are step-by-step after the initial design. Participants are expected to continue the proof process by making step-by-step inferences from the first and second sentences they wrote. When the answers were examined, it was discovered that the participants had difficulty making inferences, and that the majority of the participants were unable to make inferences fully and correctly.

In Figure 23, the inferences made by the participant named S1 during the proof process are given.



1- $f^{-1}(y) \in f^{-1}(f(X \setminus A)) \subseteq X \setminus A$	her iki tarafın ters görüntüsünü alarak ifadeyi kullanışlı hale getirdim.
2- $f^{-1}(y) \in X \wedge f^{-1}(y) \notin A$	ifadeyi daha açık yazdım.
3- $f(f^{-1}(y)) \in f(X) \subseteq Y \wedge f(f^{-1}(y)) \notin f(A)$	istediğim ifadeyi elde etmek için
4- $y \in Y \wedge y \notin f(A)$	düzenledim ki sonucu daha net göreyim.

Figure 23: S1's inference response to the second proof-of-design task

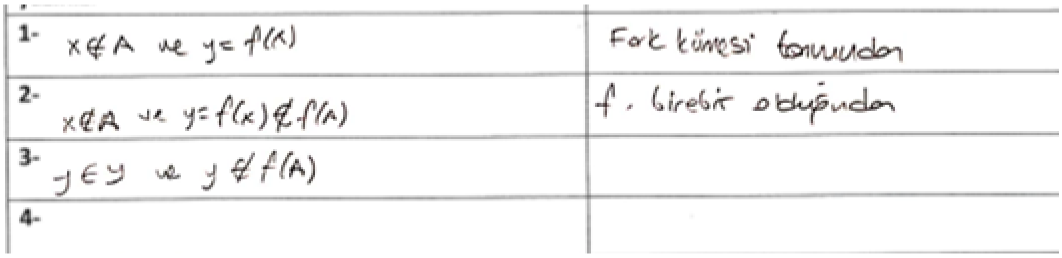
When the participant's inference response was examined, it was seen that he made the inference neatly. The participant made the following statements regarding this part in the individual interview:

> S1: *"In the first design, you had to write your first sentence, then make inferences before writing your last sentence." You expect us to make the inferences step by step, with justifications. Making a step-by-step inference by writing the reason was better and more planned for me. It was neater for us to express what we were thinking at what stage of the proof."*

With this explanation, s/he stated that making inferences step-by-step and writing them down along with the reasons allowed them to express their thoughts while also ensuring that the proof process was organized and planned.

The participant named S2 also suggested that s/he had difficulty in making inferences due to the lack of knowledge about the subject of the theorem by saying: "*I have deficiencies in the subject of functions. That's why I had a hard time making inferences.*" The mistakes s/he made at the beginning of the proof process caused him/her to have difficulty and make mistakes while making inference.

The inferences made by the participant named S3 during the proof process are given in Figure 24.



1- $x \notin A$ ve $y = f(x)$	Fark kümesi tanımından
2- $x \notin A$ ve $y = f(x) \notin f(A)$	f. birebir olduğundan
3- $y \in Y$ ve $y \notin f(A)$	
4-	

Figure 24: S3's inference response to the second proof-of-design task

The participant, on the other hand, stated that s/he was having difficulty writing his/her reasons while making inferences and that s/he assumed s/he was doing something inaccurately in this regard, as follows. S/he also stated that making inferences step by step ensures a consistent evidence process.

> *S3: "It was a little difficult to write down the reasons while making inferences. Because not all of them have a reason. I was wondering if I was making a mistake. But the step-by-step inference here, compared to the previous one, allowed us to make the proof more orderly. Otherwise, I would be confused."*

It can be stated that the participants had difficulty explaining their reasoning while making inferences, which was due to their lack of knowledge and previous mistakes.

Last sentence of the Proof Process

When the answers for the last sentence, namely writing the desired result, were analyzed, it was seen that S1 made a mistake, S2 did not give an answer, and S3 got it right.

The participant named S1 wrote inference instead of the desired result as the last sentence. While the expected answer was "$y \in Y \setminus f(A)$", writing "$y \notin Y \setminus f(A)$" caused his/her answer to be evaluated as wrong.

During the individual interview with participant S1, s/he expressed his thoughts on what he wanted to achieve while writing the last sentence and the first sentence as follows: "*Actually, I determined the last sentence in the first place. Because while I was writing the first sentence, I actually thought about what I wanted to achieve and wrote accordingly*." However, it was realized in his/her response that the reason for the last sentence was the inference of the previous theorem (the statement in the previous inference).

It was discovered that the participant S2 did not write an answer for the final sentence and made mistakes from the start of the proof process, resulting in the inability to reach a conclusion. "I did not write anything because I wasn't sure about the last sentence," s/he explained.

The participant named S3 explained that she used the definition of difference in sets by writing the expression “$y \in Y \setminus f\ (A)$” as the last sentence and that it was the desired result, saying “*I determined the last sentence according to what I wanted to see as a result.*”

Findings of the Third Design

In line with the data obtained after the implementation of the second design, it was observed that the participants had difficulties in making inferences due to the lack of information about the proposition/theorem given. For example, participant S2 stated that the inability to make inferences was due to a lack of knowledge about the function. As a result, in addition to the second design, it was thought appropriate to provide information that might be needed during the proving process, so an information box was added to the third design.

In the proof task prepared in accordance with the third design, theorem “$let \lim_{x\to a} f(x) = A\ and \lim_{x\to a} g(x) = B. So \lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = A+B$”, and the definitions and properties of mathematical concepts in this theorem are given as information.

Determining the Components of the Theorem

When the participants' answers to determining the components of the theorem were examined, it was discovered that they correctly distinguished between the hypothesis and the inference of the theorem, but the participant named S3 did not write the theorem as $p \Rightarrow q$. The answer to determining the components of the participatory theorem named S1 is shown in Figure 25.

p: $\lim_{x\to a} f(x)=A$ ve $\lim_{x\to a} g(x)=B$	**q:** $\lim_{x\to a}[f(x)+g(x)]=\lim_{x\to a} f(x)+\lim_{x\to a} g(x)=A+B$
Express the theorem in the form $p \Rightarrow q$. $\lim_{x\to a} f(x)=A$ ve $\lim_{x\to a} g(x)=B \Rightarrow \lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x)+\lim_{x\to a} g(x)=A+B$	

Figure 25: Response of S1's third proof task to determine the components of the theorem

When the participant's response was examined, s/he restated the theorem and determined the hypothesis and assumption of the theorem correctly. The explanation for this part is as follows:

> S1: *"First I read my theorem. The distinction between the propositions p and q was clear. And immediately I wrote the theorem using it, and I wrote the p and q as well."*

Participant S2 simply wrote "$p \Rightarrow q$" in restating the theorem. However, the answer that makes the distinction between hypothesis and assumption correctly is given in Figure 26.

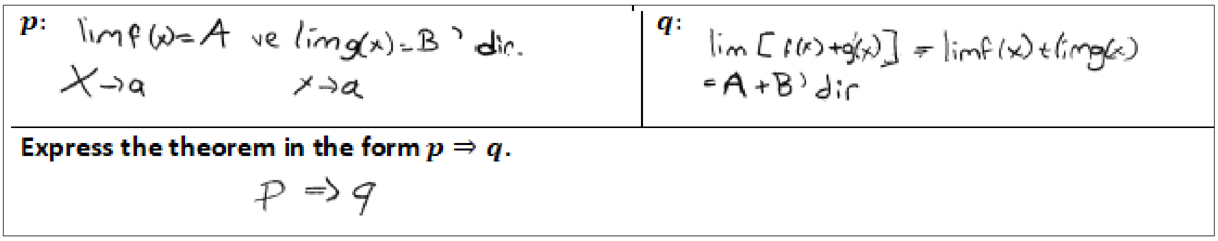


Figure 26: S2's response to the third proof task by determining the components of the theorem

The participant explained this part as follows, "*While I was reading the theorem, I separated p and q and wrote it.*"

The participant named S3 made the distinction between hypothesis and assumption correctly, similar to S2, but did not write anything in the part of restating the theorem. The participant's response is given below.

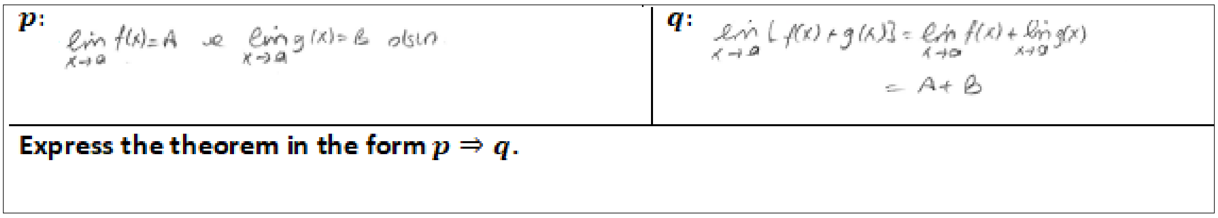


Figure 27: S3's response to the third proof task by determining the components of the theorem

The participant explained that the distinction between p and q was made while studying the theory, stating, *"I determined p and q as soon as I read the theorem at the start of the task."*

As stated by the participants, the hypothesis and assumption of the theorem were determined while reading the theorem, and thus the theorem could be rewritten. Participants

S2 and S3 wrote only the inferences p and q, but while restating the theorem, S2 wrote "$p \Rightarrow q$" while S3 did not write anything.

Introductory Sentence of the Proof Process

When the answers of the participants in the introductory sentence of the proving process, that is, the first sentence, were examined, it was determined that the participants named S1 and S2 wrote their inferences by choosing the direct proof method, but the participant named S3 made inferences instead of writing assumptions.

Participants S1 and S2 correctly wrote their assumptions in the introductory sentence of the proof process as "Let $\lim_{x \to a} f(x) = A$ and $\lim_{x \to a} g(x) = B$".

Participant S1 explained that he preferred the method of direct proof and wrote his first sentence accordingly: "*When I read the theorem, the direct proof method came to my mind first. The assumptions p and q were already very clear. I thought using direct method would be easier. I would have difficulties while using the method of proof by contradiction. I started my first sentence by accepting the proposition p as true*".

Participant S2, similar to S1, made the following explanation regarding the first sentence response: "*It was more appropriate to use the direct proof method for this theorem. For direct proof, I have to accept p as true as the first sentence*".

The response of the participant named S3 as an introductory sentence to the proof process is given in Figure 28.

Write the first sentence to prove the theorem.	**Reason:**
$\lim_{x \to a} f(x) = A \Leftrightarrow \forall \varepsilon > 0$ için $\exists \delta > 0$ $0 < \lvert x-a \rvert < \delta$ için $\lvert f(x) - A \rvert < \varepsilon$	$\lim_{x \to a} f(x) = A$ 'nın tanımından

Figure 28: S3's first sentence response to the third proof-of-design task

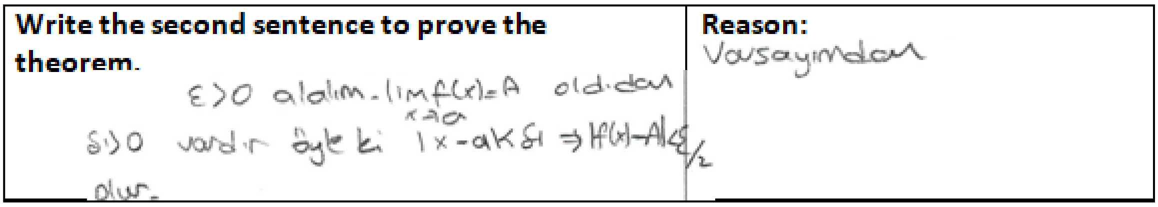
When the participant's response was examined, it was revealed that, while he was expected to write a hypothesis, he instead made inferences and did not write assumptions. As a result, it was deemed incorrect. In the individual interview, the participant stated that he realized the error as follows.

S3: *"After writing the propositions p and q, the method by which I could show q using p was the direct proof method. Accordingly, I should have started by accepting p as true. But I started to make assumptions."*

Writing the Second Sentence

In this part, the participants were asked to write a second sentence after the introductory sentence of the proof process to help make inferences.

The second sentence response of the participant named S1 is given below.

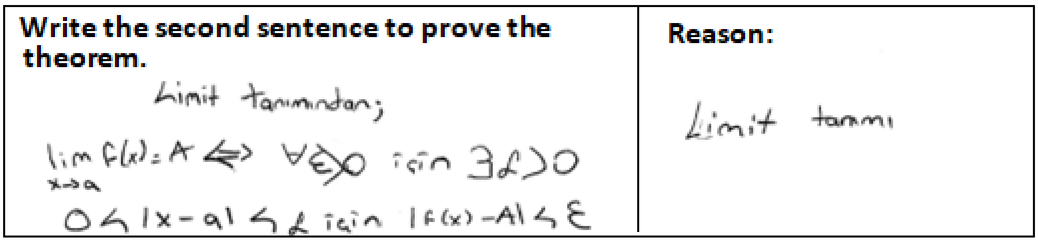


Write the second sentence to prove the theorem.	Reason:
$\varepsilon>0$ alalım. $\lim_{x\to a} f(x)=A$ olduğundan $\delta_1>0$ vardır öyle ki $\lvert x-a\rvert<\delta_1 \Rightarrow \lvert f(x)-A\rvert<\varepsilon/2$ olur.	Varsayımdan

Figure 29: S1's second sentence response to the third proof-of-design task

As in Figure 29, the participant named S1 correctly wrote his second sentence using the definition and inference, and his explanation for this was "I looked at the information and when I saw the definition of the limit; I thought I could use it in the second sentence".

The second sentence written by the participant named S2 using the definition of the limit is given in Figure 30.



Write the second sentence to prove the theorem.	Reason:
Limit tanımından; $\lim_{x\to a} f(x)=A \Leftrightarrow \forall\varepsilon>0$ için $\exists\delta>0$ $0<\lvert x-a\rvert<\delta$ için $\lvert f(x)-A\rvert<\varepsilon$	Limit tanımı

Figure 30: Second sentence response to S2's third proof-of-design task

In his individual interview, the participant stated that s/he wrote the second sentence using the definition of the limit: *"I looked at the information given. I wrote the second sentence after using the definition of the limit and following the first sentence."*

The answer of the participant named S3 that s/he continues to make inferences in his/her second sentence is given in Figure 31.

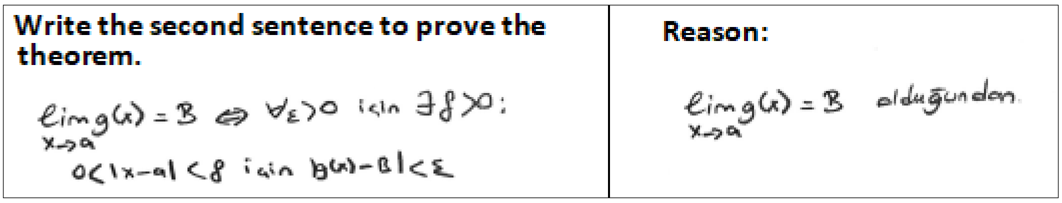


Figure 31: S3's second sentence response to the third proof-of-design task

When the participant's response was examined, it was found that he continued to make inferences, but used the symbol "δ" both for the expression $f(x)$ and for the expression $g(x)$. In the individual interview, he answered as follows regarding this part:

> S3: *"When I saw the definition of the limit in the information, I thought I should use it."*.

According to the data obtained for the second sentence, the participants stated that they wrote their second sentences using the definition of the limit in the information box and making use of the assumption (first sentence).

Inferences Made in the Proving Process

In this section, participants are expected to continue the proof process by making step-by-step inferences from the first and second sentences they wrote. The following are the inferences made by the participant S1 during the proving process.

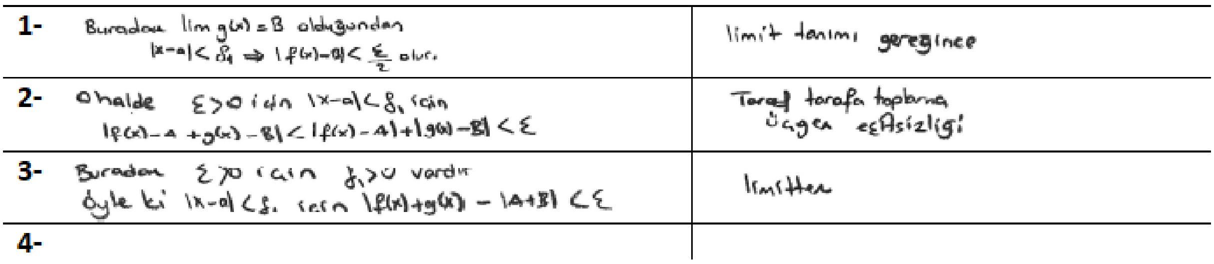


Figure 32: S1's inference response to the third proof-of-design task

As shown in Figure 32, the participant correctly assumed his/her reasons and stated that s/he benefited from the information in the information box as follows.

S1: *"It was very nice to have the information. So, I wrote my inferences using that information without any trouble."*

The answer that the participant named S2 made appropriate inferences by using the information given in the information box is given in Figure 33.

1-	$0<\|x-a\|<\delta$ için $\|f(x)-A\|<\varepsilon$	
2-	$0<\|x-a\|<\delta$ için $\|g(x)-B\|<\varepsilon$	
3-	$0<\|x-a\|<2\delta$ için $\|f(x)+g(x)-A-B\|<2\varepsilon$	
4-	$0<\|x-a\|<2\delta$ için $\|f(x)-A\|+\|g(x)-B\|<2\varepsilon$	üçgen eşitsizliği

Figure 33: S2's inference response to the third proof-of-design task

The participant made accurate inferences step by step, as shown in Figure 33, and stated this as follows.

S2: *"Having definitions of the expressions in the theorem and additional information was helpful while making inference."*

In Figure 34, the inferences made by the participant named S3 during the proving process are given.

1-	$A+B \Rightarrow \forall\varepsilon>0$ için $\exists\delta>0: 0<\|x-a\|<\delta$ için	
2-	$\|f(x)-A\|+\|g(x)-B\|<2\varepsilon$	
3-	$\|f(x)-A+g(x)-B\|<2\varepsilon$	
4-	$\|f(x)-A+g(x)-B\| \leq \|f(x)-A\|+\|g(x)-B\|<2\varepsilon$ $\leq \lim_{x\to a} f(x)=A+\lim_{x\to a} g(x)=B$	

Figure 34: S3's inference response to the third proof-of-design task

When the participant's inferences were examined, it was discovered that s/he made errors, so it was considered incorrect. However, in the individual interview, s/he stated that s/he benefited from the information as follows:

S3: "The fact that the information was given in advance made me feel comfortable when making inferences. I just used what I needed to use. For example, the triangle inequality and the definition of limit helped me a lot."

The information box added to the design has been understood to be useful when making inferences based on both the answers given and the statements of the participants.

Last sentence of the Proof Process

When the last sentence responses of the participants were examined, the answers of the participants named S1 and S2 as "$Then\ [f(x)+g(x)] = f(x) + g(x) = A+B$" were evaluated as correct since it is the desired result. The participant named S3 did not respond to this part.

In the individual interviews, no one expressed an opinion on the last sentence, but it was mentioned that the information in the information box is useful in all parts.

Participants Views on Proving Process

In the findings obtained from the interviews regarding the proving process, the participants stated that these tasks were beneficial, gave them confidence while proving, made the proof process more organized, and contributed to their proving skills, with the addition of the information box in the final design.

S1: *"I've noticed that I think faster with proof events. Previously, I always had doubts when I was proving, I always got stuck when I wrote two sentences, I couldn't move forward. I think that I was more conscious about what I will do in these tasks. With the information part, my anxiety disappeared. When I saw that I was able to prove, I became more confident."*

The participant named S1 stated that s/he was nervous while proving beforehand, but s/he thought faster with these tasks, and when s/he saw that s/he was able to prove, s/he gained self-confidence.

S2: *"I realized that I used to be very messy when I was proving. In order to get the proof right, we had to establish its systematics. Evidence is actually a discipline, I realized that in these tasks. So, I found that when I progressed regularly, I was able to prove."*

S2, the participant, stated that s/he realized s/he worked scattered before the evidence tasks, that s/he was able to prove and get into an order with these tasks.

S3: *"Before this, when I was going to prove, I didn't know where to start and how to move forward. It always seemed like I was going to do it wrong. But with these tasks, I established a regularity. Each part facilitates the proof process. The addition of the information part also gave a confidence. It reinforced the feeling that I was doing the proof right. Actually, it wasn't that hard to prove. I realized that this way I could do the proof with an task. When you get it in order, the rest comes."*

S3 also stated that s/he did not know where to begin while proving beforehand and was concerned about making mistakes. S/he stated that the proof process became more regular as a result of these tasks, and the information part also provided confidence.

Discussions

The component of determining the components of the theorem in the proof tasks was added as a solution to the difficulty of starting the proof. It is important to determine and understand the components of the given theorem in order to start the proving process (Dean, 1996, Weber, 2012). In this section, it is thought that re-expressing the theorem given as a conditional statement as q if p and determining the hypothesis (p) and the assumption (q) will help to understand the theorem. Based on the findings of this part in the proof tasks used in the research, it was determined that this part aided them in starting the proof by contributing to the understanding of the theorem. This result, like the study of Benkhalti, Selden, and Selden (2017), can be said to be a solution to the problem of starting the proving process.

It was revealed that the participants followed strategies such as looking at the key words in the theorem (if, then, in that case, etc.) and drawing a figure in order to understand the theorem. Attempting to understand the theorem by looking at the keywords in it to begin the proof process is an example of Weber's (2004) approach, which he refers to as Syntactic Proof Generation. Weber's (2004) another approach is Generating Semantic Proofs, which he describes as persuading himself to the proof process by drawing pictures suitable for the statement of the theorem for a better understanding of mathematical expressions. The participant, who tries to understand the theorem by drawing a figure, is given an example for this Generating Semantic Proofs approach.

Writing the introductory sentence of the proof process, namely the assumption for the proof, is an important step after determining the components of the theorem (Selden, Selden, & Benkhalti, 2017). As a result, writing the first sentence was added to the tasks as an introduction to the proof process. In the findings for the first sentence, it was discovered that the theorem was more easily determined once it was understood, and the assumption was mostly written correctly according to the preferred proof method.

It was discovered that the participants struggled to continue after the first design, so the writing of the second sentence was added. This section can be considered the transition phase to making assumptions, and it contributes to making assumption by connecting with the first sentence.

In the inference part, in the first stages, the participants were expected to make inferences using their prior knowledge for the proof of the theorem. However, it was observed that in some cases, the participants could not choose the appropriate one to make inferences from their existing prior knowledge and use it strategically, and sometimes they could not continue due to lack of knowledge. This situation is similar to the results of the studies of Dane (2008), Polat and Akgün (2016), Sarı Uzun and Bülbül (2013), and Weber (2001) .

In line with the evaluations and opinions received, it was thought that adding an information box to the evidence task design would positively affect the proving process, which was difficult due to lack of information. According to the findings obtained in the final design, the information box aided in making inferences during the proofing process. Selden and Selden (2009) also stated that conducting proof studies with notes containing definitions and theorems related to proof is effective in their studies. Similarly, Karaoğlu (2010) concluded in the thesis study that, the key points and ideas that will help the pre-service teachers in the proving process were implicitly given, and the pre-service teachers completed the proof process by making use of them when needed.

In order to complete the proof after the inferences, it is important to write the last sentence of the proof process, that is, to state that the proof is over. According to the findings of this section, the last sentence is the desired result, which helps participants in filling the gap between the first sentence and contributes to the proof process by directing them to the point they need to reach while making inferences. Selden, Selden, and Benkhalti (2017) also stated that determining the beginning and ending sentences in the proof frames used to prove the theorem contributes to this process.

Regarding the design of the evidence task, participants stated that it gave them confidence in proving, that it provided a more regular proof process, that it was beneficial, and that their proving skills improved. This result is similar to the result of Benkhalti, Selden, and Selden's (2017) study that proof frameworks positively affect self-confidence in the proving process.

Conclusions and Suggestions

As a result, it was seen that the results obtained in this study supported the results of the studies conducted in the field. In addition, a proof task design was developed in this study to guide the proving process and to develop proving skills.

Based on the findings of the study, it is believed that the application of such studies to a larger number of students and their dissemination in mathematics courses will contribute to overcoming the difficulties encountered in the proof and proving process, and that the bias toward proof will change in a positive way.

In this study, proof tasks were applied only to pre-service mathematics teachers at a state university and their effect on the development of their proving skills was examined. This study can also be conducted with pre-service mathematics teachers studying at different universities, and its effect on the development of their proof skills can be examined.

On the other hand, due to the wide subject area of mathematics, similar proof tasks can be designed and applied with different subjects.

In future studies, it may be recommended to apply proving tasks to students at different grade levels, from primary school to university, in accordance with their level.

Matematik Öğretmen Adaylarınn Kanıtlama Becerilerinin Geliştirilmesine Yönelik Bir Tasarım Çalışması

Özet:

Matematiksel kanıtın bir iddiayı doğrulamak ve açıklamak gibi görevlerinin yanı sıra matematiği anlamaya yardımcı olması ve yeni bilgilerin oluşumunu sağlaması matematik eğitiminde önemini artırmaktadır. Matematiği öğrenmek ve gerçek yaşamda karşılaşılan problem durumlarında etkili çözüm yolları bulabilmek için de kanıtlama becerilerinin geliştirilmesi gerekir. Kanıtın matematik eğitimindeki önemi göz önüne alındığında, özellikle matematik öğretmeni yetiştiren bölümlerde, öğretmen adaylarının kanıt bilgisini, kanıt yapabilme ve anlama düzeylerini artırmak için uygun öğretim uygulamalarının tasarlanması gelecekteki öğrencilere etkili ve kalıcı matematik öğretimi sağlamaları açısından önemlidir. Bu araştırmada tasarlanan kanıt etkinliklerinin matematik öğretmen adaylarının kanıtlama becerilerinin gelişimine etkisi incelenmiştir. Tasarım tabanlı araştırma yönteminin kullanıldığı bu araştırma, Ankara'da bulunan bir devlet üniversitesinde öğrenim gören gönüllü üç matematik öğretmen adayının katılımıyla gerçekleştirilmiştir. Araştırmada, uygulanan kanıt etkinlikleri ve katılımcılarla yapılan bireysel görüşmelerden elde edilen veriler kullanılmıştır. Nitel analiz yöntemiyle analiz edilen verilerden elde edilen bulgulara göre tasarlanan kanıt etkinlikleri katılımcıların kanıtlama becerilerinin gelişimine katkı sağlamıştır.

Anahtar kelimeler: matematiksel kanıt, kanıtlama becerileri, kanıt etkinliği, matematik öğretmen adayları.

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Research Article, Review

The Effect of Geometry Teaching Designed with the Developed Mobile Application on the Academic Achievement of the Students*

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Abstract –In order to ensure active participation of students in the lessons during distance education period, different learning tools were sought. For this purpose, within framework of the ADDIE design model, it was desired to develop a mobile application called GeoHepta to be used in learning the 7th grade mathematics course Geometry and Measurement learning field subjects. The study was carried out in the distance education process with 7th grade students in the 2020-2021 academic year. The research was carried out with a quasi-experimental design. In the study, "Geometry and Measurement Learning Field Achievement Test" was used. As a result of the research, a significant difference was reached between the post-test scores of the students in the experimental group, in which the mobile application-supported teaching was implemented, and the control group students, in which the textbook-based teaching was implemented. While there was a significant difference between the pre-test post-test scores of the experimental group students, there was no significant difference between the pre-test post-test scores of the control group students.

Key words: geometry education, mathematics achievement, ADDIE design model, mobile learning, distance education

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* This study was produced from the first author's doctoral dissertation.

Introduction

It is important to plan the activities to be carried out by addressing individual differences while teaching. Creating classroom environments that will enable individuals to develop not only their cognitive behaviors related to academic success but also their affective behaviors towards lessons supports success. Bacanlı (2006) revealed that affect has some indicators in the field of cognition. It is stated that while cognition takes action, the concept of affect affects this situation and creates awareness. In this direction, cognitive behaviors may vary according to the state of affective behaviors. Depending on the development of affective features, cognition can act as developed.

Today, generations of students studying at different levels from pre-school to university level differ. The students who will form our next generation who receive education in today's pre-school can be expressed as the Alpha generation (2010-2030), and the students who are educated at secondary school and other levels can be expressed as the Z generation (2000-2021). The generation in which the students are born affects which features of them are more developed, their perspectives on events, and their cognitive and affective behaviors. Educational environments can be arranged according to the generations of students. Considering the generation characteristics of the students, it is necessary to organize the learning environments in the school in a way that will appeal to individual differences in order to reach the specified goals. In order to ensure the effective education of generations, different strategies, methods and techniques can be used in the teaching process according to the teaching objectives and content of each branch of science. Educational designers can use generational differences as a meaningful variable in examining the effectiveness of different practices in the process of developing instructional design according to the differences between generations (Desai & Lele, 2017). It is stated in the curriculum that which dimensions of the abstract concepts in mathematics will be taught according to each grade level. In order to learn mathematics in line with the objectives of the mathematics curriculum, a learning environment can be created with different teaching methods and strategies in learning environments. Depending on the developments in education from past to present, teachers can use different teaching strategies, methods and techniques in teaching mathematics. Instructional strategies are expressed as an approach that guides the lessons in the learning process to reach the goals determined in the classroom and determines the use of methods, techniques and equipment (Yılmaz & Sünbül, 2003). Instructional strategies are generally classified as presentation strategy, discovery teaching strategy, inquiry-based teaching strategy and collaborative teaching strategy. After the teaching strategies are determined, it is determined which teaching method will be used in order to ensure that the behaviors that will be gained to the students are carried out according to a certain plan. The most commonly used teaching methods are lecture, question-answer, demonstration, group discussion, role playing, case study. Depending on the teaching methods, the ways to be used in order to realize the teaching are determined by the teaching techniques. Techniques such as brainstorming and drama are used to implement teaching methods (Ocak, 2015). Teachers can also benefit from instructional design models to address individual differences in instructional environments. Instructional designs are concerned with how learning areas are taught. Instructional design is shaped according to the design steps in order to ensure the best learning and teaching. Based on the assumption that each individual in the target audience has to learn a determined subject within the definition of instructional design, it is the detailed planning of the development of instruction with the help of the strategies, the equipment or technology that the designer will use in line with the determined objectives (Ocak, 2015). Different learning design models are used while creating instructional designs. With the dissemination and development of theories and practices related to learning, the difference has been made. Some of the instructional design models that can be applied in different ways are: Kemp, Morrison and Ross (1994) model, Dick, Carey and Carey (1996) model, Seels and Glasgow (1998) model, ADDIE model, ARCS model, ASSURE model. While deciding which model to use, attention is paid to the instructional problem addressed, the situation of the target audience, and how behavioral, cognitive and constructivist approaches are used in the models. Instructional design models can be selected depending on the learning situation. Teachers can choose the instructional design according to the learning outcome and learning situation they aim to gain in teaching. Instructional design models are generally shaped according to similar steps. Each instructional design model includes a specific instructional strategy, instructional strategy instructional methods, instructional methods instructional techniques. Although instructional design models have different features, all of them include analysis, design, development, implementation and evaluation steps. Instructional design models are shaped according to the specified steps. Instructional design models are used to give form and concreteness to an existing phenomenon, conceptual relations and methods (Ocak, 2015). Instructional design models ensure that each instructional situation is fully fulfilled by successive processes, without skipping the steps of designing the conceptual relationships that they want to convey. However, in some models, it is possible to skip some steps, while focusing more on certain steps, depending on the characteristics of the model. In this respect, instructional design models may differ. According to the comparison of instructional design models in this respect, the models used can be classified as linear and cyclic models. While linear models consist of successive steps, circular models do not mention a starting or ending point. While models such as ASSURE and the Instructional Development Institute model are examples of the linear model, models such as the Kemp model and the American Air Force model are examples of the circular model (Ocak, 2015). Although instructional designs have differences in these aspects, they have common components (Richey, 1986). In all teaching models, determining the needs of the learners, determining the goals and objectives, determining the evaluation methods, designing the presentation styles, making the pilot applications of the created system, and reviewing the system are implemented as a common component (Richey, 1986). Instructional design models can show similar components and different features of the process with visual models in order to show the whole process in detail at first glance. By looking at these models, it is possible to have information about how the design process of the model progresses. Teachers can create a teaching environment according to an instructional design model in order to create the most appropriate teaching environment in line with the goals they want to achieve. In mathematics teaching, the teaching environment can be designed according to different instructional design models, so that students can reach the desired goals in teaching the subjects. When we look at the studies on instructional design models that can be used in mathematics teaching in the literature, it is seen that there are studies that are carried out with different design models depending on features such as teaching level, goals, and problem status (Karakış, Karamete, & Okçu, 2016; Özdemir & Uyangör, 2011; Yıldız & Koçak Usluel, 2016). Özdemir and Uyangör (2011) created an instructional design model for mathematics education based on the ASSURE model and supported by the Dick and Carey Model in their study. They made suggestions for the model they developed to be used in different areas in mathematics teaching. In their study, Karakış, Karamete and Okçu (2016) examined the effects of computer-assisted instruction on students' attitudes towards mathematics lessons and computer-assisted instruction towards learning mathematics. The computer software used was designed to teach fractions to fourth graders based on the ASSURE model and the ARCS Motivation Model. As a result of the study, it was determined that the academic achievement of the students increased and their attitudes towards computer assisted education changed positively. When the studies are examined, the integration of technology in mathematics teaching is included according to different instructional design models. In this direction, in the research, it was aimed to examine the effects of a mobile application-based teaching environment by developing a mobile application named GeoHepta for the 7th grade mathematics lesson according to the ADDIE design model, which is one of the design models.

The ADDIE design model is formed by combining the initials of the steps that make up this model. These steps are Analysis, Design, Development, Implementation, Evaluation. ADDIE instructional design model; defines the stages of these five steps as the process of using instructional design to create purposeful learning lessons. The ADDIE approach, which is one of the most effective product development ways of today, is expressed as a suitable model for educational products and other learning resources (Branch, 2016). In the research carried out, a needs analysis was carried out within the scope of the research, starting from the analysis step of the ADDIE design model steps. As a result of the analysis of the interviews with the 8th grade students and mathematics teachers, it was decided to create a teaching environment by making use of technology, since they knew the subjects related to the 7th grade Geometry and Measurement learning field. As a result of the needs analysis, it was requested to use it in mathematics teaching by developing a mobile application in order to create a teaching environment suitable for the age of the students. While the content of the mobile application to be developed in the design stage was being prepared, the mobile application was developed as both a web page and a mobile application under the name GeoHepta in the development stage. The developed mobile application is designed to be used on phones or tablets. In the application step, teaching was carried out by using the mobile application. In the evaluation step, the effect of the teaching, which is carried out by using the mobile application, on the success of the students was investigated.

Problem Statement

In accordance with the purpose of the research, the problem statement was determined as follows: “What is the effect of the 7th grade students' academic achievement of the 7th grade students on the academic achievement of the subjects in the field of Geometry and Measurement designed according to the ADDIE instructional design model?”

Sub-Problems

Experiment group students who study in the learning environment designed according to the ADDIE instructional design model in the teaching of subjects related to the 7th grade Geometry and Measurement field, and the control group students where the textbook-based teaching is carried out,

i) Is there a significant difference between their academic achievements according to the pre-test measurement scores of the "7th grade Achievement Test for Geometry and Measurement Learning Field"?

ii) Is there a significant difference between their academic achievements according to the "7th grade Achievement Test for Geometry and Measurement Learning Field" post-test measurement scores?

iii) Is there a significant difference between their academic achievements according to the pre-test and post-test measurement scores of the "7th grade Achievement Test for the Learning Field of Geometry and Measurement"?

Literature

ADDIE Instructional Design Model

Instructional design; It is stated as an iterative process that includes the stages of determining performance goals, deciding on teaching strategies, selecting or creating the environment and material, and evaluating (Branch, 2016). The ADDIE model, which is one of the instructional design models, was first introduced as a general model in 1975. ADDIE design model, which is one of the systematic instructional design models consisting of five stages, consists of the stages of Analysis, Design, Development, Implementation, and Evaluation. ADDIE instructional design model; defines the stages of these five steps as the process of using instructional design to create purposeful learning lessons. It is seen that the steps of the ADDIE design model are gradual, and a relationship is established with the previous step depending on the work done after each step. As one of the most effective product development ways for educational products and other learning resources, the ADDIE approach is stated as a suitable model (Branch, 2016). In line with the applicability of the ADDIE design model, research has been carried out at different levels of education (Arkün & Akkoyunlu, 2008; Berigel, 2017; Burmabıyık, 2014; Wahab, Abdullah, Mokhtar, Atan, & Abu, 2017). To summarize these studies briefly, there are researches based on the ADDIE design model at different levels. In the research of Burmabıyık (2014), a learning material was developed for the teaching of solid objects in mathematics lesson according to the steps of the ADDIE instructional design model. As a result of the research, the students stated that they liked the developed material and wanted to use it. According to the ADDIE instructional design model for hearing-impaired students, Berigel (2017) found that learning environments created as a result of technology-supported mathematics learning environments increased students' interest in the course and had positive effects on their success. Wahab et al. (2017) stated in their studies that while learning mathematics through exploration, opportunities to experiment should be given and geometric shapes should be examined on their own. In this direction, a learning strategy called LSPE-SUM was designed to help students develop their visual spatial skills and geometric thinking, step by step, with a learning strategy using the 3D SketchUp Make dynamic software for

Geometry. As a result of the analysis of students' opinions, it was concluded that LSPE-SUM helped to develop geometric thinking and served its purposes well in a pedagogical functional way.

Mobile Learning

Mobile learning is a widely used technology-assisted teaching method. According to Semetzidis (2013), mobile learning is learning through mobile devices. Today, with mobile learning, instant communication without time and place restrictions allows individuals to carry digital files in their pockets. Educators or individuals can direct the learning process that takes place with mobile technologies. Mobile learning; It can take place both inside and outside the classroom. Mobile learning supports informal learning as it provides learning opportunities outside the classroom (Crompton, 2013). Thus, it opens the way for learning to be provided everywhere. When the literature is examined, there are studies (Kestel, 2020) on the use of mobile learning-based research in different fields of educational sciences (Berberoğlu, 2020; Sönmez, 2018) and other branches of science in the teaching and application stages. However, it has been found that there are few studies in the field of mathematics teaching among the researches on mobile assisted teaching. Yıldız (2020), one of these studies, carried out a scale development study on the examination of mobile learning acceptance of secondary school students in mathematics learning. Koparan and Kaleli Yılmaz (2020) examined the opinions of pre-service mathematics teachers about the learning environment supported by mobile learning. Again, when studies abroad are examined, Supandi, Ariyanto, Kusumaningsih, and Aini (2018) aimed to examine the role of mobile phone application in mathematics education in their studies. After the use of the mobile application, it was observed that the students found the application interesting and showed high success.

Depending on the changing conditions in current life, how students will learn and learning environments can also change. Today, where technology is an integral part of life, mobile phones are in the hands of every individual. In the conducted research, it was concluded that a learning environment should be created by utilizing technology according to the views obtained from the needs analysis according to the ADDIE design model. Therefore, it is aimed to investigate how an instructional design based on mobile learning, which has a place in everyone's life, changes the academic success of students.

Method

Research Model

In this research, a quasi-experimental design, which is one of the quantitative research methods, was used to determine the effects of the teaching carried out based on the GeoHepta mobile application developed according to the ADDIE instructional design model of 7th grade students.

Working Group

The research was carried out with 7th grade students studying in a secondary school in the Central Anatolia Region in the 2020-2021 academic year during the distance education process due to the Covid-19 pandemic, by obtaining the necessary ethics committee permission. The experimental group consisted of 26 students and the control group consisted of 21 students.

Data Collection Tools

Achievement Test for the Learning Field of Geometry and Measurement

Within the scope of the research, a multiple-choice test with 28 questions was prepared in order to determine the academic success of the students in the mathematics course. During the development process of the achievement test, a test consisting of 35 questions was created in order to examine each achievement with at least 2 questions. The pilot application of the draft test was made on 8th grade students who were learning the subjects. After the pilot application, the analysis of the items in the test was provided. As a result of item analysis, the test was finalized and the achievement test consisted of 28 questions. The KR-20 reliability coefficient of the developed test was found to be 0.746. After the pilot application, 8 questions were removed from the draft test questions prepared, and the achievement test was given its final form as 28 questions.

Analysis of Data

In the research, 7th grade Geometry and Measurement Learning Field Achievement Test was applied in order to determine the success of the experimental and control group students before and after the application and to examine whether there is a statistically significant difference between their success. While evaluating the questions in the Mathematics Course Achievement Test related to Geometry and Measurement Learning Field, 1 for each correct answer; 0 points is given for each wrong answer and unanswered question. Each student's test items were read according to the specified scoring. Accordingly, it was determined that the highest score that could be obtained from the achievement test would be 28 and the lowest score would be 0.

The scores obtained depending on the pre-test and post-test applied to the students were analyzed with the SPSS program. In the analysis of the data within the same group, the t-test was used for the related samples, and the t-test was used for the unrelated samples in the between-group analysis. Since the distribution of the pre-test and post-test scores obtained from the applied scales showed a normal distribution, the analysis was made using parametric tests. For this reason, independent samples t-test (t-test for unrelated samples) was used to compare achievement test scores between the experimental and control groups. The dependent sample t-test (t-test for related samples), which is one of the parametric tests, was used to determine the relationship between the pre-test and post-test scores of the experimental group students and the pre-test and post-test scores of the control group students. With the t-test for dependent samples, the significance between each group according to the pre-test and post-test scores was examined. From the findings obtained as a result of the analysis, statistical differences between the academic achievements of the experimental group students who had the learning process in the learning environment created with the GeoHepta mobile application developed according to the ADDIE instructional design model and the control group students who had the learning process in the textbook-based learning-teaching environment were determined.

Findings and Discussions

The research problem is "Does the 7th grade students' teaching of the subjects in the field of Geometry and Measurement designed according to the ADDIE instructional design model have an effect on the success of the students in geometry?" expressed as. Findings related to the sub-problems of the research and interpretations of the findings are given under the following headings, respectively.

Findings and Comments on the First Sub-Problem

“The 7th grade Achievement Test Related to Geometry and Measurement Learning Field" pre-test measurement scores of the Experiment group students who were educated in the learning environment designed according to the ADDIE instructional design model in teaching the subjects related to the 7th grade Geometry and Measurement learning field, and the control group students where the textbook-based teaching was carried out. Is there a significant difference between their academic achievements?

“In order to reach the findings of the sub-problem, it was examined whether all the assumptions of the t-test for unrelated samples were met. These assumptions are:

1) The measurements or scores of the dependent variable are in the interval or ratio scale, and the mean of the two groups for comparison belongs to the same variable.

2) The distribution of the measurements of the dependent variable is normal in both groups.

3) Samples to compare mean scores are unrelated. (Büyüköztürk, 2014; p. 39)

They are the measurements obtained from the pre-test application of the scores of the dependent variable. Since the measurement data obtained is in the ratio scale, the first assumption is provided.

In the comparison of the mean scores of the same variable of two independent groups; According to the Shapiro Wilk test results of the measurements in each group, the mean scores of the students in the experimental ($p=.054>.05$) and control group ($p=.236>.05$) show a normal distribution. According to the Levene test result, the variances of the distributions were found to be equal ($F=2.814$; $p>.05$). Due to its assumptions, the t-test was applied for unrelated samples. Table - 1 below shows the t-test result for the comparison of the achievement test pre-test mean scores of the experimental and control groups.

Table 1. Results of t-test for unrelated samples according to pre-application data of Achievement Test Related to Geometry and Measurement Learning Field

Test Name	**Measurement**	**n**	**Arithmetic mean**	**Ss**	**Sd**	**t**	**p**
Achievement Test	Experiment	26	9.54	3.34	45	.306	.761
	Control	21	9.81	2.56			

The results of the t-test for unrelated samples are given in Table 1. Looking at the values in the table, the difference between the experimental and control group achievement test scores ($X_d=9.54$, $X_k=9.81$) was not found statistically significant ($t=(45)=.306$, $p>.05$). From these results, it can be said that the students of the two groups are equivalent to each other in terms of achievement test pre-test scores before the application.

Findings and Comments on the Second Sub-Problem

“The 7th grade Achievement Test Related to Geometry and Measurement Learning Field" post-test measurement scores of the Experiment group students who were educated in the learning environment designed according to the ADDIE instructional design model in the teaching of subjects related to the 7th grade Geometry and Measurement learning field, and the control group students where the textbook-based teaching was carried out. Is there a significant difference between their academic achievements?

Assumptions of the t-test for unrelated samples are provided for comparing the mean scores of the same variable for unrelated samples. For this reason, unrelated samples t-test was performed. In Table 2 below, the t-test result for unrelated samples for the comparison of the achievement test post-test mean scores of the experimental and control groups is given.

Table 2. Results of the t-test for unrelated samples according to the last application data of Achievement Test Related to Geometry and Measurement Learning Field

Test Name	Measurement	n	Arithmetic mean	Ss	Sd	t	p
Achievement Test	Experiment	26	17.12	4.07	45	4.160	.000
	Control	21	12.76	2.81			

The results of the t-test for unrelated samples are given in Table 2. For unrelated samples, the difference between the mean achievement test scores according to the t-test (X_d=17.12, X_k=12.76) was found to be statistically significant (t=(45)=4.160, p<.05).

Findings and Comments on the Third Sub-Problem

The "7th grade Achievement Test for Geometry and Measurement Learning Field" pre-test and post-test were applied to the experimental group students who were educated in the learning environment designed according to the ADDIE instructional design model in teaching the subjects related to the 7th grade Geometry and Measurement learning field, and the control group students where the textbook-based teaching was carried out. -Is there a significant difference between their academic achievements according to test measurement scores?

The relationship between the pre-test and post-test measurements of the experimental group and control group students was examined separately.

Table 3. The results of the t-test for the related samples according to the pre-test post-test application data of the experimental group Achievement Test Related to Geometry and Measurement Learning Field

Test Name	Measurement	n	Arithmetic mean	Ss	Sd	t	p
Achievement Test	Pre-test	26	9.54	3.34	25	-10.093	.000
	Post-test	26	17.12	4.07			

According to Table 3, there is a significant difference between the pre-test and post-test scores of the experimental group students.

Table 4. The results of the t-test for the related samples according to the pre-test post-test application data of the Achievement Test Related to the Learning Field of Geometry and Measurement in the control group

Test name	Measurement	n	Arithmetic mean	Ss	Sd	t	p
Achievement Test	Pre-test	21	9.81	2.56	20	-4.070	.001
	Post-test	21	12.76	2.81			

According to Table 4, there is a significant difference between the pre-test and post-test scores of the control group students.

Discussion, Conclusions and Suggestions

In the conducted research, pre-test and post-test research method with control group was adopted in a quasi-experimental design. In the quasi-experimental research process, before and after the implementation process, the academic success of the students was determined by the Achievement Test in the Field of Geometry and Measurement Learning. According to the analysis of the achievement test pre-test scores of the experimental and control groups before the applications, it was understood that the two groups were equivalent in terms of success.

According to the achievement test pre-test scores, the mean score of the experimental group students was 9.54 and the mean score of the control group students was 9.81. According to the t-test results for unrelated samples, it was concluded that there was no significant difference between the pre-test scores of the two groups. After the completion of the applications in the research process, the achievement test was applied to both groups as a post-test. After the application, the average post-test achievement score of the experimental group was found to be 17.12, and the average of the post-test achievement score of the control group was 12.81. According to the t-test results for unrelated samples, there was a significant difference between the post-test scores of the two groups.

The difference between the achievement test post-test mean scores was statistically significant and found in favor of the experimental group. This result; It has been shown that the students who have undergone the learning process by using the mobile application named GeoHepta are more successful than the students who have had the learning process based on the textbook. This shows that the use of mobile applications in mathematics lessons increases the success of students. As a result of the research, it was determined that the scores of the two groups from the post-test increased compared to the scores they reached from the pre-test. To determine whether this increase in the scores of the groups was statistically significant, the t-test was applied for the related samples. From the obtained results, it was concluded that the increase in the mean scores of both groups was statistically significant. When the studies in the literature are examined, it is seen that the use of mobile applications in the learning process increases the academic success of the students in the mathematics course (Aktaş, Bulut, & Aktaş, 2018; Baş & Ulum, 2019; Çetinkaya, 2019; Fabian, Topping & Barron, 2018; Kazu, Aral & Mertoğlu, 2016; Sincuba and John, 2017) are available. In the literature, there are few studies comparing the effect of teaching by using mobile applications in the learning process on the academic success of students and the effects of using learning materials other than mobile applications on the academic achievement of students. Kazu et al. (2016); no significant difference was found in the participation and motivation of the students in the experimental and control groups of the 11th grade students in the mathematics teaching using a mobile device. In their study, Meriçelli and Uluyol (2016) examined the success and motivation of students who were taught in a mobile supported learning environment and web supported learning environment. According to the results of the study, no significant difference was found between the academic achievement and motivation scores of the students studying in web-supported blended and mobile-assisted blended learning environments.

When the results of this study are evaluated together with the research findings in the literature, it can be said that the application named GeoHepta, which provides the opportunity to learn on both the mobile application and the web page, has a positive effect on the academic success of the students in the mathematics course. Before the research application, it was determined that the experimental and control groups were equivalent to each other in terms of success. In the findings part of the second sub-problem of the study, a statistically significant difference was determined between the post-test scores of the two groups in terms of success. While there was no significant difference between the achievement test scores of the groups before the quasi-experimental study, there was a significant difference between the achievement test scores of the groups after the experimental study. In the findings of the third sub-problem of the study, there is a significant difference between the pre-test and post-test scores of the experimental group students. According to the achievement test pre-test and post-test scores, the post-test scores of the experimental group were found to have a higher average than the pre-test scores. There is also a significant difference between the pre-test and post-test scores of the control group students. The post-test and pre-test scores of the control group students were found to be closer to each other than the experimental group students' scores. From the obtained results; It is understood that the use of GeoHepta in the application process in the mathematics course within the scope of the experimental research contributes more positively to the academic success of the students. According to the results of the research, it can be stated that the positive effect of GeoHepta on success is related to the following features:

• With GeoHepta, which was developed according to the design model, the students were provided with the opportunity to reach the Geometry and Measurement learning field acquisitions in accordance with their individual learning speeds, and to examine and repeat the activities whenever they want. With this feature of GeoHepta, it can be said that students benefit from the activities and study questions in the application from anywhere in the classroom or outside of the classroom via mobile devices, which provides an advantage compared to students who have undergone a textbook-based learning process.

• Çetinkaya (2019), Gök (2020); In their study, they stated that the correct use of mobile application tools in technology-assisted mathematics education has a positive effect on students' mathematics achievement. In the developed mobile application, the dynamic mathematics software is arranged in a way that enables students to discover the conceptual features of the subjects through activities during the learning process. In this way, it can be said that students are able to establish dynamic connections between representations on the GeoGebra 6.0 software related to the concepts and make associations by showing the ability to think analytically interactively. For example; It is seen that they can discover the angle properties of polygons according to the changes in the representations by following the properties of the polygons formed according to the changing number of sides in the graphic representation on the software, by following the algebraic representation of the association between the number of sides and angle measures of the polygons. In this way, allowing students to discover the topics and enable them to learn meaningfully through similar activity questions can be a positive factor in increasing the success of the students.

• The interactive structure of GeoHepta provides students with an interactive learning environment. With this structure of GeoHepta, students progress by getting feedback on the results of their actions during the learning process. Feedback can be provided while learning the concepts through the software and solving the evaluation questions created with both web 2.0 tools and the mobile application. In this way, students have the opportunity to make instant evaluations in their learning processes, and they can reach generalizations by shaping their thoughts conceptually.

• In the experimental process, it is seen that the students have a desire to enter the GeoHepta application and solve the questions created with activities and web 2.0 tools on geometry topics. In particular, it was observed during the research that students resolved the evaluations made with web 2.0 tools such as Kahoot and Socrative, eagerly and eagerly. It is thought that this factor has positive reflections on the success of the students. In the literature, there are various studies on the positive effects of technology-supported formative assessments (Socrative, Kahoot, Plickers, …) on students' participation in mathematics lessons (Wood, Brown, & Grayson, 2017; Zengin, Bars, & Şimşek, 2017)

Suggestions

Depending on the results of the research, some suggestions were made below.

• In the study, it was determined that the mobile learning environment created through the mobile application named GeoHepta was more effective on the academic achievement of the students than the learning environment based on the textbook. Based on this result, it is recommended that teachers use mobile applications in the lessons on the subjects included in the mathematics lesson for each grade level.

• Due to the Covid-19 pandemic, the research was piloted and the main application was carried out during the distance education process. When schools switch to face-to-face education by developing similar mobile applications, similar and different effects on learning can be investigated by applying them for learning purposes.

Geliştirilen Mobil Uygulamayla Tasarlanmış Geometri Öğretiminin Öğrencilerin Akademik Başarılarına Etkisi

Özet:

Uzaktan öğretim döneminde öğrencilerin derslere aktif katılımlarını sağlamak için farklı öğrenme araçları arayışlarında bulunulmuştur. Bu amaçla ADDIE tasarım modeli çerçevesinde 7. sınıf matematik dersi Geometri ve Ölçme öğrenme alanı konularının öğrenilmesinde kullanılmak üzere GeoHepta isimli bir mobil uygulaması geliştirilmek istenilmiştir. Çalışma, 2020-2021 eğitim öğretim yılında 7. sınıf öğrencileriyle uzaktan öğretim sürecinde yapılmıştır. Araştırma yarı deneysel desen ile gerçekleştirilmiştir. Çalışmada "Geometri ve Ölçme Öğrenme Alanına İlişkin Başarı Testi" kullanılmıştır. Araştırma sonucunda mobil uygulama destekli öğretimin gerçekleştiği deney grubu öğrencileri ile ders kitabına dayalı öğretimin gerçekleştiği kontrol grubu öğrencilerinin son test puanları arasında anlamlı bir farklılığa ulaşılmıştır. Deney grubu öğrencilerinin ön-test son-test puanları arasında anlamlı bir farklılık görülürken, kontrol grubu öğrencilerinin ön-test son-test puanları arasında anlamlı farklılık görülmemiştir.

Anahtar kelimeler: geometri eğitimi, matematik başarısı, ADDIE tasarım modeli, mobil öğrenme, uzaktan öğretim

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