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INDEXES AND DATABASES

IDUNAS is indexed by 4 national and international databases.

INDEXED DATABASES	
ACARINDEX	https://www.acarindex.com/search?type=article&q=Natural+and+Applied+Sciences+Journal
Academic Resource Index ResearchBib	https://journalseeker.researchbib.com/view/issn/2645-9000
Google Scholar	https://scholar.google.com.tr/scholar?q=izmir+democracy+university+natural+and+applied+sciences+journal&hl=tr&as_sdt=0&as_vis=1&oi=scholart
Ideal Online	https://www.idealonline.com.tr/IdealOnline/
APPLIED DATABASES	
Directory of Research Journals Indexing	http://olddrji.lbp.world/
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ABOUT

Natural & Applied Sciences Journal is an international open access, peer-reviewed, free of cost academic journal that includes articles and reviews on natural and applied sciences. It is published twice a year in June and December. Our journal accepts only English content from the second issue.

AIM & SCOPE

It aims to contribute to the knowledge of the field of natural and applied sciences by publishing qualified scientific studies in the field.

Natural & Applied Sciences Journal is an international open access, peer-reviewed, free of cost academic journal that includes articles and reviews on natural and applied sciences published by İzmir Democracy University. It is published twice a year in June and December. Our journal accepts only English content from the second issue.

PERIODS

June and December

ETHICAL PRINCIPLES AND PUBLICATION POLICY

All articles submitted for publication in the journal must conform to the ethical rules of scientific research. The authors of each article are required to sign the Copyright Form confirming that they have granted permission for their work to be published in Natural and Applied Science Journal. Publication will not take place, even if the manuscript is accepted without this form. Authors are solely responsible for the content of their articles and any responsibilities they may incur regarding copyrights. The work submitted to the journal should not have been published in any language, in any journal, or in the process of being evaluated in any other publication.

Articles should be prepared in accordance with the general ethical rules specified by DOI and DOAJ. Plagiarism Control All articles submitted to Natural and Applied Science Journal are checked using the iThenticate plagiarism detection software. Based on the similarity report generated by the software, the editorial board determines whether the article should be submitted to peer review or rejected.

PRICE POLICY

No fee is charged from the author or institution under any name.

INTERNATIONAL STANDARDS FOR AUTHORS

RESPONSIBLE RESEARCH PUBLICATION

A position statement developed at the 2nd World Conference on Research Integrity, Singapore, July 22-24, 2010.

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SUMMARY

• The research being reported should have been conducted in an ethical and responsible manner and should comply with all relevant legislation.

• Researchers should present their results clearly, honestly, and without fabrication, falsification, or inappropriate data manipulation.

• Researchers should strive to describe their methods clearly and unambiguously so that their findings can be confirmed by others.

• Researchers should adhere to publication requirements that submitted work is original, is not plagiarized, and has not been published elsewhere.

• Authors should take collective responsibility for submitted and published work.

• The authorship of research publications should accurately reflect individuals' contributions to the work and its reporting.

• Funding sources and relevant conflicts of interest should be disclosed.

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INTRODUCTION

Publication is the final stage of research and therefore a responsibility for all researchers. Scholarly publications are expected to provide a detailed and permanent record of research. Because publications form the basis for both new research and the application of findings, they can affect not only the research community but also, indirectly, society at large. Researchers therefore have a responsibility to ensure that their publications are honest, clear, accurate, complete and balanced, and should avoid misleading, selective or ambiguous reporting. Journal editors also have responsibilities for ensuring the integrity of the research literature and these are set out in companion guidelines.

This document aims to establish international standards for authors of scholarly research publications and to describe responsible research reporting practice. We hope these standards will be endorsed by research institutions, funders, and professional societies; promoted by editors and publishers; and will aid in research integrity training.

Responsible research publication

1 Soundness and reliability

1.1 The research being reported should have been conducted in an ethical and responsible manner and follow all relevant legislation. [See also the Singapore Statement on Research Integrity, www.singaporestatement.org]

1.2 The research being reported should be sound and carefully executed.

1.3 Researchers should use appropriate methods of data analysis and display (and, if needed, seek, and follow specialist advice on this).

1.4 Authors should take collective responsibility for their work and for the content of their publications. Researchers should check their publications carefully at all stages to ensure methods and findings are reported accurately. Authors should carefully check calculations, data presentations, typescripts/submissions, and proofs.

2 Honesty

2.1 Researchers should present their results honestly and without fabrication, falsification, or inappropriate data manipulation. Research images (e.g. micrographs, X-rays, pictures of electrophoresis gels) should not be modified in a misleading way.

2.2 Researchers should strive to describe their methods and to present their findings clearly and unambiguously. Researchers should follow applicable reporting guidelines. Publications should provide sufficient detail to permit experiments to be repeated by other researchers.

2.3 Reports of research should be complete. They should not omit inconvenient, inconsistent, or inexplicable findings or results that do not support the authors' or sponsors' hypothesis or interpretation.

2.4 Research funders and sponsors should not be able to veto publication of findings that do not favor their product or position. Researchers should not enter agreements that permit the research sponsor to veto or control the publication of the findings (unless there are exceptional circumstances, such as research classified by governments because of security implications).

2.5 Authors should alert the editor promptly if they discover an error in any submitted, accepted or published work. Authors should cooperate with editors in issuing corrections or retractions when required.

2.6 Authors should represent the work of others accurately in citations and quotations.

2.7 Authors should not copy references from other publications if they have not read the cited work.

3 Originality

3.1 Authors should adhere to publication requirements that submitted work is original and has not been published elsewhere in any language. Work should not be submitted concurrently to more than one publication unless the editors have agreed to co-publication. If articles are co-published this fact should be made clear to readers.

3.2 Applicable copyright laws and conventions should be followed. Copyright material (e.g. tables, figures, or extensive quotations) should be reproduced only with appropriate permission and acknowledgement.

3.3 Relevant previous work and publications, both by other researchers and the authors' own, should be properly acknowledged, and referenced. The primary literature should be cited where possible.

3.4 Data, text, figures, or ideas originated by other researchers should be properly acknowledged and should not be presented as if they were the authors' own. Original wording taken directly from publications by other researchers should appear in quotation marks with the appropriate citations.

3.5 Authors should inform editors if findings have been published previously or if multiple reports or multiple analyses of a single data set are under consideration for publication elsewhere. Authors should provide copies of related publications or work submitted to other journals.

3.6 Multiple publications arising from a single research project should be clearly identified as such and the primary publication should be referenced. Translations and adaptations for different audiences should be clearly identified as such, should acknowledge the original source, and should respect relevant copyright conventions and permission requirements. If in doubt, authors should seek permission from the original publisher before republishing any work.

4 Appropriate authorship and acknowledgement

4.1 The research literature serves as a record not only of what has been discovered but also of who made the discovery. The authorship of research publications should therefore accurately reflect individuals' contributions to the work and its reporting.

4.2 In cases where major contributors are listed as authors while those who made less substantial, or purely technical, contributions to the research or to the publication are listed in an acknowledgement section, the criteria for authorship and acknowledgement should be agreed at the start of the project. Ideally, authorship criteria within a particular field should be agreed, published and consistently applied by research institutions, professional and academic societies, and funders. While journal editors should publish and promote accepted authorship criteria appropriate to their field, they cannot be expected to adjudicate in authorship disputes. Responsibility for the correct attribution of authorship lies with authors themselves working under the guidance of their institution. Research institutions should promote and uphold fair and accepted standards of authorship and acknowledgement. When required, institutions should adjudicate in authorship disputes and should ensure that due process is followed.

4.3 Researchers should ensure that only those individuals who meet authorship criteria (i.e. made a substantial contribution to the work) are rewarded with authorship and that deserving authors are not omitted. Institutions and journal editors should encourage practices that prevent guest, gift, and ghost authorship.

Note:

- Guest authors are those who do not

- Gift authors are those who do meet accepted authorship criteria but are listed because of their seniority, reputation or supposed influence not

- Ghost authors are those who meet authorship criteria but are not listed meet accepted authorship criteria but are listed as a personal favor or in return for payment

4.4 All authors should agree to be listed and should approve the submitted and accepted versions of the publication. Any change to the author list should be approved by all authors including any who have been removed from the list. The corresponding author should act as a point of contact between the editor and the other authors and should keep co-authors informed and involve them in major decisions about the publication (e.g. responding to reviewers' comments).

4.5 Authors should not use acknowledgements misleadingly to imply a contribution or endorsement by individuals who have not, in fact, been involved with the work or given an endorsement.

5 Accountability and responsibility

5.1 All authors should have read and be familiar with the reported work and should ensure that publications follow the principles set out in these guidelines. In most cases, authors will be expected to take joint responsibility for the integrity of the research and its reporting. However, if authors take responsibility only for certain aspects of the research and its reporting, this should be specified in the publication.

5.2 Authors should work with the editor or publisher to correct their work promptly if errors or omissions are discovered after publication.

5.3 Authors should abide by relevant conventions, requirements, and regulations to make materials, reagents, software, or datasets available to other researchers who request them. Researchers, institutions, and funders should have clear policies for handling such requests. Authors must also follow relevant journal standards. While proper acknowledgement is expected, researchers should not demand authorship as a condition for sharing materials.

5.4 Authors should respond appropriately to post-publication comments and published correspondence. They should attempt to answer correspondents' questions and supply clarification, or additional details where needed.

6 Adherence to peer review and publication conventions

6.1 Authors should follow publishers' requirements that work is not submitted to more than one publication for consideration at the same time.

6.2 Authors should inform the editor if they withdraw their work from review or choose not to respond to reviewer comments after receiving a conditional acceptance.

6.3 Authors should respond to reviewers' comments in a professional and timely manner.

6.4 Authors should respect publishers' requests for press embargos and should not generally allow their findings to be reported in the press if they have been accepted for publication (but not yet published) in a scholarly publication. Authors and their institutions should liaise and cooperate with publishers to coordinate media activity (e.g. press releases and press conferences) around publication. Press releases should accurately reflect the work and should not include statements that go further than the research findings.

7 Responsible reporting of research involving humans or animals

7.1 Appropriate approval, licensing or registration should be obtained before the research begins and details should be provided in the report (e.g. Institutional Review Board, Research Ethics Committee approval, national licensing authorities for the use of animals).

7.2 If requested by editors, authors should supply evidence that reported research received the appropriate approval and was carried out ethically (e.g. copies of approvals, licenses, participant consent forms).

7.3 Researchers should not generally publish or share identifiable individual data collected in the course of research without specific consent from the individual (or their representative). Researchers should remember that many scholarly journals are now freely available on the internet and should therefore be mindful of the risk of causing danger or upset to unintended readers (e.g. research participants or their families who recognize themselves from case studies, descriptions, images, or pedigrees).

7.4 The appropriate statistical analyses should be determined at the start of the study and a data analysis plan for the prespecified outcomes should be prepared and followed. Secondary or post hoc analyses should be distinguished from primary analyses and those set out in the data analysis plan.

7.5 Researchers should publish all meaningful research results that might contribute to understanding. In particular, there is an ethical responsibility to publish the findings of all clinical trials. The publication of unsuccessful studies or experiments that reject a hypothesis may help prevent others from wasting time and resources on similar projects. If findings from small studies and those that fail to reach statistically significant results can be combined to produce more useful information (e.g. by meta-analysis) then such findings should be published.

7.6 Authors should supply research protocols to journal editors if requested (e.g. for clinical trials) so that reviewers and editors can compare the research report to the protocol to check that it was carried out as planned and that no relevant details have been omitted. Researchers should follow relevant requirements for clinical trial registration and should include the trial registration number in all publications arising from the trial.

INTERNATIONAL STANDARDS FOR EDITORS

RESPONSIBLE RESEARCH PUBLICATION

A position statement developed at the 2nd World Conference on Research Integrity, Singapore, July 22-24, 2010.

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Summary

- Editors are accountable and should take responsibility for everything they publish

- Editors should make fair and unbiased decisions independent from commercial consideration and ensure a fair and appropriate peer review process

- Editors should adopt editorial policies that encourage maximum transparency and complete, honest reporting

- Editors should guard the integrity of the published record by issuing corrections and retractions when needed and pursuing suspected or alleged research and publication misconduct

- Editors should pursue reviewer and editorial misconduct

- Editors should critically assess the ethical conduct of studies in humans and animals

- Peer reviewers and authors should be told what is expected of them

- Editors should have appropriate policies in place for handling editorial conflicts of interest

Introduction

As guardians and stewards of the research record, editors should encourage authors to strive for, and adhere themselves to, the highest standards of publication ethics. Furthermore, editors are in a unique position to indirectly foster responsible conduct of research through their policies and processes. To achieve the maximum effect within the research community, ideally all editors should adhere to universal standards and good practices. While there are important differences between different fields and not all areas covered are relevant to each research community, there are important common editorial policies, processes, and principles that editors should follow to ensure the integrity of the research record.

These guidelines are a starting point and are aimed at journal editors in particular. While books and monographs are important and relevant research records in many fields, guidelines for book editors are beyond the scope of these recommendations. It is hoped that in due course such guidelines can be added to this document.

Editors should regard themselves as part of the wider professional editorial community, keep themselves abreast of relevant policies and developments, and ensure their editorial staff is trained and kept informed of relevant issues.

To be a good editor requires many more principles than are covered here. These suggested principles, policies, and processes are particularly aimed at fostering research and publication integrity.

Editorial Principles

1. Accountability and responsibility for journal content

Editors have to take responsibility for everything they publish and should have procedures and policies in place to ensure the quality of the material they publish and maintain the integrity of the published record (see paragraphs 4-8).

2. Editorial independence and integrity

An important part of the responsibility to make fair and unbiased decisions is the upholding of the principle of editorial independence and integrity.

2.1 Separating decision-making from commercial considerations

Editors should make decisions on academic merit alone and take full responsibility for their decisions. Processes must be in place to separate commercial activities within a journal from editorial processes and decisions. Editors should take an active interest in the publisher's pricing policies and strive for wide and affordable accessibility of the material they publish.

Sponsored supplements must undergo the same rigorous quality control and peer review as any other content for the journal. Decisions on such material must be made in the same way as any other journal content. The sponsorship and role of the sponsor must be clearly declared to readers.

Advertisements need to be checked so that they follow journal guidelines, should be clearly distinguishable from other content, and should not in any way be linked to scholarly content.

2.2 Editors' relationship to the journal publisher or owner

Editors should ideally have a written contract setting out the terms and conditions of their appointment with the journal publisher or owner. The principle of editorial independence should be clearly stated in this contract. Journal publishers and owners should not have any role in decisions on content for commercial or political reasons. Publishers should not dismiss an editor because of any journal content unless there was gross editorial misconduct, or an independent investigation has concluded that the editor's decision to publish was against the journal's scholarly mission.

2.3 Journal metrics and decision-making

Editors should not attempt to inappropriately influence their journal's ranking by artificially increasing any journal metric. For example, it is inappropriate to demand that references to that journal's articles are included except for genuine scholarly reasons. In general, editors should ensure that papers are reviewed on purely scholarly grounds and that authors are not pressured to cite specific publications for non- scholarly reasons.

3. Editorial confidentiality

3.1 Authors' material

If a journal operates a system where peer reviewers are chosen by editors (rather than posting papers for all to comment as a pre-print version), editors must protect the confidentiality of authors' material and remind reviewers to do so as well. In general, editors should not share submitted papers with editors of other journals, unless with the authors' agreement or in cases of alleged misconduct (see below). Editors are generally under no obligation to provide material to lawyers for court cases. Editors should not give any indication of a paper's status with the journal to anyone other than the authors. Web-based submission systems must be run in a way that prevents unauthorized access.

In the case of a misconduct investigation, it may be necessary to disclose material to third parties (e.g., an institutional investigation committee or other editors).

3.2 Reviewers

Editors should protect reviewers' identities unless operating an open peer review system. However, if reviewers wish to disclose their names, this should be permitted.

If there is alleged or suspected reviewer misconduct it may be necessary to disclose a reviewer's name to a third party.

General editorial policies

4. Encourage maximum transparency and complete and honest reporting

To advance knowledge in scholarly fields, it is important to understand why particular work was done, how it was planned and conducted and by whom, and what it adds to current knowledge. To achieve this understanding, maximum transparency and complete and honest reporting are crucial.

4.1 Authorship and responsibility

Journals should have a clear policy on authorship that follows the standards within the relevant field. They should give guidance in their information for authors on what is expected of an author and, if there are different authorship conventions within a field, they should state which they adhere to.

For multidisciplinary and collaborative research, it should be apparent to readers who has done what and who takes responsibility for the conduct and validity of which aspect of the research. Each part of the work should have at least one author who takes responsibility for its validity. For example, individual contributions and responsibilities could be stated in a contributor section. All authors are expected to have contributed significantly to the paper and to be familiar with its entire content and ideally, this should be declared in an authorship statement submitted to the journal.

When there are undisputed changes in authorship for appropriate reasons, editors should require that all authors (including any whose names are being removed from an author list) agree these in writing. Authorship disputes (i.e., disagreements on who should or should not be an author before or after publication) cannot be adjudicated by editors and should be resolved at institutional level or through other appropriate independent bodies for both published and unpublished papers. Editors should then act on the findings, for example by correcting authorship in published papers.

Journals should have a publicly declared policy on how papers submitted by editors or editorial board members are handled (see paragraph on editorial conflicts of interest: 8.2).

4.2 Conflicts of interest and role of the funding source

Editors should have policies that require all authors to declare any relevant financial and non-financial conflicts of interest and publish at least those that might influence a reader's perception of a paper, alongside the paper. The funding source of the research should be declared and published, and the role of the funding source in the conception, conduct, analysis, and reporting of the research should be stated and published.

Editors should make it clear in their information for authors if in certain sections of the journal (e.g., commissioned commentaries or review articles) certain conflicts of interest preclude authorship.

4.3 Full and honest reporting and adherence to reporting guidelines

Among the most important responsibilities of editors is to maintain a high standard in the scholarly literature. Although standards differ among journals, editors should work to ensure that all published papers make a substantial new contribution to their field. Editors should discourage so-called 'salami publications' (i.e., publication of the minimum publishable unit of research), avoid duplicate or redundant publication unless it is fully declared and acceptable to all (e.g., publication in a different language with cross-referencing), and encourage authors to place their work in the context of previous work (i.e., to state why this work was necessary/done, what this work adds or why a replication of previous work was required, and what readers should take away from it).

Journals should adopt policies that encourage full and honest reporting, for example, by requiring authors in fields where it is standard to submit protocols or study plans, and, where they exist, to provide evidence of adherence to relevant reporting guidelines. Although devised to improve reporting, adherence to reporting guidelines also makes it easier for editors, reviewers, and readers to judge the actual conduct of the research.

Digital image files, figures, and tables should adhere to the appropriate standards in the field. Images should not be inappropriately altered from the original or present findings in a misleading way.

Editors might also consider screening for plagiarism, duplicate or redundant publication by using anti-plagiarism software, or for image manipulation. If plagiarism or fraudulent image manipulation is detected, this should be pursued with the authors and relevant institutions (see paragraph on how to handle misconduct: 5.2)

5. Responding to criticisms and concerns

Reaction and response to published research by other researchers is an important part of scholarly debate in most fields and should generally be encouraged. In some fields, journals can facilitate this debate by publishing readers' responses. Criticisms may be part of a general scholarly debate but can also highlight transgressions of research or publication integrity.

5.1 Ensuring integrity of the published record - corrections

When genuine errors in published work are pointed out by readers, authors, or editors, which do not render the work invalid, a correction (or erratum) should be published as soon as possible. The online version of the paper may be corrected with a date of correction and a link to the printed erratum. If the error renders the work or substantial parts of it invalid, the paper should be retracted with an explanation as to the reason for retraction (i.e., honest error).

5.2 Ensuring the integrity of the published record – suspected research or publication misconduct

If serious concerns are raised by readers, reviewers, or others, about the conduct, validity, or reporting of academic work, editors should initially contact the authors (ideally all authors) and allow them to respond to the concerns. If that response is unsatisfactory, editors should take this to the institutional level (see below). In rare cases, mostly in the biomedical field, when concerns are very serious and the published work is likely to influence clinical practice or public health, editors should consider informing readers about these concerns, for example by issuing an 'expression of concern', while the investigation is ongoing. Once an investigation is concluded, the appropriate action needs to be taken by editors with an accompanying comment that explains the findings of the investigation. Editors should also respond to findings from national research integrity organisations that indicate misconduct relating to a paper published in their journal. Editors can themselves decide to retract a paper if they are convinced that serious misconduct has happened even if an investigation by an institution or national body does not recommend it.

Editors should respond to all allegations or suspicions of research or publication misconduct raised by readers, reviewers, or other editors. Editors are often the first recipients of information about such concerns and should act, even in the case of a paper that has not been accepted or has already been rejected. Beyond the specific responsibility for their journal's publications, editors have a collective responsibility for the research record and should act whenever they become aware of potential misconduct if at all possible. Cases of possible plagiarism or duplicate/redundant publication can be assessed by editors themselves. However, in most other cases, editors should request an investigation by the institution or other appropriate bodies (after seeking an explanation from the authors first and if that explanation is unsatisfactory).

Retracted papers should be retained online, and they should be prominently marked as a retraction in all online versions, including the PDF, for the benefit of future readers.

For further guidance on specific allegations and suggested actions, such as retractions, see the COPE flowcharts and retraction guidelines (http://publicationethics.org/flowcharts; http://publicationethics.org/files/u661/Retractions_COPE_gline_final_3_Sept_09__2_.pdf).

5.3 Encourage scholarly debate

All journals should consider the best mechanism by which readers can discuss papers, voice criticisms, and add to the debate (in many fields this is done via a print or on-line correspondence section). Authors may contribute to the debate by being allowed to respond to comments and criticisms where relevant. Such scholarly debate about published work should happen in a timely manner. Editors should clearly distinguish between criticisms of the limitations of a study and criticisms that raise the possibility of research misconduct. Any criticisms that raise the possibility of misconduct should not just be published but should be further investigated even if they are received a long time after publication. Editorial policies relevant only to journals that publish research in humans or animals.

6. Critically assess and require a high standard of ethical conduct of research

Especially in biomedical research but also in social sciences and humanities, ethical conduct of research is paramount in the protection of humans and animals. Ethical oversight, appropriate consent procedures, and adherence to relevant laws are required from authors. Editors need to be vigilant to concerns in this area.

6.1 Ethics approval and ethical conduct

Editors should generally require approval of a study by an ethics committee (or institutional review board) and the assurance that it was conducted according to the Declaration of Helsinki for medical research in humans but, in addition, should be alert to areas of concern in the ethical conduct of research. This may mean that a paper is sent to peer reviewers with particular expertise in this area, to the journal's ethics committee if there is one, or that editors require further reassurances or evidence from authors or their institutions.

Papers may be rejected on ethical grounds even if the research had ethics committee approval.

6.2 Consent (to take part in research)

If research is done in humans, editors should ensure that a statement on the consent procedure is included in the paper. In most cases, written informed consent is the required norm. If there is any concern about the consent procedure, if the research is done in vulnerable groups, or if there are doubts about the ethical conduct, editors should ask to see the consent form and enquire further from authors, exactly how consent was obtained.

6.3 Consent (for publication)

For all case reports, small case series, and images of people, editors should require the authors to have obtained explicit consent for publication (which is different from consent to take part in research). This consent should inform participants which journal the work will be published in, make it clear that, although all efforts will be made to remove unnecessary identifiers, complete anonymity is not possible, and ideally state that the person described has seen and agreed with the submitted paper.

The signed consent form should be kept with the patient file rather than sent to the journal (to maximize data protection and confidentiality, see paragraph 6.4). There may be exceptions where it is not possible to obtain consent, for example when the person has died. In such cases, a careful consideration about possible harm is needed and out of courtesy attempts should be made to obtain assent from relatives. In very rare cases, an important public health message may justify publication without consent if it is not possible despite all efforts to obtain consent and the benefit of publication outweighs the possible harm.

6.4 Data protection and confidentiality

Editors should critically assess any potential breaches of data protection and patient confidentiality. This includes requiring properly informed consent for the actual research presented consent for publication where applicable (see paragraph 6.3) and having editorial policies that comply with guidelines on patient confidentiality.

6.5 Adherence to relevant laws and best practice guidelines for ethical conduct

Editors should require authors to adhere to relevant national and international laws and best practice guidelines where applicable, for example when undertaking animal research. Editors should encourage registration of clinical trials.

Editorial Processes

7. Ensuring a fair and appropriate peer review process

One of the most important responsibilities of editors is organizing and using peer review fairly and wisely. Editors should explain their peer review processes in the information for authors and also indicate which parts of the journal are peer reviewed.

7.1 Decision whether to review

Editors may reject a paper without peer review when it is deemed unsuitable for the journal's readers or is of poor quality. This decision should be made in a fair and unbiased way. The criteria used to make this decision should be made explicit. The decision not to send a paper for peer review should only be based on the academic content of the paper and should not be influenced by the nature of the authors or the host institution.

7.2 Interaction with peer reviewers

Editors should use appropriate peer reviewers for papers that are considered for publication by selecting people with sufficient expertise and avoiding those with conflicts of interest. Editors should ensure that reviews are received in a timely manner.

Peer reviewers should be told what is expected of them and should be informed about any changes in editorial policies. In particular, peer reviewers should be asked to assess research and publication ethics issues (i.e., whether they think the research was done and reported ethically, or if they have any suspicions of plagiarism, fabrication, falsification, or redundant publication). Editors should have a policy to request a formal conflict of interest declaration from peer reviewers and should ask peer reviewers to inform them about any such conflict of interest at the earliest opportunity so that they can make a decision on whether an unbiased review is possible. Certain conflicts of interest may disqualify a peer reviewer. Editors should stress confidentiality of the material to peer reviewers and should require peer reviewers to inform them when they ask a colleague for help with a review or if they mentor a more junior colleague in conducting peer review. Editors should ideally have a mechanism to monitor the quality and timeliness of peer review and to provide feedback to reviewers.

7.3 Reviewer misconduct

Editors must take reviewer misconduct seriously and pursue any allegation of breach of confidentiality, non-declaration of conflicts of interest (financial or non-financial), inappropriate use of confidential material, or delay of peer review for competitive advantage. Allegations of serious reviewer misconduct, such as plagiarism, should be taken to the institutional level (for further guidance see: http://publicationethics.org/files/u2/07_Reviewer_misconduct.pdf).

7.4 Interaction with authors

Editors should make it clear to authors what the role of the peer reviewer is because this may vary from journal to journal. Some editors regard peer reviewers as advisors and may not necessarily follow (or even ask for) reviewers' recommendations on acceptance or rejection. Correspondence from editors is usually with the corresponding author, who should guarantee to involve co-authors at all stages. Communicating with all authors at first submission and at final acceptance stage can be helpful to ensure all authors are aware of the submission and have approved the publication. Normally, editors should pass on all peer reviewers' comments in their entirety. However, in exceptional cases, it may be necessary to exclude parts of a review, if it, for example, contains libelous or offensive remarks. It is important, however, that such editorial discretion is not inappropriately used to suppress inconvenient comments.

There should always be good reasons, which are clearly communicated to authors, if additional reviewers are sought at a late stage in the process.

The final editorial decision and reasons for this should be clearly communicated to authors and reviewers. If a paper is rejected, editors should ideally have an appeals process. Editors, however, are not obliged to overturn their decision.

8. Editorial decision-making

Editors are in a powerful position by making decisions on publications, which makes it very important that this process is as fair and unbiased as possible, and is in accordance with the academic vision of the particular journal.

8.1 Editorial and journal processes

All editorial processes should be made clear in the information for authors. In particular, it should be stated what is expected of authors, which types of papers are published, and how papers are handled by the journal. All editors should be fully familiar with the journal policies, vision, and scope. The final responsibility for all decisions rests with the editor-in-chief.

8.2 Editorial conflicts of interest

Editors should not be involved in decisions about papers in which they have a conflict of interest, for example if they work or have worked in the same institution and collaborated with the authors, if they own stock in a particular company, or if they have a personal relationship with the authors. Journals should have a defined process for handling such papers. Journals should also have a process in place to handle papers submitted by editors or editorial board members to ensure unbiased and independent handling of such papers. This process should be stated in the information for authors. Editorial conflicts of interests should be declared, ideally publicly.

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Investigation of the Tensile and Mixed Mode (tensile and shear) Fracture Properties of Cement-stabilized Soils by Numerical Analysis

Research Article

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Abstract

In this study, crack initiation, crack propagation, and fracture failure of soil specimens stabilized with cement, an elasto-plastic material, are investigated by numerical analyses. There is no international standard recommended in the literature to find the mode I and mixed mode I-II (tensile and shear) failure values of reinforced soil materials. The aim of this study is to investigate the applicability of ASTM C78, an international standard recommended for concrete specimens, for both indirect tensile and tensile-compression strength tests. Stress and crack analyses in beam specimens were performed using FRANC2D software. The indirect tensile fracture toughness (KIC) value of the modelled beam specimens was found to be 0.32 MPa√m. Similarly, the indirect tensile and shear fracture toughness values were found to be 0.38 MPa√m.

Both non-cohesive and cohesive crack analyses were performed in numerical modeling. Numerical analysis results showed that the most significant slipping between the cohesive crack surfaces was observed in the specimen under mixed mode I-II loading. Moreover, "wing crack" growth in cement-stabilized soil specimens was obtained in numerical modeling in accordance with the principles of fracture mechanics. It is believed that the results of this study will lead to a new international standard for the determination of mode I and mixed mode I-II fracture toughness of cement-stabilized soil specimens.
Keywords: Failure of cement stabilized soil, fractures and reinforced soil, FRANC2D and reinforced soil, cohesive fracture of cement stabilized soil

1. INTRODUCTION

Pavements are a material that distributes the stresses caused by external loads to the lower sections of the road. It is also a layer that reflects deformations and cracks from the road substrates to the surface. Axial

stresses occurring under wheel load cause compressive stress in areas close to the surface. Stresses in the vertical direction in the area immediately below the wheel are diametrical compressive stresses and these stresses cause indirect tensile stresses in the horizontal direction. Compressive stresses in the surface pavement cause rutting deformations, while indirect tensile stresses under the pavement cause tensile and fatigue cracks and deformation. At this point, it is obvious that the principles of fracture mechanics will be very useful in asphalt pavement and road foundation research. In particular, mode I (tensile) and mixed mode I-II (tensile-shear) cracks/defects in fracture mechanics have been reported in many studies in the literature to be very useful in asphalt pavement and asphalt concrete research where such both compressive and indirect tensile and shear stresses occur (Daneshfar et al. 2023).
Stabilized soils are generally used as foundation and sub-base material in road construction. Soil reinforcement with cement is a strengthening process to improve the load-bearing properties of the soil. There are various soil stabilization methods such as mechanical stabilization, chemical stabilization, and compaction to stabilize weak soil (Savran, 1988, Prasad et al. 2015, Fondjo et al., 2021, Zada et al., 2023). However, chemical stabilization with cement is an effective and widely used method to improve weak soil properties. The main aim of soil stabilization with cement is to achieve the necessary improvement in the mechanical properties of the soils in terms of environmental and loading conditions, mostly for transportation structures, water storage structures, building foundations, solid waste storage facilities, etc.

Reinforced soil with cement is an elastoplastic material composed to static loading (Davis 1991, Aliha et al. 2021, Xia et al. 2022, Zada et al. 2023, Xiushan 2023). It is known that indirect tensile and shear stresses develop in asphalt bituminous base and subbase materials under compressive stress applied by the wheel load and the asphalt pavement and foundations material loading (Crockford and Little 1987, Sophan and Das 2007, Paul and Gnanendran 2017, Mashaan et al. 2021, Takahassi et al. 2021, Guo et al. 2021, Erarslan 2023, Pietras et al. 2023, Mousavi et al.2024). Therefore, cement stabilized soil is used commonly in road base and compacted subbase materials, and it is very important to investigate its mixed mode (tensile-shear) strength and fatigue properties (Rezaeian et al. 2019, Chen et al. 2020). Fracture mechanics is the study of defects in materials such as notches, cracks and voids that increase the stress intensity and the damage that occurs due to them. The theory of Linear Elastic Fracture Mechanics (LEFM) was first proposed by Griffith (Lajtai 1971). In the theories used in crack analysis, the stress field near the crack tip of an isotropic linear elastic material is given in Fig.1. The maximum tensile stress criterion called 'Maximum Tangential Stress Criterion', which is mainly used in crack analysis, is given in Eq.1 (Aliha et al. 2022):

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} cos\left(\frac{\theta}{2}\right)\left[K_I cos^2\left(\frac{\theta}{2}\right) - \frac{3}{2}K_{II} sin(\theta)\right] + \frac{T}{2}\left(1 - cos(2\theta)\right)$$

$$\theta_c = 2tan^{-1}\left(\frac{1}{4}\left[\frac{K_I}{K_{II}} - sin(K_{II})\sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8}\right]\right) \tag{1}$$

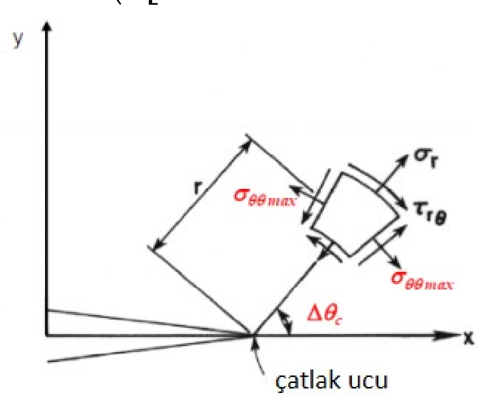


Fig.1 Stress field near the crack tip of an isotropic elastic material

The results obtained in this research will be very important in cement stabilized soil mechanics research. Because cement and lime stabilized soils are semi-friable soils and it will be essential to determine the cracking properties of such materials. It will be very important to be able to intervene in the structure before the final failure, especially in deformation properties under repeated loads and stable crack investigations, which are known as fissure cracks.

1.1 Cohesive Fracturing

Crack formation, propagation and fracture are complementary concepts. The Griffith relation gives values close to reality in glass and brittle materials (Lajtai 2002). However, permanent deformation occurs in metallic materials. Therefore, the energy released in Griffith's criterion is spent in permanent deformation while forming new surfaces. In the 1960s, experimental research on notched concrete specimens showed that parameters such as KIc (fracture toughness) varied depending on the size and geometry of the specimen. These defects of LEFM in practice are due to the presence of the fracture process zone at the tip of the crack, which occupies a larger area compared to other materials. For this reason, some non-linear fracture mechanics approaches have been proposed by some researchers to characterize this region. These models are divided into two categories: cohesive crack models and equivalent elastic crack approaches. Cohesive crack approaches model the fracture process region with a stress block at the crack tip that decreases and compresses the crack, while equivalent elastic crack approaches model it using an effective crack length. A fracture model is developed to determine the critical crack strain at peak load, defined as $\Delta a = a_c - a_0$ (where a_c is the crack length and a_0 is the pre-existing crack length). However, a_c depends on the size of the structure since the critical crack length decreases with increasing specimen size (Bazant 2002). Therefore, nonlinear fracture mechanics approaches recommend the use of at least two parameters for the fracture of concrete.

Stress-induced crack initiation in composite materials, such as cement stabilized soil and reinforced concrete, typically leads to unstable crack growth due to the plastic deformation and the fracture process zobe (FPZ) (Fig. 2) (Dugdale, 1960; Hillerborg, 1977; Behnam, 2021; Ma et al., 2022, Shahbazian and Mirsayar 2023).

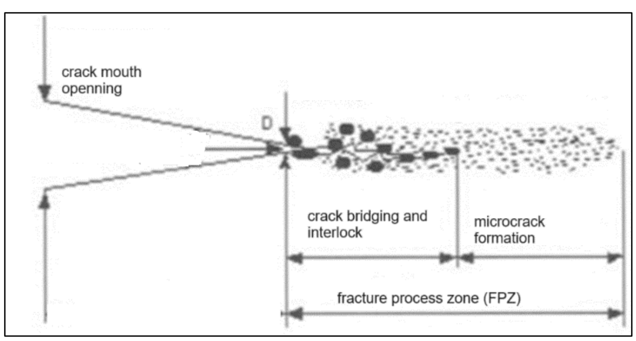


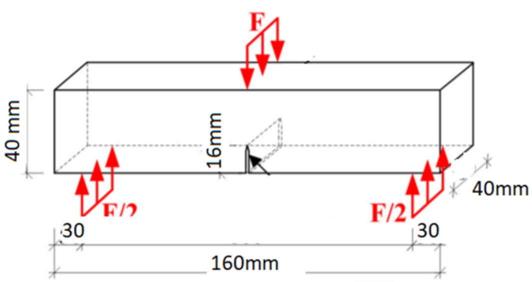
Fig. 2 Fracture process zone (FPZ) at the tip of a crack

2. MATERIAL AND METHODS

In this research, both stress analyses in the specimen and crack analyses were performed by a discrete finite element analysis program FRANC2D. In order to determine the mode I fracture mode, the notch crack was placed at the centre of the beam specimens. For shear and indirect tensile-shear loading, two notches were placed at specified distances from the centre. A specimen prepared with the beam specimen geometry and notch dimensions used is shown in Fig.3. Thus, a three-point bending test was performed by placing two supports below and a single support above the centre.

In this study, numerical analysis and modelling were carried out using the Fracture Analysis Code (FRANC2D) software. Tangential stress concentration theory is used in FRANC2D analyses (Erdoğan and Sih, 1963). While experimental studies reveal the final failure plane and surface cracks that lead to failure, such programs are highly valuable as they allow for the observation of the initiation and propagation trends of microscale cracks and regions with high stress concentrations within the tensioned sample through numerical analyses like the FRANC2D program.

CASCA software was used for modelling specimen shape and mesh structure. Beam specimens were modelled as 30mmx30mmx160mm consistent with the indirect tensile tests (Fig. 3). The modeled beam geometry was fixed in both the x-horizontal and y-vertical directions at two support roller locations beneath the specimen in the experiments (Fig.3).



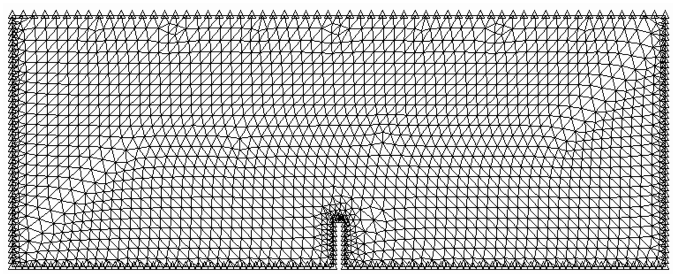


Fig.3. Beam specimen dimensions, specimen geometry and mesh generation using CASCA

Fracture toughness of beam specimens is calculated by three-point bending test and fracture toughness equation is as floows:

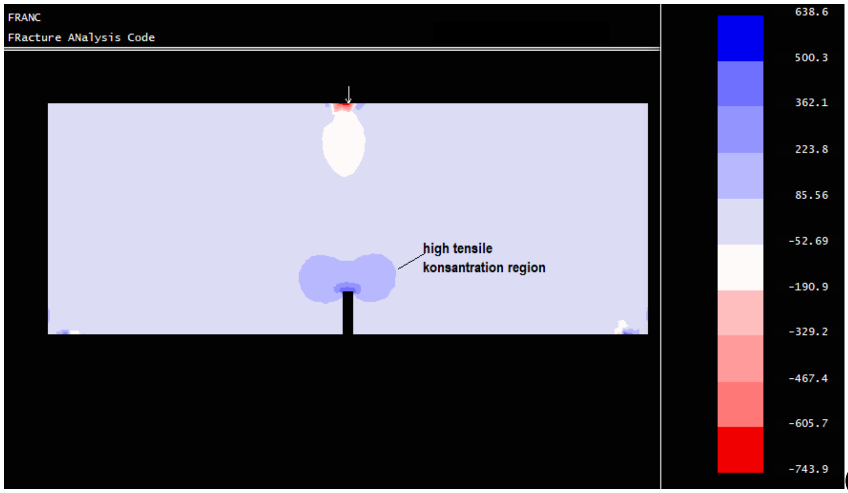
$$K_{IC} = 3(P_{max} + 0.5W)\frac{S(\pi a_c)^{\frac{1}{2}}F(\alpha)}{2d^2 b} \quad [\mathrm{Nm}^{-3/2}] \tag{2}$$

$$F(\alpha) = \frac{1.99 - \alpha(1-\alpha)(2.15 - 3.93\alpha + 2.7\alpha^2)}{\sqrt{\pi^{1/2}}(1+2\alpha)(1-\alpha)^{3/2}}$$

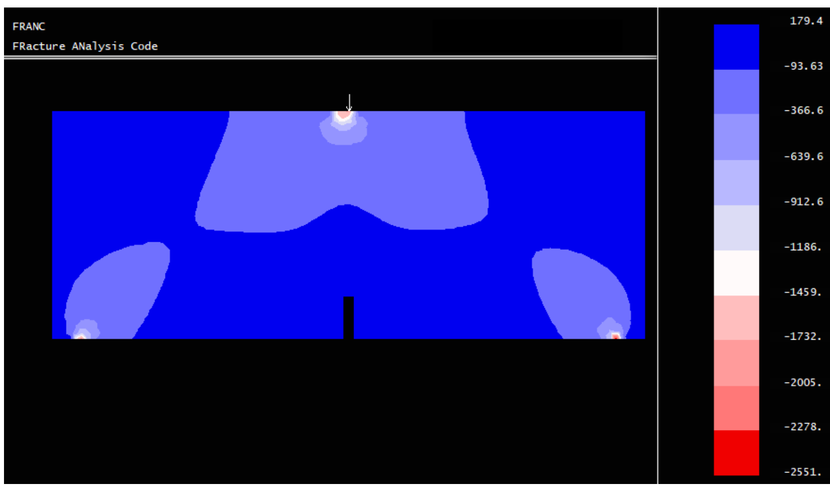
Where: α=ac/d, ac: notch crack length (mm), P_{max} ultimate failure load [N], d: beam width (mm), L: beam length (mm), W: beam thickness (mm), S: distance between two supports (mm).

RESULTS AND DISCUSSION

The first series of modellings were performed to analyze the conjugate stresses around the pre-existing cracks in the beam specimen before the fracture analyses. In numerical modelling with FRANC2D, the load applied to the beam specimen was applied to obtain indirect tensile stresses. The results of mode I and mixed mode I-II stress distribution analysis for beam geometry specimens are shown in Fig.4 and Fig.5 respectively. When the results of the stress analyses were examined, tensile stress concentration was determined at the notch crack tip under indirect compressive stress in mode I condition. When the minimum principal stress analyses are examined under the same loading condition, compressive stress concentration in areas other than the crack tip and the shear stress is expected to occur in these areas (Fig.4 a and b). In Fig.4c and d, these stress concentrations are shown with 'stress bars' which is one of the FRANC2D post process options.



(a)



(b)

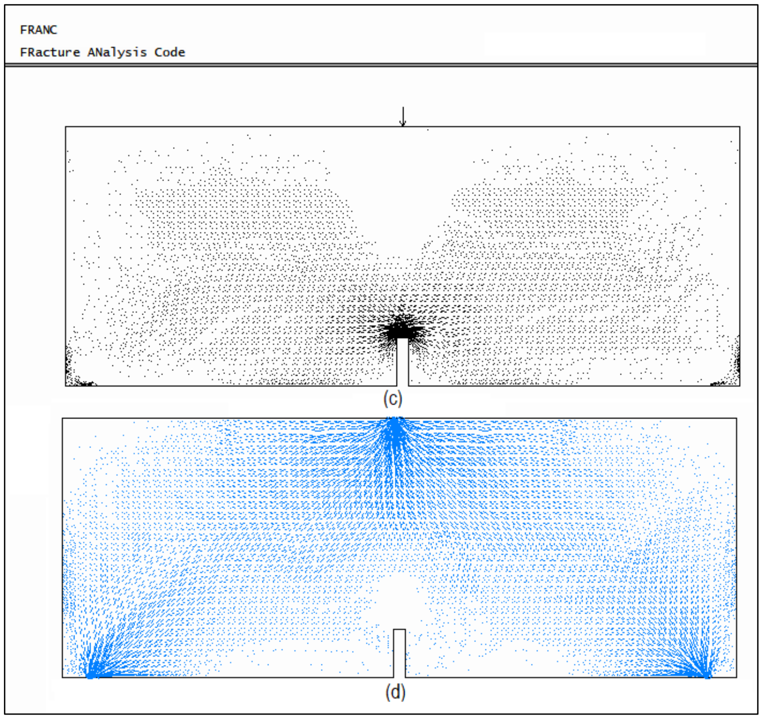
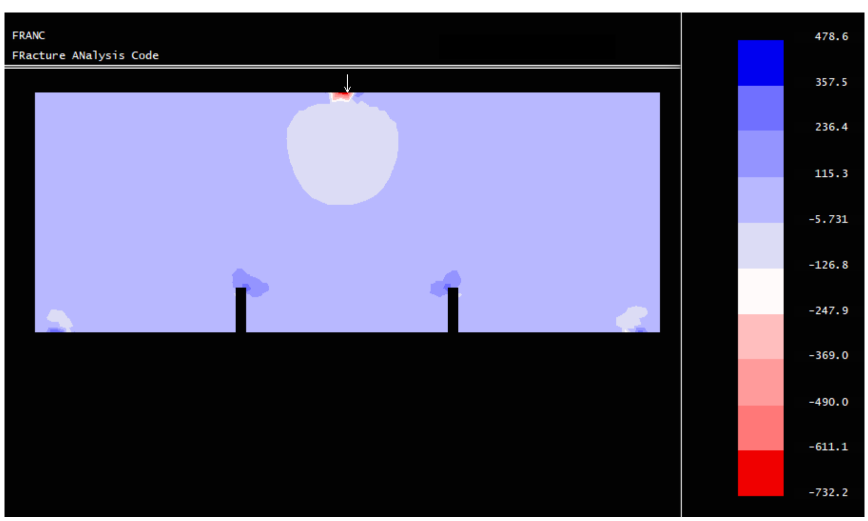
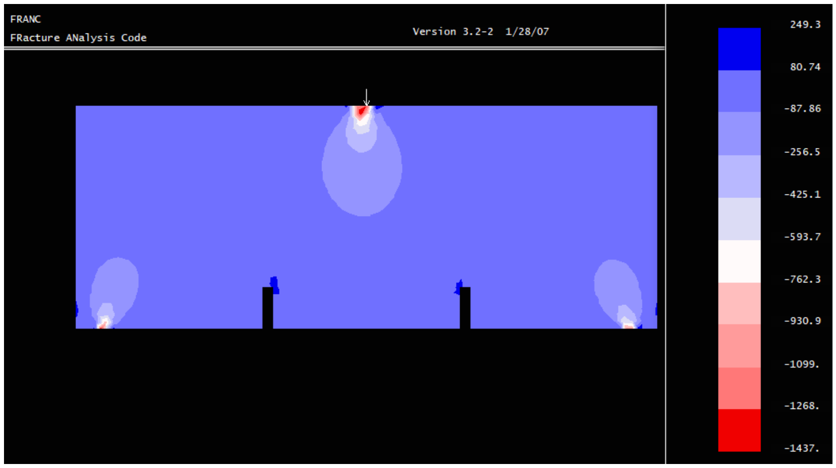


Fig.4 Mode I stress distribution in the beam specimens obtained by FRANC2D program (+: tensile; -: compressive) (a) maximum principal stress (tensile) distribution, (b) minimum principal stress, c) indirect tensile stress zone, d) compressive stress concentration zones

On the other hand, when the stress analysis results were examined, it was determined that indirect tensile stress was developed at the tip of crack under indirect compressive stress in mixed mode I-II, but these regions were shifted from the center to the loading axis at the crack tip (Fig. 5a-b). When the minimum principal stress analyses are examined under the same loading condition, compressive stress concentration in areas other than the crack tip and the shear stress is expected to occur in these areas. In this case, 'wing cracks' mentioned in fracture mechanics are expected to form in these regions (Whittaker et al. 1998). In Fig.5c and d, these stress concentrations are shown with 'stress bars' which is one of the FRANC2D post process options.



(a)



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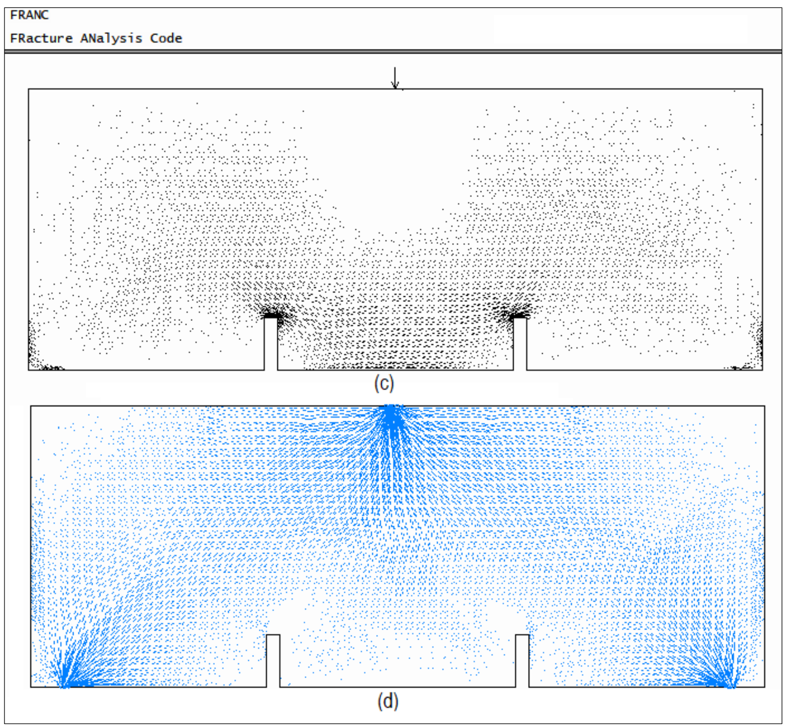
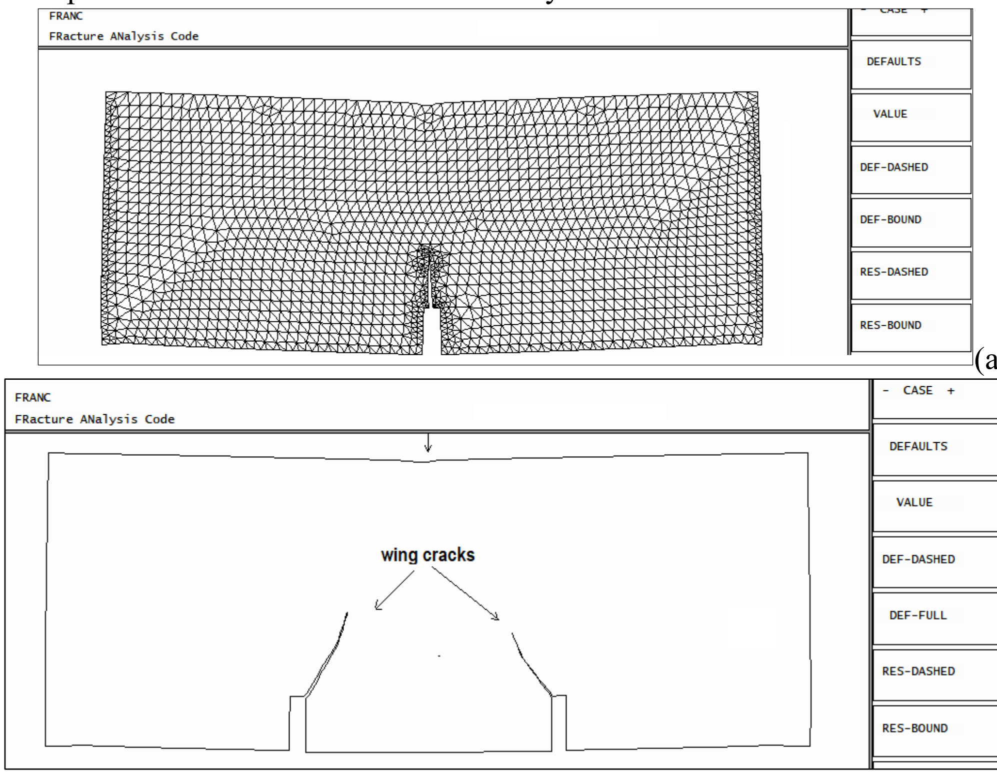


Fig.5 Mixed mode I-II stress distribution in the beam specimens obtained by FRANC2D program (a) maximum principal stress (tensile) distribution, (b) minimum principal stress distribution, c) indirect tensile concentration zone, d) compressive stress concentration zones

The second series modelling to analyze the fracture characteristic of material were conducted. One of the notable advantages of FRANC2D is that during fracture analysis, the mesh generated at the crack tip is removed at each crack propagation step. Subsequently, the program automatically generates a new mesh structure around the crack tip based on the new stress state. FRANC2D is a program that can successfully model elastoplastic fracture analyses.The fracture characteristic of the beam specimen under mode I and mixed mode I-II loading condition is shown in Fig.6a and b respectively.
The crack formation at the pre-existing crack tip in the material under mode I loading by considering the principles of fracture mechanics moves towards the loading axis. As e seen in Figure 6a, the crack formed and propagated in accordance with the principles of fracture mechanics. On the other hand, in the material under mixed mode I-II loading, the crack that develops at the pre-existing crack tip grows towards the

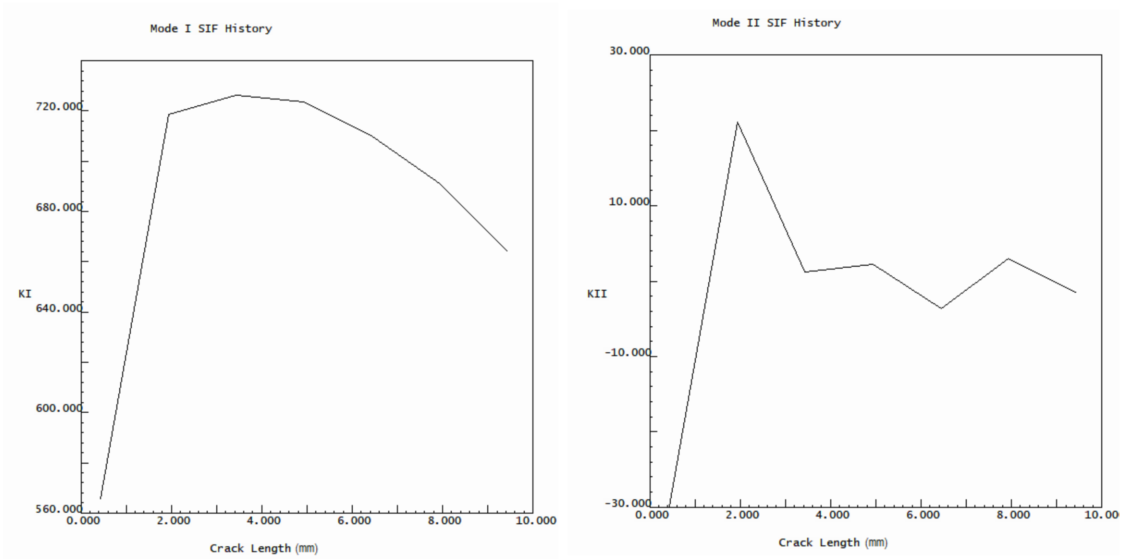
loading axis and propagated in the form of a 'wing crack'. As seen in Fig. 6b, the wing cracks propagated within the beam specimen in accordance with the theory.



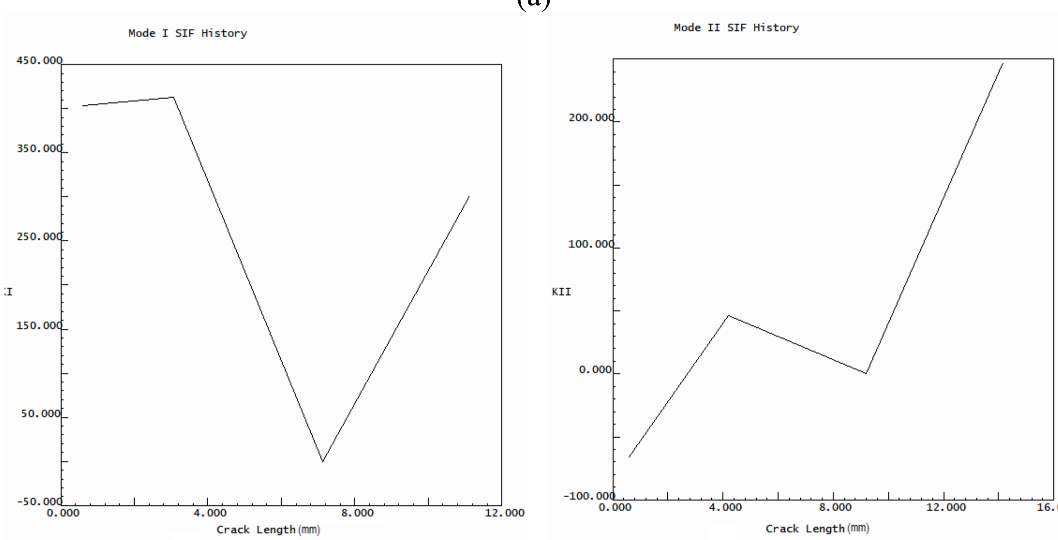
(a)
(b)

Fig.6 Crack propagation with a) mode I loading and b) mixed mode I-II loading

Alternative of as seen it is stated that As seen in both the colored contour representation in Fig. 4 and the graphical representation in Figure 7, the peak indirect tensile stress was obtained by modelling disinclined crack. It is observed that the maximum tensile SIF value (KI) obtained in the mixed mode I-II case of the modelled crack is considerably lower than the maximum SIF value obtained with mode I (Fig.7a). This result is in accordance with the principles of fracture mechanics because mode I loading develops very high tensile stresses while mixed mode loading develops shear stresses as well as tensile stresses. During mixed mode I-II tests, the tensile force applied to the specimen forces the crack tip to damage in both the opening and sliding directions. This compound loading causes the plastic zone to be more inclined than in the Mode I loading case. Although the plastic zone in Mode I/II is geometrically different from Mode I, its reactions to the change in crack length and deformation rate are similar to those in Mode I loading. On one hand On the other hand, the maximum shear SIF value (KII) obtained under mixed mode I-II loading is considerably higher than the shear stress value obtained under mode I loading (Fig.7b). The mode I (tensile) fracture toughness (KIC) value of the cement stabilized soil specimens was found to be 0.32 MPa√m. On the other hand, the mixed mode I-II fracture toughness value was found to be 0.38 MPa√m.
It is believed that the calculation of fracture toughness of cement stabilized soil will lead to experimental research when fracture mechanics tests are used with these research results. It will be possible to form specimens using molds with cement stabilized soil specimens and it will be possible to prepare specimens by creating notch cracks as in this research or semi-circular bending specimens as in the literature.



(a)



(b)

Fig.7 SIF values in font of the notch crack for a) mode I loading condition and b) mixed mode I-II loading condition

Linear elastic fracture mechanics (LEFM) is confined to the elastic zones, including very small plastic damage. However, NEFM fracture mechanics approaches are necessary when the inelastic/plastic damage is large enough to affect the relative elasto-plastic dimensions, such as the FPZ in front of a notch crack tip. Stress-induced crack initiation in composite materials, such as informal, alternatives: including, for example. such as cement stabilized soil and reinforced concrete, typically leads to unstable crack growth due to the plastic deformation and the FPZ (Fig. 8a). The cohesive crack model is a well-known fictitious crack approach (Figure 8b) used to model cohesive fracture in numerical analyses (Dugdale, 1960; Hillerborg, 1976; Behnam, 2021; Ma et al., 2022, Bittencort, 1993). FRANC2D is a fracture mechanics programme that can model plastic deformation and strain softening in front of the crack tip. Following Dugdale's work, Barenblatt (1959) studied the combined forces at the molecular scale that occur in the region pointed out by Dugdale (1960). In 1976, Hillerborg et al. (1976) proposed a model similar to the one developed by Barenblatt (1959). However, the concept of tensile strength has been introduced instead of the molecular scale solution. Hillerborg's model allowed existing cracks to grow and, more importantly, initiate new cracks. This model is called the "Fictitious Crack Model" (Hillerborg et al., 1976). This developed model is considered the beginning of the development of the cohesive interface model to simulate sudden crack growth in brittle solids.

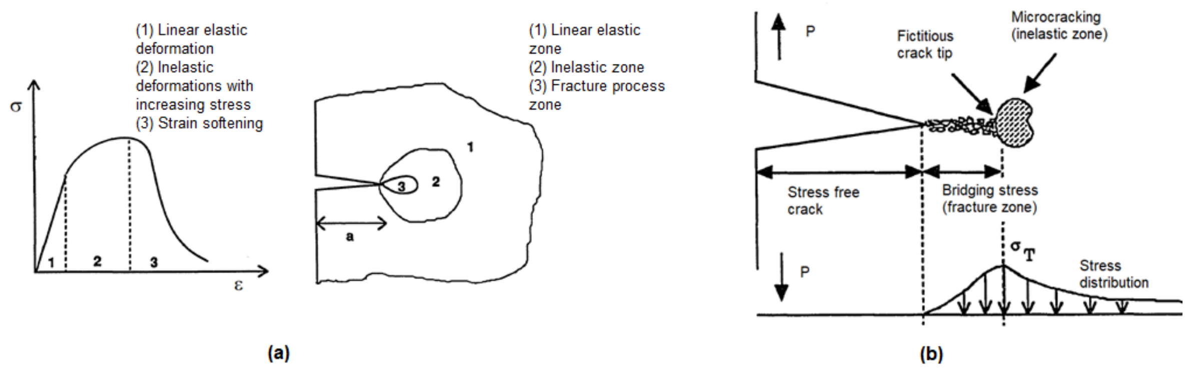


Fig. 8. (a) Fictitious damage zones in front of a crack (b) Fictitious crack model

In numerical analyses, non-linear analyses are very important for modelling the elastoplastic damage zone. In numerical non-linear (fictitious cracking) analyses, NL interface elements for mixed mode I-II fracture are predefined in the program. With these NL elements, the initiation of new cracks and the propagation of existing cracks are modelled. The nonlinear interface elements of FRANC2D are used to model plastic deformation under external load (Fig. 9). The fictitious crack propagation with FRANC2D occurs when the maximum circumferential stress at the tip is exceeded.

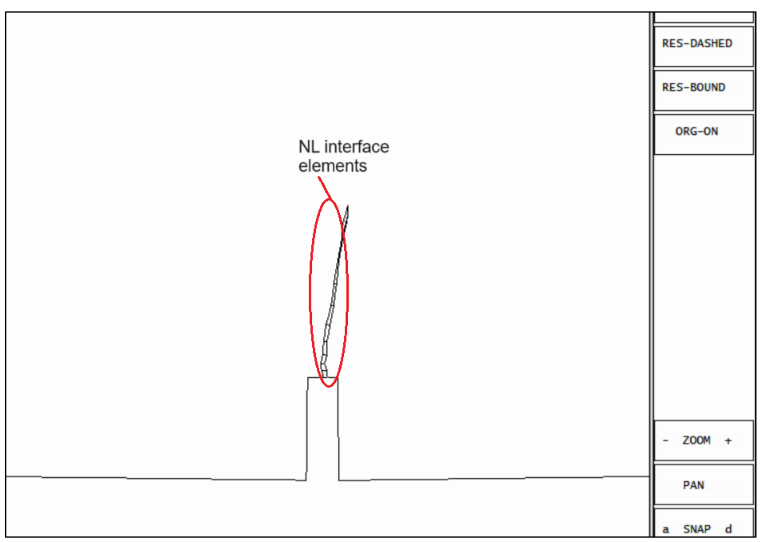


Fig.9 Modelling of fictitious crack propagation

The fictitious crack lengths and the sliding between crack plains for the beam specimen under indirect tensile loading modeling is shown in Fig.10. As seen in Figure 10, cohesion due to friction on the crack surfaces increases up to 3 mm crack length and then continues to decrease. In this case, it is explained that the increase in KI given in Fig. 7 up to this crack length is due to these cohesive forces. When the same analysis was performed for beam specimens under shear-tensile stress condition, it was found that the sliding value between the cohesive crack surfaces was about ten times higher than in the mode I case

(Fig.11). This result is expected and it is stated that the sliding values are higher due to the higher shear forces in the mixed mode I-II case.

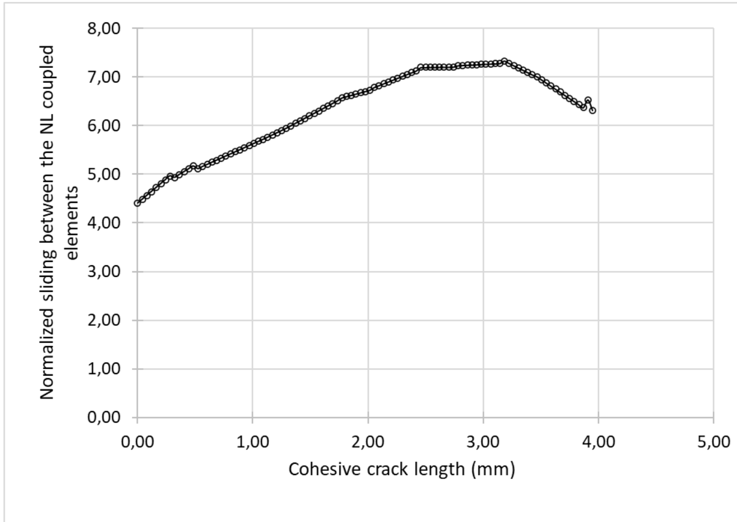


Fig.10 Sliding between crack surface plains for mode I loading

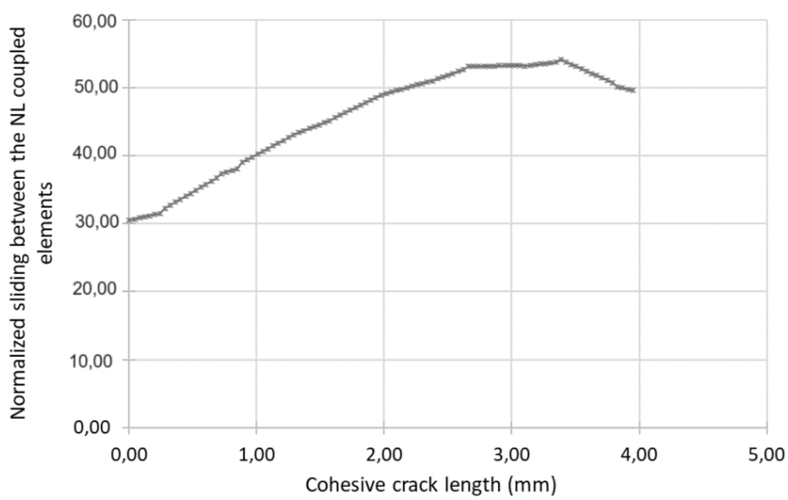


Fig.11 Sliding between crack surface plains for shear-tensile loading

According to the results obtained after cohesive fracture analyses and SIF analyses, it will be very important at this point to investigate and introduce appropriate mixed-mode fracture models and theories for the prediction of fracture properties of soils or soft geo-materials. Because is not proper. Thus, Because, although there are many international standard test methods to find the fracture toughness of cement and metals, no international standard test has been developed for the fracture toughness test of cement stabilized soils, which is a deficiency in the field of mechanics of materials. It is possible to develop mode I and mixed mode I-II fracture models and theories for the prediction of fracture properties of stabilized soils or soft geo-materials using the notched beam specimen or semi-circular disc specimens mentioned in this research.

It is recommended to study different types of industrial and environmentally friendly sustainable cement mixtures in future cement stabilized soil studies. Because cement stabilized soils are mainly used in road construction, but also in deep soil mixing (DSM) and jet grouting (JG) applications. Therefore, considering that millions of tons of cement will be used, it is essential to investigate new generation environmentally friendly cements in terms of fracture mechanics principles in terms of climate change and sustainability. For example, OYAK Cement Ltd., which is the producer of enrironment friendly cement NOVOCEM, has kept the reduction of CO_2 emission with calcined clay technology in the first place among its targets in parallel with borderline carbon applications and produces NOVOCEM© cement with 40% lower CO2 emission and 35% less energy consumption with 20% renewable fuel.The cement sector is responsible for 8% of CO_2 emissions from greenhouse gases in the world. Global climate change and sustainability require the cement industry to seek innovative solutions. Innovative methods aiming to reduce the environmental footprint of cement production have a critical role in the development of the sector.

Soil reinforcement with cement is also very common in underground and tunneling applications. For example, in the so-called umbrella reinforcement, it is a technique that aims to improve the ground around the tunnel during the excavation of the tunnel. It is usually applied using a series of holes into which a cement mixture is injected. In the method, cement is injected into the pipes placed in the ground to increase its strength. Moreoevr, the combination of Sodium Silicate injection and cement would provide effective results in soil improvement and water management processes within tunneling projects. This method is particularly applicable in tunnels where the cement-water mixture cannot control the pressure of groundwater. The cement-water mix can reduce its density due to ground-borne water action and can be washed before setting when in contact with water.

5. CONCLUSION

In this study, the fracture toughness and cracking behavior of soil specimens stabilized with cement, an elasto-plastic material, were investigated. Numerical analyses of stress condition and fracture behaviour were performed using FRANC2D program. Stress and crack analyses in beam specimens were performed using FRANC2D software. The indirect tensile fracture toughness (KIC) value of the modelled beam specimens was found to be 0.32 MPa√m. Similarly, the indirect tensile and shear fracture toughness values were found to be 0.38 MPa√m.

The failure and fracture behaviour of the beam samples under indirect tensile and indirect tensile-shear loading were analyzed for both non-cohesive and cohesive fracture by numerical modeling with FRANC2D. The results of the analyses showed that the most significant slippage between the cohesive crack surfaces was observed in the specimen under indirect tensile-shear loading. The "wing crack" growth in the reinforced soil specimens was obtained compatible with the fracture mechanics applications. The results obtained in this study are expected to bridge over in the development of a new international standard test method for the determination of fracture toughness of reinforced soil specimens under tensile and shear loads.

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A New Soft Set Operation: Complementary Extended Gamma Operation

Research Article

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Abstract

Since its inception, soft set theory has shown to be a useful mathematical framework for addressing problems involving uncertainty, proving its usefulness in a variety of academic and practical disciplines. The operations of soft sets are at the very core concept of this theory. In this regard, a new kind of soft set operation known as the complementary extended gamma operation for soft sets is presented in order to improve the theory and theoretically contribute to it in this study. To shed light on the relationship between the complementary extended gamma operation and other soft set operations, a thorough analysis of this operation's attributes, including its distributions across other soft set operations, has been conducted. Additionally, this paper aims to contribute to the literature on soft sets by examining the algebraic structure of soft sets from the perspective of soft set operations, which provides a thorough grasp of their use as well as an appreciation of the ways in which soft sets can be applied to both classical and nonclassical logical thought.
Keywords: Soft Set, Soft Set Operations, Complementary Extended Soft Set Operations.

1. INTRODUCTION

In our daily lives, we often encounter subjective concepts which lack the objectivity of scientific knowledge and vary from person to person. To address the complexities of the uncertainty we face, people have sought various solutions over time. However, existing methods have shown discrepancies in tackling new complex problems arising from changing conditions. Among the theories proposed to handle uncertain situations, Zadeh's theory of fuzzy sets stands out as the most prominent. Fuzzy sets are defined by their membership functions. As fuzzy set theory rapidly developed, certain structural issues came to light. In response, Molodtsov [1] introduced soft set theory as a solution to these structural problems.

The soft set has been applied in many theoretical and practical situations since its debut, and several additional studies have been published in the literature. Maji et al. [2] paved the path for more study on the subject of soft set theory by defining the equality of two soft sets, the subset and superset of a soft set, the complement of a soft set, and soft binary operations like and/or, union, and intersection operations for soft sets. Set theory concepts led Pei and Miao [3] to redefine the concepts of "soft subset" and "intersection of two soft sets." Ali et al. [4] then suggested a number of other soft set operations, which Sezgin and Atagün [5] and Ali et al. [6] carefully investigated. Sezgin et al. [7] and Stojanovic [8] described the extended difference and extended symmetric difference of soft sets, respectively, and their properties were carefully investigated in relation to other operations on soft sets.

Soft set operations may be broadly classified into two types: restricted soft set operations and extended soft set operations, according to an analysis of the research conducted thus far. Eren and Çalışıcı [9] developed and evaluated the soft binary piecewise difference operation for soft sets, and Sezgin and Çalışıcı [10] carried out an in-depth investigation of the properties of this soft set operation. The inclusive and exclusive complement of sets is a novel concept in set theory that was presented in 2021 by Çağman [11]. Sezgin et al. [12] provided five new concepts pertaining to binary complement operations that were already described by Çağman [11]. Aybek [13] explored the properties of numerous more restricted and extended soft set operations with the motivation of [11,12]. Moreover, the soft binary piecewise operation form-of which Eren and Çağman [9] were the pioneers-was modified somewhat by taking the complement of the image set in the first row. Consequently, several researchers have studied the complementary soft binary piecewise operations in great detail [14–22]. On the other hand, Akbulut [23] and Demirci [24] altered the form of the existing extended soft set operations in the literature by taking the complement of the image set in the first and second rows, and defining the complementary extended difference, lambda and union, plus and theta, respectively, and giving their algebraic properties and relations with other soft set operations. For more on soft equality, please see [25-32] and for other applications of soft sets to algebraic structures, see the following [32-46].

Analyzing the characteristics of designated operations on sets as well as the sets themselves is a crucial component of algebraic structures, which is to categorize mathematical structures. This analysis is important within the context of algebra. There are two main kinds of soft set collections to be aware of when thinking about soft sets as algebraic structures: collections with a fixed set of parameters and collections with changing parameter sets. Depending on the extra actions performed, these collections exhibit different behaviors. Concepts related to soft set operations are just as essential as the operations of classical set theory, which form the basis of soft sets.

In this study, a novel soft set operation named "complementary extended gamma" is introduced and its properties are thoroughly examined, with the goal of advancing the theory of soft sets. In addition, an analysis is conducted to investigate the relationship between different kinds of soft set operations and the complemented extended gamma operation in order to clarify them. This topic is significant within the framework because knowledge of the algebraic structures of soft sets in relation to novel operations is necessary to comprehend their applications.

2. PRELIMINARIES

Definition 2.1. Let U be the universal set, E be the parameter set, P(U) be the power set of U, and let $D \subseteq E$. A pair (F, D) is called a soft set on U. Here, F is a function given by $F: D \rightarrow P(U)$ [1].

The notation of the soft set (F,D) is also shown as F_D, however, we prefer to use the notation of (F,D) as is used by Molodtsov [1] and Maji et al. [2].
The set of all soft sets over U is denoted by $S_E(U)$. Let K be a fixed subset of E, then the set of all soft sets over U with the fixed parameter set K is denoted by $S_K(U)$. In other words, in the collection $S_K(U)$, only soft sets with the parameter set K are included, while in the collection $S_E(U)$, soft sets over U with any parameter set can be included. Clearly, the set $S_K(U)$ is a subset of the set $S_E(U)$.
Definition 2.2. Let (F,D) be a soft set over U. If F(ℵ)=∅ for all ℵ∈D, then the soft set (F,D) is called a null soft set with respect to D, denoted by $\emptyset_D$. Similarly, let (F,E) be a soft set over U. If F(e)=∅ for all ℵ∈E, then the soft set (F,E) is called a null soft set with respect to E, denoted by $\emptyset_E$ [4].
A soft set can be defined as F: ∅ ⟶ P(U), where U is a universal set. Such a soft set is called an empty soft set and is denoted as $\emptyset_\emptyset$. Thus, $\emptyset_\emptyset$ is the only soft set with an empty parameter set [6].

Definition 2.3. Let (F,D) be a soft set over U. If F(ℵ)=U for all ℵ∈D, then the soft set (F,K) is called a relative whole soft set with respect to D, denoted by U_D. Similarly, let (F,E) be a soft set over U. If F(e)=U for all ℵ∈E, then the soft set (F,E) is called a whole soft set with respect to E, denoted by U_E [4].
Definition 2.4. Let (F,D) and (G,Y) be soft sets over U. If D⊆Y and F(ℵ) ⊆G(ℵ) for all ℵ∈D, then (F,D) is said to be a soft subset of (G,Y), denoted by (F,D)$\tilde{\subseteq}$(G,Y). If (G,Y) is a soft subset of (F,D), then (F,D) is said to be a soft superset of (G,Y), denoted by (F,D)$\tilde{\supseteq}$(G,Y). If (F,D)$\tilde{\subseteq}$(G,Y) and (G,Y)$\tilde{\subseteq}$(F,D), then (F,D) and (G,Y) are called soft equal sets [3].
Definition 2.5. Let (F,D) be a soft set over U. The soft complement of (F,D), denoted by $(F,D)^r=(F^r,D)$, is defined as follows: for all ℵ∈D, F^r(ℵ)=U-F(ℵ) [4].
Çağman [11] introduced two new complements as a novel concept in set theory, termed as the inclusive complement and exclusive complement. For ease of representation, we denote these binary operations as + and θ, respectively. For two sets D and Y, these binary operations are defined as D+Y=D'∪Y, DθY=D'∩Y'. Sezgin et al. [12] examined the relations between these two operations and also defined three new binary operations and analyzed their relations with each other. Let D and Y be two sets D*Y=D'∪Y', DγY= D'∩Y, and DλY=D∪Y'.

As a summary for soft set operations, we can categorize all types of soft set operations as follows: Let "⍟" be used to represent the set operations (i.e., here ⍟ can be ∩, ∪,\, ∆, +,θ, *, λ,γ), then all type of soft set operations are defined as follows:

Definition 2.6. Let $(F,D),(G,Y)\in S_E(U)$. The restricted ⍟ operation of (F,D) and (G,Y) is the soft set (H,P), denoted to be $(F,D)\circledast_{\mathfrak{R}}(G,Y)=(H,P)$, where $P=D\cap Y\neq\emptyset$ and for all ℵ∈P, H(ℵ) = F(ℵ)⍟G(ℵ). Here, if P=D∩Y=∅, then $(F,D)\circledast_{R}(G,Y)=\emptyset_\emptyset$ [4,5,6,13]
Definition 2.7. Let $(F,D),(G,Y)\in S_E(U)$. The extended ⍟ operation (F, D) and (G, Y) is the soft set (H, P), denoted by $(F,D)\circledast_{\varepsilon}(G,Y)=(H,P)$, where $P=D\cup Y$ and for all $\aleph\in P$,

$$H(\aleph)=\begin{cases}F(\aleph), & \aleph\in D-Y\\ G(\aleph), & \aleph\in Y-D\\ F(\aleph)\circledast G(\aleph), & \aleph\in D\cap Y\end{cases}$$

[2,4,6,7,8,13]
Definition 2.8. Let $(F,D),(G,Y)\in S_E(U)$. The complementary extended ⍟ operation (F, D) and (G, Y) is the soft set (H, P), denoted by $(F,D)\overset{*}{\circledast}_{\varepsilon}(G,Y)=(H,P)$, where $P=D\cup Y$ and for all $\aleph\in P$,

$$H(\aleph)=\begin{cases}F'(\aleph), & \aleph\in D-Y\\ G'(\aleph), & \aleph\in Y-D\\ F(\aleph)\circledast G(\aleph), & \aleph\in D\cap Y\end{cases}$$

[23,24]

Definition 2.9. Let $(F, D), (G, Y) \in S_E(U)$. The soft binary piecewise ⍟ of (F,D) and (G,Y) is the soft set (H,D), denoted by $(F, D)\underset{\circledast}{\sim}(G, Y) = (H, D)$, where for all $\aleph \in D$,

$$H(\aleph) = \begin{cases} F(\aleph), & \aleph \in D - Y \\ F(\aleph) \circledast G(\aleph), & \aleph \in D \cap Y \end{cases}$$

[9,10,47,48]

Definition 2.10. Let $(F, D), (G, Y) \in S_E(U)$. The complementary soft binary piecewise ⍟ of (F,D) and (G,Y) is the soft set (H,D), denoted by $(F, D) \overset{*}{\underset{\circledast}{\sim}} (G, Y) = (H, D)$, where for all $\aleph \in D$,

$$H(\aleph) = \begin{cases} F'(\aleph), & \aleph \in D - Y \\ F(\aleph) \circledast G(\aleph), & \aleph \in D \cap Y \end{cases}$$

[14-22].

Definition 2.11. Let (S, ⋆) be an algebraic structure. An element s ∈S is called idempotent if $s^2=s$. If $s^2=s$ for all s∈S, then the algebraic structure (S,⋆) is said to be idempotent. An idempotent semigroup is called a band, an idempotent and commutative semigroup is called a semilattice, an idempotent and commutative monoid is called a bounded semilattice [49].

In a monoid, although the identity element is unique, a semigroup/groupoid can have one or more left identities, however, if it has more than one left identity, it does not have a right identity element, thus it does not have an identity element. Similarly, a semigroup/groupoid can have one or more right identities, however, if it has more than one right identity, it does not have a left identity element, thus it does not have an identity element [50].

Similarly, in a group, although each element has a unique inverse, in a monoid, an element can have one or more left inverses, however, if an element has more than one left inverse, it does not have a right inverse, thus it does not have an inverse. Similarly, in a monoid, an element can have one or more right inverses, however, if an element has more than one right inverse, it does not have a left inverse, thus it does not have an inverse [50]. We refer to [51] for the potential future graph applications and network analysis with respect to soft sets.

3. COMPLEMENTARY EXTENDED GAMMA OPERATION

In this section, a new soft set operation called the complementary extended gamma operation of soft sets is introduced with its example, and its full algebraic properties are analyzed.

Definition 3.1. Let (F, Z), (G,C) be soft sets over U. The complementary extended gamma operation (γ) of (F, Z) and (G, C) is the soft set (H, K), denoted by $(F, Z) \overset{*}{\gamma_\varepsilon} (G,C) = (H, K)$, where for all $\aleph \in K = Z \cup C$,

$$H(\aleph) = \begin{cases} F'(\aleph), & \aleph \in Z - C \\ G'(\aleph), & \aleph \in C - Z \\ F(\aleph)\gamma G(\aleph), & \aleph \in Z \cap C \end{cases}$$

where $F(\aleph)\gamma G(\aleph) = F'(\aleph) \cap G(\aleph)$ for all $\aleph \in Z \cap C$.

Example 3.2. Let $E=\{e_1,e_2,e_3,e_4\}$ be the parameter set, and $Z=\{e_1, e_3\}$ and $C=\{e_2, e_3, e_4\}$ be two subsets of E, and $U=\{h_1,h_2,h_3,h_4,h_5\}$ be the universal set. Assume that $(F,Z)=\{(e_1,\{h_2,h_5\}),(e_3,\{h_1,h_2,h_5\})\}$, $(G,C)=\{(e_2,\{h_1,h_4,h_5\}),(e_3,\{h_2,h_3,h_4\}),(e_4,\{h_3,h_5\})\}$ be two soft sets over U. Let $(F,Z) \overset{*}{\gamma_\varepsilon} (G,C)=(L,Z\cup C)$, where for all $\aleph \in Z\cup C$,

$$L(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z\text{-}C \\ G'(ℵ), & ℵ\in C\text{-}Z \\ F'(ℵ)\cap G(ℵ), & ℵ\in Z\cap C \end{cases}$$

Here, since $Z\cup C=\{e_1,e_2,e_3,e_4\}$, $Z\text{-}C=\{e_1\}$, $C\text{-}Z=\{e_2,e_4\}$, $Z\cap C=\{e_3\}$, thus $L(e_1)=F'(e_1)=\{h_1,h_3,h_4\}$, $L(e_2)=G'(e_2)=\{h_2,h_3\}$, $L(e_4)=G'(e_4)=\{h_1,h_2,h_4\}$, $L(e_3)=F'(e_3)\cap G(e_3)=\{h_3,h_4\}\cap\{h_2,h_3,h_4\}=\{h_3,h_4\}$. Hence,

$$(F,Z)\underset{\gamma_\varepsilon}{*}(G,C)=\{(e_1,\{h_1,h_3,h_4\}),(e_2,\{h_2,h_3\}),(e_3,\{h_3,h_4\}),(e_4,\{h_1,h_2,h_4\})\}.$$

Theorem 3.3. (Algebraic Properties of Operation)

1) The set $S_E(U)$ is closed under $\underset{\gamma_\varepsilon}{*}$.

Proof: It is clear that $\underset{\gamma_\varepsilon}{*}$ is a binary operation on $S_E(U)$. Indeed,

$$\underset{\gamma_\varepsilon}{*}: S_E(U)\times S_E(U)\rightarrow S_E(U)$$

$$((F,Z),(G,C))\rightarrow (F,Z)\underset{\gamma_\varepsilon}{*}(G,C)=(L,Z\cup C)$$

Similarly,

$$\underset{\gamma_\varepsilon}{*}: S_Z(U)\times S_Z(U)\rightarrow S_Z(U)$$

$$((F,Z),(G,Z))\rightarrow (F,Z)\underset{\gamma_\varepsilon}{*}(G,Z)=(T,Z\cup Z)=(T,Z)$$

That is, when Z is a fixed subset of the set E and (F,Z) and (G,Z) are elements of $S_Z(U)$, then so is $(F,Z)\underset{\gamma_\varepsilon}{*}(G,Z)$. Namely, $S_Z(U)$ is closed under $\underset{\gamma_\varepsilon}{*}$, too.

2) $[(F,Z)\underset{\gamma_\varepsilon}{*}G,C)]\underset{\gamma_\varepsilon}{*}(H,R)\neq(F,Z)\underset{\gamma_\varepsilon}{*}[(G,C)\underset{\gamma_\varepsilon}{*}(H,R)]$.

Proof: Firstly, let's handle the left hand side (LHS). Let $(F,Z)\underset{\gamma_\varepsilon}{*}(G,C)=(T,Z\cup C)$, where for all $ℵ\in Z\cup C$,

$$T(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z\text{-}C \\ G'(ℵ), & ℵ\in C\text{-}Z \\ F'(ℵ)\cap G(ℵ), & ℵ\in Z\cap C \end{cases}$$

Let $(T,Z\cup C)\underset{\gamma_\varepsilon}{*}(H,R)=(M,Z\cup C\cup R)$, where for all $ℵ\in Z\cup C\cup R$,

$$M(ℵ)=\begin{cases} T'(ℵ), & ℵ\in (Z\cup C)\text{-}R \\ H'(ℵ), & ℵ\in R\text{-}(Z\cup C) \\ T'(ℵ)\cap H(ℵ), & ℵ\in (Z\cup C)\cap R \end{cases}$$

Thus,

$$M(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z\text{-}C)\text{-}R=Z\cap C'\cap R' \\ G(\aleph), & \aleph\in(C\text{-}Z)\text{-}R=Z'\cap C\cap R' \\ F(\aleph)\cup G'(\aleph), & \aleph\in(Z\cap C)\text{-}R=Z\cap C\cap R' \\ H'(\aleph) & \aleph\in R\text{-}(Z\cup C)=Z'\cap C'\cap R \\ F(\aleph)\cap H(\aleph), & \aleph\in(Z\text{-}C)\cap R=Z\cap C'\cap R \\ G(\aleph)\cap H(\aleph), & \aleph\in(C\text{-}Z)\cap R=Z'\cap C\cap R \\ \big(F(\aleph)\cup G'(\aleph)\big)\cap H(\aleph), & \aleph\in(Z\cap C)\cap R=Z\cap C\cap R \end{cases}$$

Now let's handle the right hand side (RFS) of the equation, Let $(G,C)\overset{*}{\gamma_\varepsilon}(H,R)=(K,C\cup R)$. Here, for all $\aleph\in C\cup R$,

$$K(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C\text{-}R \\ H'(\aleph), & \aleph\in R\text{-}C \\ G'(\aleph)\cap H(\aleph), & \aleph\in C\cap R \end{cases}$$

Assume that $(F,Z)\overset{*}{\gamma_\varepsilon}(K,C\cup R)=(S,Z\cup C\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$S(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}(C\cup R) \\ K'(\aleph), & \aleph\in(C\cup R)\text{-}Z \\ F'(\aleph)\cap K(\aleph), & \aleph\in Z\cap(C\cup R) \end{cases}$$

Thus,

$$S(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}(C\cup R)=Z\cap C'\cap R' \\ G(\aleph), & \aleph\in(C\text{-}R)\text{-}Z=Z'\cap C\cap R' \\ H(\aleph), & \aleph\in(R\text{-}C)\text{-}Z=Z'\cap C'\cap R \\ G(\aleph)\cup H'(\aleph), & \aleph\in(C\cap R)\text{-}Z=Z'\cap C\cap R \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap(C\text{-}R)=Z\cap C\cap R' \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap(R\text{-}C)=Z\cap C'\cap R \\ F'(\aleph)\cap(G(\aleph)\cup H'(\aleph)), & \aleph\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

It is seen that $M\neq S$. That is, in the set $S_E(U)$, $\overset{*}{\gamma_\varepsilon}$ does not have associative property.

3) $[(F,Z)\overset{*}{\gamma_\varepsilon}(G,Z)]\overset{*}{\gamma_\varepsilon}(H,Z)\neq(F,Z)\overset{*}{\gamma_\varepsilon}[(G,Z)\overset{*}{\gamma_\varepsilon}(H,Z)]$

Proof : Firstly, let's look at the LHS. Let $(F,Z)\overset{*}{\gamma_\varepsilon}(G,Z)=(T,Z\cup Z)$, where for all $\aleph\in Z\cup Z=Z$,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ G'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Let $(T,Z)\overset{*}{\gamma_\varepsilon}(H,Z)=(M,Z\cup Z)$, where for all $\aleph\in Z$,

$$M(\aleph)=\begin{cases} T'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ H'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ T'(\aleph)\cap H(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Thus,

$$M(\aleph)=\begin{cases} T'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ H'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ (F(\aleph)\cup G'(\aleph))\cap H(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Now let's handle RHS. Let (G,Z) $\underset{\gamma_\varepsilon}{*}$ (H,Z)=(L, Z∪Z), where for all $\aleph \in Z$,

$$L(\aleph)=\begin{cases} G'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ H'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ G'(\aleph)\cap H(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Let (F,Z) $\underset{\gamma_\varepsilon}{*}$ (L,Z) = (N,Z∪Z), where for all $\aleph \in Z$,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ L'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph)\cap L(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Hence,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ L'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph)\cap(G'(\aleph)\cap H(\aleph)), & \aleph\in Z\cap Z=Z \end{cases}$$

It is seen that M≠N. That is, in the set $S_Z(U)$, $\underset{\gamma_\varepsilon}{*}$ does not have associative property.

4) (F,Z) $\underset{\gamma_\varepsilon}{*}$(G,C) ≠ (G,C) $\underset{\gamma_\varepsilon}{*}$ (F,Z).

Proof: Firstly, we observe that the parameter set of the soft set on both sides of the equation is Z∪C, and thus the first condition of the soft equality is satisfied. Now let us look at the LHS. Let (F,Z) $\underset{\gamma_\varepsilon}{*}$ (G,C)=(H,Z∪C), where for all $\aleph \in Z\cup C$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}C \\ G'(\aleph), & \aleph\in C\text{-}Z \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap C \end{cases}$$

Now let's handle the RHS. Assume that (G,C) $\underset{\gamma_\varepsilon}{*}$(F,Z)=(T,C∪Z), where for all $\aleph \in C\cup Z$,

$$T(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C\text{-}Z \\ F'(\aleph), & \aleph\in Z\text{-}C \\ G'(\aleph)\cap F(\aleph), & \aleph\in C\cap Z \end{cases}$$

Thus, it is seen that H≠T. Similarly, it is easily seen that (F,Z) $\underset{\gamma_\varepsilon}{*}$(G,Z) ≠ (G,Z) $\underset{\gamma_\varepsilon}{*}$(F,Z). That is, $\underset{\gamma_\varepsilon}{*}$ is commutative neither in $S_E(U)$ nor in $S_Z(U)$.

5) (F,Z) $\underset{\gamma_\varepsilon}{*}$(F,Z)= $\emptyset_Z$

***Proof*:** Let (F,Z) $\underset{\gamma_\varepsilon}{*}$ (F,Z)=(H,Z∪Z), where for all $\aleph \in Z$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph)\cap F(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Hence, for all $\aleph \in Z$, H(ℵ)= F'(ℵ)∩F(ℵ)=∅, and so (H,Z)= $\emptyset_Z$. That is, $\underset{\gamma_\varepsilon}{*}$ is not idempotent in $S_E(U)$.

6) $\emptyset_Z$ $\underset{\gamma_\varepsilon}{*}$(F,Z)= (F,Z)

Proof: Let $\emptyset_Z$=(T,Z). Thus, for all ℵ∈Z, T(ℵ)=∅. Let $(T, Z) \underset{\gamma_\varepsilon}{*} (F, Z)$ =(H,Z∪Z), where for all ℵ∈Z,

$$H(ℵ)=\begin{cases} T'(ℵ), & ℵ\in Z\text{-}Z=\emptyset \\ F'(ℵ), & ℵ\in Z\text{-}Z=\emptyset \\ T'(ℵ)\cap F(ℵ), & ℵ\in Z\cap Z=Z \end{cases}$$

Hence, for all ℵ∈Z, H(ℵ)= T'(ℵ)∩F(ℵ)= U∩F(ℵ)=F(ℵ) and (H,Z)=(F,Z). That is, in S_Z(U), the left identity element of $\underset{\gamma_\varepsilon}{*}$ is the soft set.

7) (F,Z) $\underset{\gamma_\varepsilon}{*}\emptyset_Z=\emptyset_Z$.

Proof: Let $\emptyset_Z$ =(S,Z). Thus, for all ℵ∈Z, S(ℵ)= ∅. Let (F,Z) $\underset{\gamma_\varepsilon}{*}$ (S,Z)=(H,Z∪Z), where for all ℵ∈Z,

$$H(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z\text{-}Z=\emptyset \\ S'(ℵ), & ℵ\in Z\text{-}Z=\emptyset \\ F'(ℵ)\cap S(ℵ), & ℵ\in Z\cap Z=Z \end{cases}$$

Hence, for all ℵ∈Z, H(ℵ)= F'(ℵ)∩S(ℵ)= F'(ℵ)∩∅=∅ and (H,Z)= $\emptyset_Z$. That is, the right absorbing element of $\underset{\gamma_\varepsilon}{*}$ in S_Z(U) is the soft set $\emptyset_Z$.

8) (F,Z) $\underset{\gamma_\varepsilon}{*}$ $\emptyset_\emptyset$= $\emptyset_\emptyset$ $\underset{\gamma_\varepsilon}{*}$(F,Z) = $(F,Z)^r$.

Proof: Let $\emptyset_\emptyset$= (K, ∅) and (F,Z) $\underset{\gamma_\varepsilon}{*}$ (K, ∅) = (Q,Z ∪ ∅) = (Q,Z), where for all ℵ∈Z,

$$Q(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z\text{-}\emptyset=Z \\ K'(ℵ), & ℵ\in \emptyset\text{-}Z=\emptyset \\ F'(ℵ)\cap K(ℵ), & ℵ\in Z\cap\emptyset=\emptyset \end{cases}$$

Hence, for all ℵ∈Z, Q(ℵ)= F'(ℵ) and thus (Q,Z) = $(F,Z)^r$.

Similarly, $\emptyset_\emptyset$ $\underset{\gamma_\varepsilon}{*}$(F,Z)= (W, ∅ ∪ Z) = (W,Z). where for all ℵ∈Z,

$$W(ℵ)=\begin{cases} K'(ℵ), & ℵ\in \emptyset\text{-}Z=\emptyset \\ F'(ℵ), & ℵ\in Z\text{-}\emptyset=Z \\ K'(ℵ)\cap F(ℵ), & ℵ\in \emptyset\cap Z=\emptyset \end{cases}$$

Hence, for all ℵ∈Z, W(ℵ)= F'(ℵ) and thus (W,Z) = $(F,Z)^r$.

9) U_Z $\underset{\gamma_\varepsilon}{*}$(F,Z)= $\emptyset_Z$

Proof: Let U_Z =(H,Z), where for all ℵ∈Z, H(ℵ)=U. Let (H,Z) $\underset{\gamma_\varepsilon}{*}$(F,Z)=(T,Z∪Z), where for all ℵ∈Z,

$$T(ℵ)=\begin{cases} H'(ℵ), & ℵ\in Z\text{-}Z=\emptyset \\ F'(ℵ), & ℵ\in Z\text{-}Z=\emptyset \\ H'(ℵ)\cap F(ℵ), & ℵ\in Z\cap Z=Z \end{cases}$$

Here for all ℵ∈Z, T(ℵ)= H'(ℵ)∩F(ℵ)= ∅ ∩F(ℵ)=∅, and thus (T,Z)= $\emptyset_Z$.

10) (F,Z) $\underset{\gamma_\varepsilon}{*}$ U_Z=$(F,Z)^r$.

Proof: Let U_Z =(H,Z), where for all ℵ∈Z, H(ℵ)=U. Let (F,Z) $\underset{\gamma_\varepsilon}{*}$(H,Z)=(T,Z∪Z), where for all ℵ∈Z,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ H'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Here for all $\aleph\in Z$, $T(\aleph)= F'(\aleph)\cap H(\aleph)= F'(\aleph)\cap U=F'(\aleph)$, and thus $(T,Z)= (F,Z)^r$

11) $U_E \overset{*}{\gamma_\varepsilon} (F,Z) =\emptyset_E$

Proof: Let $U_E =(H,E)$, where for all $\aleph\in Z$, $H(\aleph)=U$. Let $(H,E) \overset{*}{\gamma_\varepsilon}(F,Z)=(T,E\cup Z)$, where for all $\aleph\in E$,

$$T(\aleph)=\begin{cases} H'(\aleph), & \aleph\in E\text{-}Z=Z' \\ F'(\aleph), & \aleph\in Z\text{-}E=\emptyset \\ H'(\aleph)\cap F(\aleph), & \aleph\in E\cap Z=Z \end{cases}$$

Here for all $\aleph\in E$, $T(\aleph)= H'(\aleph)\cap F(\aleph)= \emptyset \cap F(\aleph)=\emptyset$, and

$$T(\aleph)=\begin{cases} \emptyset, & \aleph\in E\text{-}Z=Z' \\ F'(\aleph), & \aleph\in Z\text{-}E=\emptyset \\ \emptyset, & \aleph\in Z\cap E=Z \end{cases}$$

Thus, for all $\aleph\in E$, $T(\aleph)=\emptyset$, therefore $(T,E)= \emptyset_E$.

12) $(F,Z) \overset{*}{\gamma_\varepsilon}(F,Z)^r = (F,Z)^r$.

Proof: Let $(F,Z)^r =(H,Z)$, where for all $\aleph\in Z$, $H(\aleph)=F'(\aleph)$. Let $(F,Z) \overset{*}{\gamma_\varepsilon}(H,Z)=(T,Z\cup Z)$, where for all $\aleph\in Z$,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ H'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Here for all $\aleph\in Z$, $T(\aleph)= F'(\aleph)\cap H(\aleph)= F'(\aleph) \cap F'(\aleph)=F'(\aleph)$, and thus $(T,Z)= (F,Z)^r$

That is, the right absorbing element of $\overset{*}{\gamma_\varepsilon}$ in $S_Z(U)$ is the soft set $(F,Z)^r$.

13) $(F,Z)^r \overset{*}{\gamma_\varepsilon}(F,Z)= (F,Z)$.

Proof: Let $(F,Z)^r =(H,Z)$, where for all $\aleph\in Z$, $H(\aleph)=F'(\aleph)$. Let $(H,Z) \overset{*}{\gamma_\varepsilon}(F,Z)=(T,Z\cup Z)$, where for all $\aleph\in Z$,

$$T(\aleph)=\begin{cases} H'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ H'(\aleph)\cap F(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Here for all $\aleph\in Z$, $T(\aleph)= H'(\aleph)\cap F(\aleph)= F(\aleph)\cap F'(\aleph)=F(\aleph)$, and thus $(T,Z)= (F,Z)$

That is, the left unit element of $\overset{*}{\gamma_\varepsilon}$ in $S_Z(U)$ is the soft set $(F,Z)^r$.

14) $[(F,Z) \overset{*}{\gamma_\varepsilon}(G,C)]^r =(F,Z)\, \lambda_\varepsilon(G,C)$.

Proof: Let $(F,Z) \overset{*}{\gamma_\varepsilon} (G,C)=(H,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}C \\ G'(\aleph), & \aleph\in C\text{-}Z \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(H,Z\cup C)^r =(T,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in Z-C \\ G(\aleph), & \aleph\in C-Z \\ F(\aleph)\cup G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Hence, $(T,Z\cup C)=(F,Z)\ \lambda_{\varepsilon}(G,C)$.

15) $(F,Z) \overset{*}{\gamma_{\varepsilon}} (G,Z)=U_Z \Leftrightarrow (F,Z)=\emptyset_Z$ and $(G,Z)=U_Z$

Proof: Let $(F,Z) \overset{*}{\gamma_{\varepsilon}} (G,Z)=(T,Z\cup Z)$, where for all $\aleph\in Z$,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-Z=\emptyset \\ G'(\aleph), & \aleph\in Z-Z=\emptyset \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Since $(T,Z)=U_Z$, $T(\aleph)=U$ for all $\aleph\in Z$ and thus, $F'(\aleph)\cap G(\aleph)=U$ for all $\aleph\in Z$. So, $F'(\aleph)=G(\aleph)=U$ for all $\aleph\in Z$. Thus, $(F,Z)=\emptyset_Z$, and $(G,Z)=U_Z$.

16) $\emptyset_Z \cong\!\!\subseteq (F,Z) \overset{*}{\gamma_{\varepsilon}} (G,C)$, $\emptyset_C \cong\!\!\subseteq (F,Z) \overset{*}{\gamma_{\varepsilon}} (G,C)$, $\emptyset_{Z\cup C} \cong\!\!\subseteq (F,Z) \overset{*}{\gamma_{\varepsilon}} (G,C)$, $(F,Z) \overset{*}{\gamma_{\varepsilon}} (G,C) \cong\!\!\subseteq U_{Z\cup C}$.

17) $(F,Z) \overset{*}{\gamma_{\varepsilon}} (G,Z) \cong\!\!\subseteq (F,Z)^r$ and $(F,Z) \overset{*}{\gamma_{\varepsilon}} (G,Z) \cong\!\!\subseteq (G,Z)$.

Proof: Let $(F,Z) \overset{*}{\gamma_{\varepsilon}} (G,Z)=(H,Z\cup Z)$, where for all $\aleph\in Z$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-Z=\emptyset \\ G'(\aleph), & \aleph\in Z-Z=\emptyset \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Since $H(\aleph)=F'(\aleph)\cap G(\aleph)\subseteq F'(\aleph)$ for all $\aleph\in Z$, $(F,Z) \overset{*}{\gamma_{\varepsilon}} (G,Z) \cong\!\!\subseteq (F,Z)^r$. Similarly, $H(\aleph)=F'(\aleph)\cap G(\aleph)\subseteq G(\aleph)$ for all $\aleph\in Z$. Thus, $(F,Z) \overset{*}{\gamma_{\varepsilon}} (G,Z) \cong\!\!\subseteq (G,Z)$.

18) If $(F,Z)\cong\!\!\subseteq(G,Z)$, then $(G,Z) \overset{*}{\gamma_{\varepsilon}} (H,C) \cong\!\!\subseteq (F,Z) \overset{*}{\gamma_{\varepsilon}} (H,C)$.

Proof: Let $(F,Z)\cong\!\!\subseteq(G,Z)$ then, $F(\aleph)\subseteq G(\aleph)$ for all $\aleph\in Z$ and so $G'(\aleph)\subseteq F'(\aleph)$. Let $(G,Z) \overset{*}{\gamma_{\varepsilon}} (H,C)=(W,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$W(\aleph)=\begin{cases} G'(\aleph), & \aleph\in Z-C \\ H'(\aleph), & \aleph\in C-Z \\ G'(\aleph)\cap H(\aleph), & \aleph\in Z\cap C \end{cases}$$

$(F,Z) \overset{*}{\gamma_{\varepsilon}} (H,C)=(L,Z\cup C)$. where for all $\aleph\in Z\cup C$,

$$L(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ H'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap C \end{cases}$$

Thus, $W(\aleph)=G'(\aleph)\subseteq G'(\aleph)=L(\aleph)$ for all $\aleph\in Z-C$, $W(\aleph)=H'(\aleph)\subseteq H'(\aleph)=L(\aleph)$ for all $\aleph\in C-Z$, and $W(\aleph)=H'(\aleph)\cap F(\aleph)\subseteq H'(\aleph)\cap G(\aleph)=L(\aleph)$ for all $\aleph\in Z\cap C$. Thus, $(G,Z) \overset{*}{\gamma_{\varepsilon}} (H,C) \cong\!\!\subseteq (F,Z) \overset{*}{\gamma_{\varepsilon}} (H,C)$.

19) If $(G,Z) \overset{*}{\gamma_{\varepsilon}} (H,C) \cong\!\!\subseteq (F,Z) \overset{*}{\gamma_{\varepsilon}} (H,C)$, then $(F,Z)\cong\!\!\subseteq(G,Z)$ need not have to be true. That is, the converse of Theorem 3.3. (18) is not true.

Proof: Let us give an example to show that the converse of Theorem 3.3. (18) is not true. Let $E=\{e_1,e_2,e_3,e_4,e_5\}$ be the parameter set, $A=\{e_1,e_3\}$, $C=\{e_1,e_3,e_5\}$ be the subset of E, and $U=\{h_1,h_2,h_3,h_4,h_5\}$ be the universal set.

Let $(F,Z)=\{(e_1,\{h_2,h_5\}),(e_3,\{h_1,h_2,h_5\})\}$, $(G,Z)=\{(e_1,\{h_2\}),(e_3,\{h_1,h_2\})\}$, $(H,C)=\{(e_1,\emptyset),(e_3,\emptyset),(e_5,U)\}$ be soft sets over U.

Let $(G,Z)\overset{*}{\gamma_\varepsilon}(H,C)=(L,Z\cup C)$, then $(L,Z\cup C)=\{(e_1,\emptyset),(e_3,\emptyset),(e_5,\emptyset)\}$ and let $(F,Z)\overset{*}{\gamma_\varepsilon}(H,C)=(K,Z\cup C)$ $(K,Z\cup C)=\{(e_1,\emptyset),(e_3,\emptyset),(e_5,\emptyset)\}$. Hence, $(G,Z)\overset{*}{\gamma_\varepsilon}(H,C)\,\tilde{\subseteq}\,(F,Z)\overset{*}{\gamma_\varepsilon}(H,C)$ but (F,Z) is not a soft subset of (G,Z).

20) If $(G,Z)\,\tilde{\subseteq}\,(F,C)$ and $(K,Z)\,\tilde{\subseteq}\,(L,C)$, then $(F,Z)\overset{*}{\gamma_\varepsilon}(K,Z)\,\tilde{\subseteq}\,(G,C)\overset{*}{\gamma_\varepsilon}(L,C)$.

Proof: Let $(G,Z)\,\tilde{\subseteq}\,(F,C)$ and $(K,Z)\,\tilde{\subseteq}\,(L,C)$. Hence, $Z\subseteq C$ and for all $\aleph\in Z$, $G(\aleph)\subseteq F(\aleph)$ and $K(\aleph)\subseteq L(\aleph)$. Let $(F,Z)\overset{*}{\gamma_\varepsilon}(K,Z)=(W,Z)$. Thus, for all $\aleph\in Z$,

$$W(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ K'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph)\cap K(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Let $(G,C)\overset{*}{\gamma_\varepsilon}(L,C)=(S,C)$. Thus, for for all $\aleph\in C$,

$$S(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C\text{-}C=\emptyset \\ L'(\aleph), & \aleph\in C\text{-}C=\emptyset \\ G'(\aleph)\cap L(\aleph), & \aleph\in C\cap C=C \end{cases}$$

Hence, sinec for all $\aleph\in Z$, $G(\aleph)\subseteq F(\aleph)$ since $F'(\aleph)\subseteq G'(\aleph)$ $W(\aleph)=F'(\aleph)\cap K(\aleph)\subseteq G'(\aleph)\cap L(\aleph)=S(\aleph)$, $(F,Z)\overset{*}{\gamma_\varepsilon}(K,Z)\,\tilde{\subseteq}\,(G,C)\overset{*}{\gamma_\varepsilon}(L,C)$.

Theorem 3.4. The complementary extended gamma operation has the following distributions over other soft set operations:

Theorem 3.4.1. The complementary extended gamma operation has the following distributions over restricted soft set operations:

i) LHS Distributions of the Complementary Extended Gamma Operation on Restricted Soft Set Operations:

1) If $Z\cap C\cap R=\emptyset$ then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)\cap_R(H,R)]=[(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)]\cup_R[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]$.

Proof: Consider first the LHS. Let $(G,C)\cap_R(H,R)=(M,C\cap R)$, where for all $\aleph\in C\cap R$, $M(\aleph)=G(\aleph)\cap H(\aleph)$. Let $(F,Z)\overset{*}{\gamma_\varepsilon}(M,C\cap R)=(N,Z\cup(C\cap R))$, where for all $\aleph\in Z\cup(C\cap R)$,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}(C\cap R) \\ M'(\aleph), & \aleph\in(C\cap R)\text{-}Z \\ F'(\aleph)\cap M(\aleph), & \aleph\in Z\cap(C\cap R) \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}(C\cap R)=Z\text{-}(C\cap R) \\ G'(\aleph)\cup H'(\aleph), & \aleph\in(C\cap R)\text{-}Z=Z'\cap C\cap R \\ F'(\aleph)\cap(G(\aleph)\cap H(\aleph)), & \aleph\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Now lets handle the RHS i.e. $[(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)]\cup_R[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)=(V,Z\cup C)$, where, for all $\aleph\in Z\cup C$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap C \end{cases}$$

Assume that $(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)=(W,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$W(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Let $(V,Z\cup C)\cup_R(W,Z\cup R)=(T,(Z\cup C)\cap(Z\cup R)))$, where, for all $\aleph\in Z\cup(C\cap R)$, $T(\aleph)=V(\aleph)\cap W(\aleph)$. Hence,

$$T(\aleph)=\begin{cases} F'(\aleph)\cup F'(\aleph), & \aleph\in(Z-C)\cap(Z-R)=Z\cap C'\cap R' \\ F'(\aleph)\cup H'(\aleph), & \aleph\in(Z-C)\cap(R-Z)=\emptyset \\ F'(\aleph)\cup(F'(\aleph)\cap H(\aleph)) & \aleph\in(Z-C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cup F'(\aleph), & \aleph\in(C-Z)\cap(Z-R)=\emptyset \\ G'(\aleph)\cup H'(\aleph), & \aleph\in(C-Z)\cap(R-Z)=Z'\cap C\cap R \\ G'(\aleph)\cup\big(F'(\aleph)\cap H(\aleph)\big), & \aleph\in(C-Z)\cap(Z\cap R)=\emptyset \\ (F'(\aleph)\cap G(\aleph))\cup F'(\aleph), & \aleph\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R' \\ (F'(\aleph)\cap G(\aleph))\cup H'(\aleph), & \aleph\in(Z\cap C)\cap(R-Z)=\emptyset \\ (F'(\aleph)\cap G(\aleph))\cup(F'(\aleph)\cap H(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in(Z-C)\cap(Z-R)=Z\cap C'\cap R' \\ F'(\aleph) & \aleph\in(Z-C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cup H'(\aleph), & \aleph\in(C-Z)\cap(R-Z)=Z'\cap C\cap R \\ F'(\aleph), & \aleph\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R' \\ (F'(\aleph)\cap G(\aleph))\cup(F'(\aleph)\cap H(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Here, when considering $Z-(C\cap R)$ in the function N, since $Z-(C\cap R)=Z\cap(C\cap R)'$, if an element is in the complement of $(C\cap R)$, it is either in $C-R$, in $R-C$, or in $(C\cup R)'$. Thus, if $\aleph\in Z-(C\cap R)$, then $\aleph\in Z\cap C\cap R'$ or $\aleph\in Z\cap C'\cap R$ or $\aleph\in Z\cap C'\cap R'$. Thus, N=T under $Z\cap C\cap R=\emptyset$.

2) If $Z'\cap C\cap R=\emptyset$ then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)\cup_R(H,R)]=[(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)]\cup_R[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]$.

3) If $Z'\cap C\cap R=\emptyset$ then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)*_R(H,R)]=[(F,Z)\overset{*}{\theta_\varepsilon}(G,C)]\cup_R[(F,Z)\overset{*}{\theta_\varepsilon}(H,R)]$.

4) If $Z'\cap C\cap R=Z\cap C\cap R=\emptyset$ then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)\,\theta_R(H,R)]=[(F,Z)\overset{*}{\theta_\varepsilon}(G,C)]\cup_R[(F,Z)\overset{*}{\theta_\varepsilon}(H,R)]$

ii) RHS Distribution of Complementary Extended Gamma Operation on Restricted Soft Set Operations

1) If $(Z\Delta C)\cap R=\emptyset$ if $[(F,Z)\cup_R(G,C)]\overset{*}{\gamma_\varepsilon}(H,R)=[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]\cap_R[(G,C)\overset{*}{\gamma_\varepsilon}(H,R)]$.

Proof: Consider first LHS. Let $(F,Z)\cup_R(G,C)=(M,Z\cap C)$, where for all $\aleph\in Z\cap C$, $M(\aleph)=F(\aleph)\cup G(\aleph)$. Let $(M,Z\cap C)\overset{*}{\cap_\varepsilon}(H,R)=(N,(Z\cap C)\cup R))$, where $\aleph\in(Z\cap C)\cup R$,

$$N(\aleph)=\begin{cases} M'(\aleph), & \aleph\in(Z\cap C)-R \\ H'(\aleph), & \aleph\in R-(Z\cap C) \\ M'(\aleph)\cap H(\aleph), & \aleph\in Z\cap(C\cap R) \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph)\cap G'(\aleph), & \aleph\in(Z\cap C)-R=Z\cap C\cap R' \\ H'(\aleph), & \aleph\in R-(Z\cap C)=R-(Z\cap C) \\ (F'(\aleph)\cap G'(\aleph))\cap H(\aleph), & \aleph\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Now consider RHS. i.e. $[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]\cap_R[(G,C)\overset{*}{\gamma_\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)=(V,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Now, let $(G,C)\overset{*}{\gamma_\varepsilon}(H,R)=(W, C\cup R)$, where for all $\aleph\in C\cup R$,

$$W(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C-R \\ H'(\aleph), & \aleph\in R-C \\ G'(\aleph)\cap H(\aleph), & \aleph\in C\cap R \end{cases}$$

Let $(V,Z\cup R)\cap_R(W,C\cup R)=(T,(Z\cup R)\cap(C\cup R))$. Here, for all $\aleph\in(Z\cap C)\cup R$, $T(\aleph)=V(\aleph)\cap W(\aleph)$. . Thus,

$$T(\aleph)=\begin{cases} F'(\aleph)\cap G'(\aleph), & \aleph\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ F'(\aleph)\cap H'(\aleph), & \aleph\in(Z-R)\cap(R-C)=\emptyset \\ F'(\aleph)\cap\big(G'(\aleph)\cap H(\aleph)\big), & \aleph\in(Z-R)\cap(C\cap R)=\emptyset \\ H'(\aleph)\cap G'(\aleph), & \aleph\in(R-Z)\cap(C-R)=\emptyset \\ H'(\aleph)\cap H'(\aleph), & \aleph\in(R-Z)\cap(R-C)=Z'\cap C'\cap R \\ H'(\aleph)\cap(G'(\aleph)\cap H(\aleph)), & \aleph\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ \big(F'(\aleph)\cap H(\aleph)\big)\cap G'(\aleph), & \aleph\in(Z\cap R)\cap(C-R)=\emptyset \\ \big(F'(\aleph)\cap H(\aleph)\big)\cap H'(\aleph) & \aleph\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ (F'(\aleph)\cap H(\aleph))\cap(G'(\aleph)\cap H(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F'(\aleph)\cap G'(\aleph), & \aleph\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ H'(\aleph) & \aleph\in(R-Z)\cap(R-C)=Z'\cap C'\cap R \\ \emptyset, & \aleph\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ \emptyset, & \aleph\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ (F'(\aleph)\cap H(\aleph))\cap(G'(\aleph)\cap H(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Here, if we consider $R-(Z\cap C)$ in the function N, since $R-(Z\cap C)=R\cap(Z\cap C)'$, if an element is in the complement of $(Z\cap C)$, it is either in Z-C, in C-Z, or in $(Z\cup C)'$. Thus, if $\aleph\in R-(Z\cap C)$, then $\aleph\in R\cap Z\cap C'$ or $\aleph\in R\cap Z'\cap C$ or $\aleph\in R\cap Z'\cap C'$. Hence, N=T is satisfied under the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$. The condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$ implies that $(Z\Delta C)\cap R=\emptyset$ is obvious

2) If $Z\cap C\cap R'=(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\cap_R(G,C)]\overset{*}{\gamma_\varepsilon}(H,R)=[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]\cup_R[(G,C)\overset{*}{\gamma_\varepsilon}(H,R)]$.

3) If $Z\cap C\cap R'=(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\theta_R(G,C)]\overset{*}{\gamma_\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap_\varepsilon}(H,R)]\cup_R[(G,C)\overset{*}{\cap_\varepsilon}(H,R)]$.

4) If $Z\cap C\cap R'=(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)*_R (G,C)] \underset{\gamma_\varepsilon}{*} (H,R)=[(F,Z) \underset{\cap_\varepsilon}{*} (H,R)] \cap_R [(G,C) \underset{\cap_\varepsilon}{*} (H,R)]$.

Theorem 3.4.2. The following distributions of the complementary extended gamma operation over extended soft set operations hold:

i)LHS Distributions of the Complementary Extended Gamma Operation on Extended Soft Set Operations

1)If $Z\cap C\cap R'=(Z\Delta C)\cap R=\emptyset$, then $(F,Z) \underset{\gamma_\varepsilon}{*} [(G,C)*_\varepsilon(H,R)]=[(F,Z) \underset{\theta_\varepsilon}{*} (G,C)]\cup_\varepsilon[(F,Z) \underset{\theta_\varepsilon}{*} (H,R)]$.

Proof: Consider first LHS. Let $(G,C)*_\varepsilon (H,R)=(M,C\cup R)$, where for all $\aleph \in C\cup R$,

$$M(\aleph)=\begin{cases} G(\aleph), & \aleph\in C-R \\ H(\aleph), & \aleph\in R-C \\ G'(\aleph)\cup H'(\aleph), & \aleph\in C\cap R \end{cases}$$

Let $(F,Z) \underset{\gamma_\varepsilon}{*} (M,C\cup R)=(N,Z\cup(C\cup R))$, where for all $\aleph\in Z\cup C\cup R$,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-(C\cup R) \\ M'(\aleph), & \aleph\in (C\cup R)-Z \\ F'(\aleph)\cap M(\aleph), & \aleph\in Z\cap(C\cup R) \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-(C\cup R)=Z\cap C'\cap R' \\ G'(\aleph), & \aleph\in (C-R)-Z=Z'\cap C\cap R' \\ H'(\aleph), & \aleph\in (R-C)-Z=Z'\cap C'\cap R \\ G(\aleph)\cap H(\aleph), & \aleph\in (C\cap R)-Z=Z'\cap C\cap R \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap(C-R)=Z\cap C\cap R' \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap(R-C)=Z\cap C'\cap R \\ F'(\aleph)\cap(G'(\aleph)\cup H'(\aleph)), & \aleph\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Now consider the RHS, i.e. $[(F,Z) \underset{\theta_\varepsilon}{*} (G,C)]\cup_\varepsilon[(F,Z) \underset{\theta_\varepsilon}{*} (H,R)]$. Let $(F,Z) \underset{\theta_\varepsilon}{*} (G,C)=(V,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cap G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(F,Z) \underset{\theta_\varepsilon}{*} (H,R)=(W,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$W(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ F'(\aleph)\cap H'(\aleph), & \aleph\in Z\cap R \end{cases}$$

Let $(V,Z\cup C) \cup_\varepsilon (W,Z\cup R)=(T,(Z\cup C)\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$T(\aleph)=\begin{cases} V(\aleph), & \aleph\in (Z\cup C)-(Z\cup R) \\ W(\aleph), & \aleph\in (Z\cup R)-(Z\cup C) \\ V(\aleph)\cup W(\aleph), & \aleph\in (Z\cup C)\cap(Z\cup R) \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases}
F'(\aleph), & \aleph\in(Z-C)-(Z\cup R)=\emptyset\\
G'(\aleph), & \aleph\in(C-Z)-(Z\cup R)=Z'\cap C\cap R'\\
F'(\aleph)\cap G'(\aleph), & \aleph\in(Z\cap C)-(Z\cup R)=\emptyset\\
F'(\aleph), & \aleph\in(Z-R)-(Z\cup C)=\emptyset\\
H'(\aleph), & \aleph\in(R-Z)-(Z\cup C)=Z'\cap C'\cap R\\
F'(\aleph)\cap H'(\aleph), & \aleph\in(Z\cap R)-(Z\cup C)=\emptyset\\
F'(\aleph)\cup F'(\aleph), & \aleph\in(Z-C)\cap(Z-R)=Z\cap C'\cap R'\\
F'(\aleph)\cup H'(\aleph), & \aleph\in(Z-C)\cap(R-Z)=\emptyset\\
F'(\aleph)\cup(F'(\aleph)\cap H'(\aleph)), & \aleph\in(Z-C)\cap(Z\cap R)=Z\cap C'\cap R\\
G'(\aleph)\cup F'(\aleph), & \aleph\in(C-Z)\cap(Z-R)=\emptyset\\
G'(\aleph)\cup H'(\aleph), & \aleph\in(C-Z)\cap(R-Z)=Z'\cap C\cap R\\
G'(\aleph)\cup(F'(\aleph)\cap H'(\aleph)), & \aleph\in(C-Z)\cap(Z\cap R)=\emptyset\\
(F'(\aleph)\cap G'(\aleph))\cup F'(\aleph), & \aleph\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R'\\
(F'(\aleph)\cap G'(\aleph))\cup H'(\aleph), & \aleph\in(Z\cap C)\cap(R-Z)=\emptyset\\
(F'(\aleph)\cap G'(\aleph))\cup(F'(\aleph)\cap H'(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R
\end{cases}$$

Hence,

$$T(\aleph)=\begin{cases}
G'(\aleph), & \aleph\in(C-Z)-(Z\cup R)=Z'\cap C\cap R'\\
H'(\aleph), & \aleph\in(R-Z)-(Z\cup C)=Z\cap C'\cap R\\
F'(\aleph), & \aleph\in(Z-C)\cap(Z-R)=Z\cap C'\cap R'\\
F'(\aleph), & \aleph\in(Z-C)\cap(Z\cap R)=Z\cap C'\cap R\\
G'(\aleph)\cup H'(\aleph), & \aleph\in(C-Z)\cap(R-Z)=Z'\cap C\cap R\\
F'(\aleph), & \aleph\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R\\
(F'(\aleph)\cap G'(\aleph))\cup(F'(\aleph)\cap H'(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R
\end{cases}$$

It is seen that N=T is satisfied under the condition $Z'\cap C\cap R=Z\cap C'\cap R=Z\cap C\cap R'=\emptyset$. It is obvious that the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$ is equal to $(Z\Delta C)\cap R=\emptyset$.

2) If $Z'\cap C\cap R=\emptyset$, then $(F,Z)\underset{\gamma_\varepsilon}{*}[(G,C)\cap_\varepsilon(H,R)]=[(F,Z)\underset{\gamma_\varepsilon}{*}(G,C)]\cap_\varepsilon[(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)]$.

3) If $Z\cap C\cap R'=(Z\Delta C)\cap R=\emptyset$, then $(F,Z)\underset{\gamma_\varepsilon}{*}[(G,C)\cup_\varepsilon(H,R)]=[(F,Z)\underset{\gamma_\varepsilon}{*}(G,C)]\cup_\varepsilon[(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)]$.

4) If $Z\cap C\cap R'=(Z\Delta C)\cap R=\emptyset$, then $(F,Z)\underset{\gamma_\varepsilon}{*}[(G,C)\theta_\varepsilon(H,R)]=[(F,Z)\underset{\theta_\varepsilon}{*}(G,C)]\cap_\varepsilon[(F,Z)\underset{\theta_\varepsilon}{*}(H,R)]$.

ii) RHS Distributions of Complementary Extended Gamma Operation over Extended Soft Set Operations

1)If $(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\cup_\varepsilon(G,C)]\underset{\gamma_\varepsilon}{*}(H,R)=[(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)]\cap_\varepsilon[(G,C)\underset{\gamma_\varepsilon}{*}(H,R)]$.

Proof: Consider first the LHS. Let $(F,Z)\cup_\varepsilon(G,C)=(M,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$M(\aleph)=\begin{cases}
F(\aleph), & \aleph\in Z-C\\
G(\aleph), & \aleph\in C-Z\\
F(\aleph)\cup G(\aleph), & \aleph\in Z\cap C
\end{cases}$$

Let $(M,Z\cup C)\underset{\gamma_\varepsilon}{*}(H,R)=(N,(Z\cup C)\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$N(\aleph)=\begin{cases} M'(\aleph), & \aleph\in(Z\cup C)-R \\ H'(\aleph), & \aleph\in R-(Z\cup C) \\ M'(\aleph)\cap H(\aleph), & \aleph\in(Z\cup C)\cap R \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in(Z-C)-R=Z\cap C'\cap R' \\ G'(\aleph), & \aleph\in(C-Z)-R=Z'\cap C\cap R' \\ F'(\aleph)\cap G'(\aleph), & \aleph\in(Z\cap C)-R=Z\cap C\cap R' \\ H'(\aleph), & \aleph\in R-(Z\cup C)=Z'\cap C'\cap R \\ F'(\aleph)\cap H(\aleph), & \aleph\in(Z-C)\cap R=Z\cap C'\cap R \\ G'(\aleph)\cap H(\aleph), & \aleph\in(C-Z)\cap R=Z'\cap C\cap R \\ (F'(\aleph)\cap G'(\aleph))\cap H(\aleph), & \aleph\in(Z\cap C)\cap R=Z\cap C\cap R \end{cases}$$

Now consider the RHS, i.e. $[(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)]\cap_\varepsilon[(G,C)\underset{\gamma_\varepsilon}{*}(H,R)]$. Let $(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)=(V,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Let $(G,C)\underset{\gamma_\varepsilon}{*}(H,R)=(W,C\cup R)$, where for all $\aleph\in C\cup R$,

$$W(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C-R \\ H'(\aleph), & \aleph\in R-C \\ G'(\aleph)\cap H(\aleph), & \aleph\in C\cap R \end{cases}$$

Let $(V,Z\cup R)\cap_\varepsilon(W,C\cup R)=(T,Z\cup C\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$T(\aleph)=\begin{cases} V(\aleph), & \aleph\in(Z\cup R)-(C\cup R) \\ W(\aleph), & \aleph\in(C\cup R)-(Z\cup R) \\ V(\aleph)\cap W(\aleph), & \aleph\in(Z\cup R)\cap(C\cup R) \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in(Z-R)-(C\cup R)=Z\cap C'\cap R' \\ H'(\aleph), & \aleph\in(R-Z)-(C\cup R)=\emptyset \\ F'(\aleph)\cap H(\aleph), & \aleph\in(Z\cap R)-(C\cup R)=\emptyset \\ G'(\aleph), & \aleph\in(C-R)-(Z\cup R)=Z'\cap C\cap R' \\ H'(\aleph), & \aleph\in(R-C)-(Z\cup R)=\emptyset \\ G'(\aleph)\cap H(\aleph), & \aleph\in(C\cap R)-(Z\cup R)=\emptyset \\ F'(\aleph)\cap G'(\aleph), & \aleph\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ F'(\aleph)\cap H'(\aleph), & \aleph\in(Z-R)\cap(R-C)=\emptyset \\ F'(\aleph)\cap(G'(\aleph)\cap H(\aleph)), & \aleph\in(Z-R)\cap(C\cap R)=\emptyset \\ H'(\aleph)\cap G'(\aleph), & \aleph\in(R-Z)\cap(C-R)=\emptyset \\ H'(\aleph)\cap H'(\aleph), & \aleph\in(R-Z)\cap(R-C)=Z\cap C'\cap R \\ H'(\aleph)\cap\big(G'(\aleph)\cap H(\aleph)\big) & \aleph\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ (F'(\aleph)\cap H(\aleph))\cap G'(\aleph), & \aleph\in(Z\cap R)\cap(C-R)=\emptyset \\ (F'(\aleph)\cap H(\aleph))\cap H'(\aleph) & \aleph\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ (F'(\aleph)\cap H(\aleph))\cap(G'(\aleph)\cap H(\aleph)) & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in(Z\text{-}R)\text{-}(C\cup R)=Z\cap C'\cap R' \\ G'(\aleph), & \aleph\in(C\text{-}R)\text{-}(Z\cup R)=Z'\cap C\cap R' \\ F'(\aleph)\cap G'(\aleph), & \aleph\in(Z\text{-}R)\cap(C\text{-}R)=Z\cap C\cap R' \\ H'(\aleph), & \aleph\in(R\text{-}Z)\cap(R\text{-}C)=Z'\cap C'\cap R \\ \emptyset & \aleph\in(R\text{-}Z)\cap(C\cap R)=Z'\cap C\cap R \\ \emptyset & \aleph\in(Z\cap R)\cap(R\text{-}C)=Z\cap C'\cap R \\ (F'(\aleph)\cap H(\aleph))\cap(G'(\aleph)\cap H(\aleph)) & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Hence, under the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$, N=T is satisfied. It is obvious that the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$ is equivalent to the condition $(Z\Delta C)\cap R=\emptyset$.

2) If $(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\cap_\varepsilon(G,C)]\overset{*}{\gamma_\varepsilon}(H,R)=[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]\cup_\varepsilon[(G,C)\overset{*}{\gamma_\varepsilon}(H,R)]$.

3) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $[(F,Z)\theta_\varepsilon(G,C)]\overset{*}{\gamma_\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap_\varepsilon}(H,R)]\cup_\varepsilon[(G,C)\overset{*}{\cap_\varepsilon}(H,R)]$.

4) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $[(F,Z)*_\varepsilon(G,C)]\overset{*}{\gamma_\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap_\varepsilon}(H,R)]\cap_\varepsilon[(G,C)\overset{*}{\cap_\varepsilon}(H,R)]$.

Theorem 3.4.3. The following distributions of the complementary extended gamma operation over complementary extended operations hold:

i) LHS Distributions of Complementary Extended Gamma Operations over Complementary Extended Soft Set Operations

1) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)\overset{*}{\cap_\varepsilon}(H,R)]=[(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)]\overset{*}{\cap_\varepsilon}[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]$.

Proof: Consider first LHS. Let $(G,C)\overset{*}{\cap_\varepsilon}((H,R)=(M,C\cup R)$, where for all $\aleph\in C\cup R$,

$$M(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C\text{-}R \\ H'(\aleph), & \aleph\in R\text{-}C \\ G(\aleph)\cap H(\aleph), & \aleph\in C\cap R \end{cases}$$

Let $(F,Z)\overset{*}{\gamma_\varepsilon}(M,C\cup R)=(N,Z\cup(C\cup R))$, where for all $\aleph\in Z\cup C\cup R$,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}(C\cup R) \\ M'(\aleph), & \aleph\in(C\cup R)\text{-}Z \\ F'(\aleph)\cap M(\aleph), & \aleph\in Z\cap(C\cup R) \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}(C\cup R)=Z\cap C'\cap R' \\ G(\aleph), & \aleph\in(C\text{-}R)\text{-}Z=Z'\cap C\cap R' \\ H(\aleph), & \aleph\in(R\text{-}C)\text{-}Z=Z'\cap C'\cap R \\ G'(\aleph)\cup H'(\aleph), & \aleph\in(C\cap R)\text{-}Z=Z'\cap C\cap R \\ F'(\aleph)\cap G'(\aleph), & \aleph\in Z\cap(C\text{-}R)=Z\cap C\cap R' \\ F'(\aleph)\cap H'(\aleph), & \aleph\in Z\cap(R\text{-}C)=Z\cap B'\cap R \\ F'(\aleph)\cap(G(\aleph)\cap H(\aleph)), & \aleph\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Now consider the RHS, i.e. $[(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)]\overset{*}{\cap_\varepsilon}[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)=(V,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$V(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z\text{-}C \\ G'(ℵ), & ℵ\in C\text{-}Z \\ F'(ℵ)\cap G(ℵ), & ℵ\in Z\cap C \end{cases}$$

Let $(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)=(W,Z\cup R)$, where for all $ℵ\in Z\cup R$,

$$W(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z\text{-}R \\ H'(ℵ), & ℵ\in R\text{-}Z \\ F'(ℵ)\cap H(ℵ), & ℵ\in Z\cap R \end{cases}$$

Let $(V,Z\cup C)\underset{\cap_\varepsilon}{*}(W,Z\cup R)=(T,(Z\cup C)\cup R)$, where for all $ℵ\in Z\cup C\cup C\cup R$

$$T(ℵ)=\begin{cases} V'(ℵ), & ℵ\in(Z\cup C)\text{-}(Z\cup R) \\ W'(ℵ), & ℵ\in(Z\cup R)\text{-}(Z\cup C) \\ V(ℵ)\cap W(ℵ), & ℵ\in(Z\cup C)\cap(Z\cup R) \end{cases}$$

Thus,

$$T(ℵ)=\begin{cases} F(ℵ), & ℵ\in(Z\text{-}C)\text{-}(Z\cup R)=\emptyset \\ G(ℵ), & ℵ\in(C\text{-}Z)\text{-}(Z\cup R)=Z'\cap C\cap R' \\ F(ℵ)\cup G'(ℵ), & ℵ\in(Z\cap C)\text{-}(Z\cup R)=\emptyset \\ F(ℵ), & ℵ\in(Z\text{-}R)\text{-}(Z\cup C)=\emptyset \\ H(ℵ), & ℵ\in(R\text{-}Z)\text{-}(Z\cup C)=Z'\cap C'\cap R \\ F(ℵ)\cup H'(ℵ), & ℵ\in(Z\cap R)\text{-}(Z\cup C)=\emptyset \\ F'(ℵ)\cap F'(ℵ), & ℵ\in(Z\text{-}C)\cap(Z\text{-}R)=Z\cap C'\cap R' \\ F'(ℵ)\cap H'(ℵ), & ℵ\in(Z\text{-}C)\cap(R\text{-}Z)=\emptyset \\ F'(ℵ)\cap(F'(ℵ)\cap H(ℵ)), & ℵ\in(Z\text{-}C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(ℵ)\cap F'(ℵ), & ℵ\in(C\text{-}Z)\cap(Z\text{-}R)=\emptyset \\ G'(ℵ)\cap H'(ℵ), & ℵ\in(C\text{-}Z)\cap(R\text{-}Z)=Z'\cap C\cap R \\ G'(ℵ)\cap(F'(ℵ)\cap H(ℵ)), & ℵ\in(C\text{-}Z)\cap(Z\cap R)=\emptyset \\ (F'(ℵ)\cap G(ℵ))\cap F'(ℵ), & ℵ\in(Z\cap C)\cap(Z\text{-}R)=Z\cap C\cap R' \\ (F'(ℵ)\cap G(ℵ))\cap H'(ℵ), & ℵ\in(Z\cap C)\cap(R\text{-}Z)=\emptyset \\ (F'(ℵ)\cap G(ℵ))\cap(F'(ℵ)\cap H(ℵ)), & ℵ\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Thus,

$$T(ℵ)=\begin{cases} G(ℵ), & ℵ\in(C\text{-}Z)\text{-}(Z\cup R)=Z'\cap C\cap R' \\ H(ℵ), & ℵ\in(R\text{-}Z)\text{-}(Z\cup C)=Z'\cap C'\cap R \\ F'(ℵ) & ℵ\in(Z\text{-}C)\cap(Z\text{-}R)=Z\cap C'\cap R' \\ F'(ℵ)\cap H(ℵ), & ℵ\in(Z\text{-}C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(ℵ)\cap H'(ℵ), & ℵ\in(C\text{-}Z)\cap(R\text{-}Z)=Z'\cap C\cap R \\ F'(ℵ)\cap G(ℵ)) & ℵ\in(Z\cap C)\cap(Z\text{-}R)=Z\cap C\cap R' \\ (F'(ℵ)\cap G(ℵ))\cap(F'(ℵ)\cap H(ℵ)), & ℵ\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

N=T is satisfied under the condition $Z'\cap C\cap R=Z\cap C'\cap R=Z\cap C\cap R'=\emptyset$. It is obvious that the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$ is equivalent to $(Z\Delta C)\cap R=\emptyset$.

2) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $(F,Z)\underset{\gamma_\varepsilon}{*}[(G,C)\underset{\cup_\varepsilon}{*}(H,R)]=[(F,Z)\underset{\gamma_\varepsilon}{*}(G,C)]\underset{\cup_\varepsilon}{*}[(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)]$.

3) If $Z'\cap C\cap R=Z\cap C\cap R=\emptyset$, then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)\overset{*}{*_\varepsilon}(H,R)]=[(F,Z)\overset{*}{\theta_\varepsilon}(G,C)]\overset{*}{\cap_\varepsilon}[(F,Z)\overset{*}{\theta_\varepsilon}(H,R)]$.

4) If $Z'\cap C\cap R=\emptyset$, then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)\overset{*}{\theta_\varepsilon}(H,R)]=[(F,Z)\overset{*}{\theta_\varepsilon}(G,C)]\overset{*}{\cap_\varepsilon}[(F,Z)\overset{*}{\theta_\varepsilon}(H,R)]$.

ii) RHS Distributions of Complementary Extended Gamma Operation over Complementary Extended Operations

1) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $[(F,Z)\overset{*}{\theta_\varepsilon}(G,C)]\overset{*}{\gamma_\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap_\varepsilon}(H,R)]\overset{*}{\cup_\varepsilon}[(G,C)\overset{*}{\cap_\varepsilon}(H,R)]$.

Proof: Consider first LHS. Let $(F,Z)\overset{*}{\theta_\varepsilon}(G,C)=(M,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$M(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cap G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(M,Z\cup C)\overset{*}{\gamma_\varepsilon}(H,R)=(N,(Z\cup C)\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$N(\aleph)=\begin{cases} M'(\aleph), & \aleph\in(Z\cup C)-R \\ H'(\aleph), & \aleph\in R-(Z\cup C) \\ M'(\aleph)\cap H(\aleph), & \aleph\in(Z\cup C)\cap R \end{cases}$$

Hence,

$$N(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z-C)-R=Z\cap C'\cap R' \\ G(\aleph), & \aleph\in(C-Z)-R=Z'\cap C\cap R' \\ F(\aleph)\cup G(\aleph), & \aleph\in(Z\cap C)-R=Z\cap C\cap R' \\ H'(\aleph), & \aleph\in R-(Z\cup C)=Z'\cap C'\cap R \\ F(\aleph)\cap H(\aleph), & \aleph\in(Z-C)\cap R=Z\cap C'\cap R \\ G(\aleph)\cap H(\aleph), & \aleph\in(C-Z)\cap R=Z'\cap C\cap R \\ (F(\aleph)\cup G(\aleph))\cap H(\aleph), & \aleph\in(Z\cap C)\cap R=Z\cap C\cap R \end{cases}$$

Now consider the RHS, i.e. $[(F,Z)\overset{*}{\cap_\varepsilon}(H,R)]\overset{*}{\cup_\varepsilon}[(G,C)\overset{*}{\cap_\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{\cap_\varepsilon}(H,R)=(V,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ F(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Let $(G,C)\overset{*}{\cap_\varepsilon}(H,R)=(W,C\cup R)$, where for all $\aleph\in C\cup R$,

$$W(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C-R \\ H'(\aleph), & \aleph\in R-C \\ G(\aleph)\cap H(\aleph), & \aleph\in C\cap R \end{cases}$$

Let $(V,Z\cup R)\overset{*}{\cup_\varepsilon}(W,C\cup R)=(T,Z\cup C\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$T(\aleph)=\begin{cases} V'(\aleph), & \aleph\in(Z\cup R)-(C\cup R) \\ W'(\aleph), & \aleph\in(C\cup R)-(Z\cup R) \\ V(\aleph)\cup W(\aleph), & \aleph\in(Z\cup R)\cap(C\cup R) \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z\text{-}R)\text{-}(C\cup R)=Z\cap C'\cap R' \\ H(\aleph), & \aleph\in(R\text{-}Z)\text{-}(C\cup R)=\emptyset \\ F'(\aleph)\cup H'(\aleph), & \aleph\in(Z\cap R)\text{-}(C\cup R)=\emptyset \\ G(\aleph), & \aleph\in(C\text{-}R)\text{-}(Z\cup R)=Z'\cap C\cap R' \\ H(\aleph), & \aleph\in(R\text{-}C)\text{-}(Z\cup R)=\emptyset \\ G'(\aleph)\cup H'(\aleph), & \aleph\in(C\cap R)\text{-}(Z\cup R)=\emptyset \\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z\text{-}R)\cap(C\text{-}R)=Z\cap C\cap R' \\ F'(\aleph)\cup H'(\aleph), & \aleph\in(Z\text{-}R)\cap(R\text{-}C)=\emptyset \\ F'(\aleph)\cup(G(\aleph)\cap H(\aleph)), & \aleph\in(Z\text{-}R)\cap(C\cap R)=\emptyset \\ H'(\aleph)\cup G'(\aleph), & \aleph\in(R\text{-}Z)\cap(C\text{-}R)=\emptyset \\ H'(\aleph)\cup H'(\aleph), & \aleph\in(R\text{-}Z)\cap(R\text{-}C)=Z'\cap C'\cap R \\ H'(\aleph)\cup\big(G(\aleph)\cap H(\aleph)\big) & \aleph\in(R\text{-}Z)\cap(C\cap R)=Z'\cap C\cap R \\ (F(\aleph)\cap H(\aleph))\cup G'(\aleph), & \aleph\in(Z\cap R)\cap(C\text{-}R)=\emptyset \\ \big(F(\aleph)\cap H(\aleph)\big)\cup H'(\aleph) & \aleph\in(Z\cap R)\cap(R\text{-}C)=Z\cap C'\cap R \\ (F(\aleph)\cap H(\aleph))\cup(G(\aleph)\cap H(\aleph)) & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Therefore,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z\text{-}R)\text{-}(C\cup R)=Z\cap C'\cap Z' \\ G(\aleph), & \aleph\in(C\text{-}R)\text{-}(Z\cup R)=Z'\cap C\cap Z \\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z\text{-}R)\cap(C\text{-}R)=Z\cap C\cap Z' \\ H'(\aleph), & \aleph\in(R\text{-}Z)\cap(R\text{-}C)=Z'\cap C'\cap R \\ H'(\aleph)\cup G(\aleph), & \aleph\in(R\text{-}Z)\cap(C\cap R)=Z'\cap C\cap R \\ F(\aleph)\cup H'(\aleph) & \aleph\in(Z\cap R)\cap(R\text{-}C)=Z\cap C'\cap R \\ (F(\aleph)\cap H(\aleph))\cup(G(\aleph)\cap H(\aleph)) & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

It is seen that N=T under the condition $Z'\cap C\cap R=Z\cap C'\cap R=Z\cap C\cap R'=\emptyset$. It is obvious that the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$ is equivalent to the condition $(Z\Delta C)\cap R=\emptyset$.

2) If $(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\overset{*}{\cup}_{\varepsilon}(G,C)]\overset{*}{\gamma}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\gamma}_{\varepsilon}(H,R)]\overset{*}{\cap}_{\varepsilon}[(G,C)\overset{*}{\gamma}_{\varepsilon}(H,R)]$.

3) If $(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,C)]\overset{*}{\gamma}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\gamma}_{\varepsilon}(H,R)]\overset{*}{\cap}_{\varepsilon}[(G,C)\overset{*}{\gamma}_{\varepsilon}(H,R)]$.

4) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $[(F,Z)\overset{*}{*}_{\varepsilon}(G,C)]\overset{*}{\gamma}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,R)]\overset{*}{\cap}_{\varepsilon}[(G,C)\overset{*}{\cap}_{\varepsilon}(H,R)]$.

Theorem 3.4.4. The following distributions of the complementary extended gamma operation over soft binary piecewise operations hold:

i) LHS Distributions of the Complementary Extended Gamma Operation on Soft Binary Pievewise Operations

1) If $(Z\Delta C)\cap R=\emptyset$, then $(F,Z)\overset{*}{\gamma}_{\varepsilon}[(G,C)\overset{\sim}{\cap}(H,R)]=[(F,Z)\overset{*}{\gamma}_{\varepsilon}(G,C)]\overset{\sim}{\cap}[(F,Z)\overset{*}{\gamma}_{\varepsilon}(H,R)]$.

Proof *:* Consider first the LHS. Let $(G,C)\overset{\sim}{\cap}(H,R)=(M,C)$. Hence for all $\aleph\in C$,

$$M(\aleph)=\begin{cases} G(\aleph), & \aleph\in C-R \\ G(\aleph)\cap H(\aleph), & \aleph\in C\cap R \end{cases}$$

Let $(F,Z)\underset{\gamma_\varepsilon}{*}(M,C)=(N,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ M'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cap M(\aleph), & \aleph\in Z\cap C \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in (C-R)-Z=Z'\cap C\cap R' \\ G'(\aleph)\cup H'(\aleph), & \aleph\in (C\cap R)-Z=Z'\cap C\cap R \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap (C-R)=Z\cap C\cap R' \\ F'(\aleph)\cap (G(\aleph)\cap H(\aleph)), & \aleph\in Z\cap (C\cap R)=Z\cap C\cap R \end{cases}$$

Now consider the rhs, i.e. $[(F,Z)\underset{\gamma_\varepsilon}{*}(G,C)]\tilde{\cap}[(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)]$. Let $(F,Z)\underset{\gamma_\varepsilon}{*}(G,C)=(V,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)=(W,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$W(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Let $(V,Z\cup C)\tilde{\cap}(W,Z\cup R)=(T,(Z\cup C))$, where for all $\aleph\in Z\cup C$,

$$T(\aleph)=\begin{cases} V(\aleph), & \aleph\in (Z\cup C)-(Z\cup R) \\ V(\aleph)\cap W(\aleph), & \aleph\in (Z\cup C)\cap (Z\cup R) \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in (Z-C)-(Z\cup R)=\emptyset \\ G'(\aleph), & \aleph\in (C-Z)-(Z\cup R)=Z'\cap C\cap R' \\ F'(\aleph)\cap G(\aleph), & \aleph\in (Z\cap C)-(Z\cup R)=\emptyset \\ F'(\aleph)\cap F'(\aleph), & \aleph\in (Z-C)\cap (Z-R)=Z\cap C'\cap R' \\ F'(\aleph)\cap H'(\aleph), & \aleph\in (Z-C)\cap (R-Z)=\emptyset \\ F'(\aleph)\cap (F'(\aleph)\cap H(\aleph)), & \aleph\in (Z-C)\cap (Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cap F'(\aleph), & \aleph\in (C-Z)\cap (Z-R)=\emptyset \\ G'(\aleph)\cap H'(\aleph), & \aleph\in (C-Z)\cap (R-Z)=Z'\cap C\cap R \\ G'(\aleph)\cap (F'(\aleph)\cap H(\aleph)), & \aleph\in (C-Z)\cap (Z\cap R)=\emptyset \\ (F'(\aleph)\cap G(\aleph))\cap F'(\aleph), & \aleph\in (Z\cap C)\cap (Z-R)=Z\cap C\cap R' \\ (F'(\aleph)\cap G(\aleph))\cap H'(\aleph), & \aleph\in (Z\cap C)\cap (R-Z)=\emptyset \\ (F'(\aleph)\cap G(\aleph))\cap (F'(\aleph)\cap H(\aleph)), & \aleph\in (Z\cap C)\cap (Z\cap R)=Z\cap C\cap R \end{cases}$$

Therefore,

$$T(\aleph)=\begin{cases} G'(\aleph), & \aleph\in(C-Z)-(Z\cup R)=Z'\cap C\cap R' \\ F'(\aleph), & \aleph\in(Z-C)\cap(Z-R)=Z\cap C'\cap R' \\ F'(\aleph)\cap H(\aleph), & \aleph\in(Z-C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cap H'(\aleph), & \aleph\in(C-Z)\cap(R-Z)=Z'\cap C\cap R \\ F'(\aleph)\cap G(\aleph), & \aleph\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R' \\ (F'(\aleph)\cap G(\aleph))\cap(F'(\aleph)\cap H(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Here, if we consider Z-C in the function N, since Z-C=Z∩C' if an element is in the complement of C, it is either in R-C or (C ∪ R)'. Thus, if $\aleph \in$ Z- C, then $\aleph\in Z\cap C'\cap R$ or $\aleph\in Z\cap C'\cap R'$. Thus, it is seen that N=T under the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$. It is obvious that the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$ is equivalent to the condition $(Z\Delta C)\cap R=\emptyset$.

2) If $(Z\Delta R)\cap C=\emptyset$, then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)\,\tilde{\cup}\,(H,R)]=[(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)]\,\tilde{\cup}\,[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]$.

3) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)\,\tilde{*}\,(H,R)]=[(F,Z)\overset{*}{\theta_\varepsilon}(G,C)]\,\tilde{\cup}\,[(F,Z)\overset{*}{\theta_\varepsilon}(H,R)]$.

4) If $(Z\Delta C)\cap R=Z\cap C'\cap R=\emptyset$, then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)\,\tilde{\theta}\,(H,R)]=[(F,Z)\overset{*}{\theta_\varepsilon}(G,C)]\,\tilde{\cap}\,[(F,Z)\overset{*}{\theta_\varepsilon}(H,R)]$.

ii) RHS Distributions of the Complementary Extended Gamma Operation over Soft Binary Piecewise Operations

1) If $(Z\Delta C)\cap R=Z\cap C'\cap R=\emptyset$, then $[(F,Z)\,\tilde{\theta}\,(G,C)]\overset{*}{\gamma_\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap_\varepsilon}(H,R)]\,\tilde{\cup}\,[(G,C)\overset{*}{\cap_\varepsilon}(H,R)]$.

Proof: Consider first LHS. Let $(F,Z)\,\tilde{\theta}\,(G,C)=(M,Z)$, where for all $\aleph\in Z$,

$$M(\aleph)=\begin{cases} F(\aleph), & \aleph\in Z-C \\ F'(\aleph)\cap G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(M,Z)\overset{*}{\gamma_\varepsilon}(H,R)=(N,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$N(\aleph)=\begin{cases} M'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ M'(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in(Z-C)-R=Z\cap C'\cap R' \\ F(\aleph)\cup G(\aleph), & \aleph\in(Z\cap C)-R=Z\cap C\cap R' \\ H'(\aleph), & \aleph\in R-Z \\ F'(\aleph)\cap H(\aleph), & \aleph\in(Z-C)\cap R=Z\cap C'\cap R \\ (F(\aleph)\cup G(\aleph))\cap H(\aleph), & \aleph\in(Z\cap C)\cap R=Z\cap C\cap R \end{cases}$$

Now consider the RHS, that is, $[(F,Z)\overset{*}{\cap_\varepsilon}(H,R)]\,\tilde{\cup}\,[(G,C)\overset{*}{\cap_\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{\cap_\varepsilon}(H,R)=(V,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ F(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Now let $(G,C)\overset{*}{\cap_\varepsilon}(H,R)=(W,C\cup R)$, where for all $\aleph\in C\cup R$,

$$W(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C-R \\ H'(\aleph), & \aleph\in R-C \\ G(\aleph)\cap H(\aleph), & \aleph\in C\cap R \end{cases}$$

Let $(V,Z\cup R)\ \tilde{\cup}\ (W,C\cup R)=(T,(Z\cup R))$, where for all $\aleph\in Z\cup R$,

$$T(\aleph)=\begin{cases} V(\aleph), & \aleph\in(Z\cup R)-(C\cup R) \\ V(\aleph)\cup W(\aleph), & \aleph\in(Z\cup R)\cap(C\cup R)\end{cases}$$

Thus,,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in(Z-R)-(C\cup R)=Z\cap C'\cap R' \\ H'(\aleph), & \aleph\in(R-Z)-(C\cup R)=\emptyset \\ F(\aleph)\cap H(\aleph), & \aleph\in(Z\cap R)-(C\cup R)=\emptyset \\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ F'(\aleph)\cup H'(\aleph), & \aleph\in(Z-R)\cap(R-C)=\emptyset \\ F'(\aleph)\cup\big(G(\aleph)\cap H(\aleph)\big), & \aleph\in(Z-R)\cap(C\cap R)=\emptyset \\ H'(\aleph)\cup G'(\aleph), & \aleph\in(R-Z)\cap(C-R)=\emptyset \\ H'(\aleph)\cup H'(\aleph), & \aleph\in(R-Z)\cap(R-C)=Z'\cap C'\cap R \\ H'(\aleph)\cup\big(G(\aleph)\cap H(\aleph)\big) & \aleph\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ \big(F(\aleph)\cap H(\aleph)\big)\cup G'(\aleph), & \aleph\in(Z\cap R)\cap(C-R)=\emptyset \\ \big(F(\aleph)\cap H(\aleph)\big)\cup H'(\aleph) & \aleph\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ \big(F'(\aleph)\cap H(\aleph)\big)\cup\big(G(\aleph)\cap H(\aleph)\big), & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Hence

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in(Z-R)-(C\cup R)=Z\cap C'\cap R' \\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ H'(\aleph), & \aleph\in(R-Z)\cap(R-C)=Z'\cap C'\cap R \\ H'(\aleph)\cup G(\aleph), & \aleph\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ F(\aleph)\cup H'(\aleph), & \aleph\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ \big(F'(\aleph)\cap H(\aleph)\big)\cup\big(G'(\aleph)\cap H(\aleph)\big), & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Under the condition $Z'\cap C\cap R=Z\cap C'\cap R=Z\cap C\cap R'=\emptyset$, it can be seen that N=T. It is obvious that the condition $Z'\cap C\cap R= Z\cap C'\cap R=\emptyset$ is equivalent to the condition $(Z\Delta C)\cap R=\emptyset$.

2) If $(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\ \tilde{\cup}\ (G,C)]\ \overset{*}{\gamma_\varepsilon}\ (H,R)=[(F,Z)\ \overset{*}{\gamma_\varepsilon}\ (H,R)]\ \tilde{\cap}\ [(G,C)\ \overset{*}{\gamma_\varepsilon}\ (H,R)]$.

3) If $(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\ \tilde{\cap}\ (G,C)]\ \overset{*}{\gamma_\varepsilon}\ (H,R) =[(F,Z)\ \overset{*}{\gamma_\varepsilon}\ (H,R)]\ \tilde{\cup}\ [(G,C)\ \overset{*}{\gamma_\varepsilon}\ (H,R)]$.

4) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $[(F,Z)\ \tilde{*}\ (G,C)]\ \overset{*}{\gamma_\varepsilon}(H,R) =[(F,Z)\ \overset{*}{\cap_\varepsilon}\ (H,R)]\ \tilde{\cap}\ [(G,C)\ \overset{*}{\cap_\varepsilon}(H,R)]$.

Theorem 3.4.5. The following distributions of the complementary extended gamma operation over the complementary soft binary piecewise operations exist:

i) LHS Distribution of the Complementary Extended Gamma Operation on Complementary Soft Binary Piecewise Operations

1)If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $(F,Z)\ \overset{*}{\gamma_\varepsilon}\ [(G,C)\overset{*}{\underset{\cap}{\sim}}(H,R)]=[(F,Z)\ \overset{*}{\gamma_\varepsilon}(G,C)]\ \overset{*}{\underset{\cap}{\sim}}\ [(F,Z)\ \overset{*}{\gamma_\varepsilon}(H,R)]$.

Proof: Consider first LHS. Let $(G,C)\overset{*}{\underset{\cap}{\sim}}(H,R)=(M,C)$, where for all $\aleph\in C$,

$$M(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C-R \\ G(\aleph)\cap H(\aleph), & \aleph\in C\cap R\end{cases}$$

Let (F,Z) $\underset{\gamma_\varepsilon}{*}$ (M,C)=(N,Z∪C), where for all ℵ∈Z∪C,

$$N(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z-C \\ M'(ℵ), & ℵ\in C-Z \\ F'(ℵ)\cap M(ℵ), & ℵ\in Z\cap C \end{cases}$$

Thus,

$$N(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z-C \\ G(ℵ), & ℵ\in(C-R)-Z=Z'\cap C\cap R' \\ G'(ℵ)\cup H'(ℵ), & ℵ\in(C\cap R)-Z=Z'\cap C\cap R \\ F'(ℵ)\cap G'(ℵ), & ℵ\in Z\cap(C-R)=Z\cap C\cap R' \\ F'(ℵ)\cap(G(ℵ)\cap H(ℵ)), & ℵ\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Now consider RHS, i.e. [(F,Z) $\underset{\gamma_\varepsilon}{*}$ (G,C)]$\underset{\cap}{\overset{*}{\sim}}$[(F,Z) $\underset{\gamma_\varepsilon}{*}$ (H,R)]. Let (F,Z) $\underset{\gamma_\varepsilon}{*}$ (G,C)=(V,Z∪C), where for all ℵ∈Z∪C,

$$V(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z-C \\ G'(ℵ), & ℵ\in C-Z \\ F'(ℵ)\cap G(ℵ), & ℵ\in Z\cap C \end{cases}$$

Let (F,Z) $\underset{\gamma_\varepsilon}{*}$ (H,R)=(W,Z∪R), where for all ℵ∈Z∪R,

$$W(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z-R \\ H'(ℵ), & ℵ\in R-Z \\ F'(ℵ)\cap H(ℵ), & ℵ\in Z\cap R \end{cases}$$

Let (V,Z∪C) $\underset{\cap}{\overset{*}{\sim}}$ (W,Z∪R)=(T,(Z∪C)), where for all ℵ∈Z∪C,

$$T(ℵ)=\begin{cases} V'(ℵ), & ℵ\in(Z\cup C)-(Z\cup R) \\ V(ℵ)\cap W(ℵ), & ℵ\in(Z\cup C)\cap(Z\cup R) \end{cases}$$

Thus,

$$T(ℵ)=\begin{cases} F(ℵ), & ℵ\in(Z-C)-(Z\cup R)=\emptyset \\ G(ℵ), & ℵ\in(C-Z)-(Z\cup R)=Z'\cap C\cap R' \\ F(ℵ)\cup G'(ℵ), & ℵ\in(Z\cap C)-(Z\cup R)=\emptyset \\ F'(ℵ)\cap F'(ℵ), & ℵ\in(Z-C)\cap(Z-R)=Z\cap C'\cap R' \\ F'(ℵ)\cap H'(ℵ), & ℵ\in(Z-C)\cap(R-Z)=\emptyset \\ F'(ℵ)\cap(F'(ℵ)\cap H(ℵ)), & ℵ\in(Z-C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(ℵ)\cap F'(ℵ), & ℵ\in(C-Z)\cap(Z-R)=\emptyset \\ G'(ℵ)\cap H'(ℵ), & ℵ\in(C-Z)\cap(R-Z)=Z'\cap C\cap R \\ G'(ℵ)\cap(F'(ℵ)\cap H(ℵ)), & ℵ\in(C-Z)\cap(Z\cap R)=\emptyset \\ (F'(ℵ)\cap G(ℵ))\cap F'(ℵ), & ℵ\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R' \\ (F'(ℵ)\cap G(ℵ))\cap H'(ℵ), & ℵ\in(Z\cap C)\cap(R-Z)=\emptyset \\ (F'(ℵ)\cap G(ℵ))\cap(F'(ℵ)\cap H(ℵ)), & ℵ\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Hence,

$$T(\aleph)=\begin{cases} G(\aleph), & \aleph\in(C\text{-}Z)\text{-}(Z\cup R)=Z'\cap C\cap R' \\ F'(\aleph), & \aleph\in(Z\text{-}C)\cap(Z\text{-}R)=Z\cap C'\cap R' \\ F'(\aleph)\cap H(\aleph), & \aleph\in(Z\text{-}C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cap H'(\aleph), & \aleph\in(C\text{-}Z)\cap(R\text{-}Z)=Z'\cap C\cap R \\ F'(\aleph)\cap G(\aleph), & \aleph\in(Z\cap C)\cap(Z\text{-}R)=Z\cap C\cap R' \\ (F'(\aleph)\cap G(\aleph))\cap(F'(\aleph)\cap H(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Here, if we consider Z-C in the function N, since Z-C=Z∩C', if an element in the complement of C it is either in R-C or $(C\cup R)'$. Thus, if $\aleph\in Z-C$, then $\aleph\in Z\cap C'\cap R$ or $\aleph\in Z\cap C'\cap R'$. Hence, it is seen that N=T under the condition Z'∩C∩R=Z∩C'∩R=Z∩C∩R'=∅. It is obvious that the condition Z'∩C∩R=Z∩C'∩R=∅ is equivalent to the condition (ZΔC)∩R=∅.

2) If (ZΔR)∩C=∅, then $(F,Z)\underset{\gamma_\varepsilon}{*}[(G,C)\underset{\cup}{\overset{*}{\sim}}(H,R)]=[(F,Z)\underset{\gamma_\varepsilon}{*}(G,C)]\underset{\cup}{\overset{*}{\sim}}[(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)]$.

3)If (ZΔR)∩C= ∅, then $(F,Z)\underset{\gamma_\varepsilon}{*}[(G,C)\underset{\cup}{\overset{*}{\sim}}(H,R)]=[(F,Z)\underset{\theta_\varepsilon}{*}(G,C)]\underset{\cup}{\overset{*}{\sim}}[(F,Z)\underset{\theta_\varepsilon}{*}(H,R)]$.

4)If (ZΔC)∩R=∅, then $(F,Z)\underset{\gamma_\varepsilon}{*}[(G,C)\underset{\theta}{\overset{*}{\sim}}(H,R)]=[(F,Z)\underset{\theta_\varepsilon}{*}(G,C)]\underset{\cap}{\overset{*}{\sim}}[(F,Z)\underset{\theta_\varepsilon}{*}(H,R)]$.

ii) RHS Distributions of Complementary Extended Gamma Operation over Complementary Soft Binary Piecewise Operations

1) If (ZΔC)∩R=Z∩C∩R'=∅, then $[(F,Z)\underset{\theta}{\overset{*}{\sim}}(G,C)]\underset{\gamma_\varepsilon}{*}(H,R)=[(F,Z)\underset{\cap_\varepsilon}{*}(H,R)]\underset{\cup}{\overset{*}{\sim}}[(G,C)\underset{\cap_\varepsilon}{*}(H,R)]$.

Proof: Consider first LHS. Let $(F,Z)\underset{\theta}{\overset{*}{\sim}}(G,C)=(M,Z)$, where for all ℵ∈Z,

$$M(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}C \\ F'(\aleph)\cap G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(M,Z)\underset{\gamma_\varepsilon}{*}(H,R)=(N,Z\cup R)$, where for all ℵ∈Z∪R,

$$N(\aleph)=\begin{cases} M'(\aleph), & \aleph\in Z\text{-}R \\ H'(\aleph), & \aleph\in R\text{-}Z \\ M'(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z\text{-}C)\text{-}R=Z\cap C'\cap R' \\ F(\aleph)\cup G(\aleph), & \aleph\in(Z\cap C)\text{-}R=Z\cap C\cap R' \\ H'(\aleph), & \aleph\in R\text{-}Z \\ F(\aleph)\cap H(\aleph), & \aleph\in(Z\text{-}C)\cap R=Z\cap C'\cap R \\ (F(\aleph)\cup G(\aleph))\cap H(\aleph), & \aleph\in(Z\cap C)\cap R=Z\cap C\cap R \end{cases}$$

Now consider the RHS, that is, $[(F,Z)\underset{\cap_\varepsilon}{*}(H,R)]\underset{\cup}{\overset{*}{\sim}}[(G,C)\underset{\cap_\varepsilon}{*}(H,R)]$. Let $(F,Z)\underset{\cap_\varepsilon}{*}(H,R)=(V,Z\cup R)$, where for all ℵ∈Z∪R,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}R \\ H'(\aleph), & \aleph\in R\text{-}Z \\ F(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Now let $(G,C)\underset{\cap_\varepsilon}{*}(H,R)=(W,C\cup R)$, where for all ℵ∈CUR,

$$W(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C\text{-}R \\ H'(\aleph), & \aleph\in R\text{-}C \\ G(\aleph)\cap H(\aleph), & \aleph\in C\cap R \end{cases}$$

Let $(V,Z\cup R)\overset{*}{\underset{\cup}{\sim}}(W,C\cup R)=(T,(Z\cup R))$, where for all $\aleph\in Z\cup R$,

$$T(\aleph)=\begin{cases} V'(\aleph), & \aleph\in(Z\cup R)\text{-}(C\cup R) \\ V(\aleph)\cup W(\aleph), & \aleph\in(Z\cup R)\cap(C\cup R) \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z\text{-}R)\text{-}(C\cup R)=Z\cap C'\cap R' \\ H(\aleph), & \aleph\in(R\text{-}Z)\text{-}(C\cup R)=\emptyset \\ F'(\aleph)\cup H'(\aleph), & \aleph\in(Z\cap R)\text{-}(C\cup R)=\emptyset \\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z\text{-}R)\cap(C\text{-}R)=Z\cap C\cap R' \\ F'(\aleph)\cup H'(\aleph), & \aleph\in(Z\text{-}R)\cap(R\text{-}C)=\emptyset \\ F'(\aleph)\cup\big(G(\aleph)\cap H(\aleph)\big), & \aleph\in(Z\text{-}R)\cap(C\cap R)=\emptyset \\ H'(\aleph)\cup G'(\aleph), & \aleph\in(R\text{-}Z)\cap(C\text{-}R)=\emptyset \\ H'(\aleph)\cup H'(\aleph), & \aleph\in(R\text{-}Z)\cap(R\text{-}C)=Z'\cap C'\cap R \\ H'(\aleph)\cup\big(G(\aleph)\cap H(\aleph)\big) & \aleph\in(R\text{-}Z)\cap(C\cap R)=Z'\cap C\cap R \\ \big(F(\aleph)\cap H(\aleph)\big)\cup G'(\aleph), & \aleph\in(Z\cap R)\cap(C\text{-}R)=\emptyset \\ \big(F(\aleph)\cap H(\aleph)\big)\cup H'(\aleph) & \aleph\in(Z\cap R)\cap(R\text{-}C)=Z\cap C'\cap R \\ \big(F(\aleph)\cap H(\aleph)\big)\cup\big(G(\aleph)\cap H(\aleph)\big), & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Therefore,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z\text{-}R)\text{-}(C\cup R)=Z\cap C'\cap R' \\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z\text{-}R)\cap(C\text{-}R)=Z\cap C\cap R' \\ H'(\aleph), & \aleph\in(R\text{-}Z)\cap(R\text{-}C)=Z'\cap C'\cap R \\ H'(\aleph)\cup G(\aleph), & \aleph\in(R\text{-}Z)\cap(C\cap R)=Z'\cap C\cap R \\ F(\aleph)\cup H'(\aleph), & \aleph\in(Z\cap R)\cap(R\text{-}C)=Z\cap C'\cap R \\ \big(F(\aleph)\cap H(\aleph)\big)\cup\big(G(\aleph)\cap H(\aleph)\big), & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Under the condition $Z'\cap C\cap R=Z\cap C'\cap R=Z\cap C\cap R'=\emptyset$, N=T is satisfied. It is obvious that the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$ is equivalent to $(Z\Delta C)\cap R=\emptyset$.

2) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $[(F,Z)\overset{*}{\underset{\cup}{\sim}}(G,C)]\underset{\gamma_\varepsilon}{*}(H,R)=[(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)]\overset{*}{\underset{\cap}{\sim}}[(G,C)\underset{\gamma_\varepsilon}{*}(H,R)]$.

3)I

f $(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\overset{*}{\underset{\cap}{\sim}}(G,C)]\underset{\gamma_\varepsilon}{*}(H,R)=[(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)]\overset{*}{\underset{\cup}{\sim}}[(G,C)\underset{\gamma_\varepsilon}{*}(H,R)]$.

4) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $[(F,Z)\overset{*}{\underset{*}{\sim}}(G,C)]\underset{\gamma_\varepsilon}{*}(H,R)=[(F,Z)\underset{\cap_\varepsilon}{*}(H,R)]\overset{*}{\underset{\cap}{\sim}}[(G,C)\underset{\cap_\varepsilon}{*}(H,R)]$.

4. CONCLUSION

Soft set operations play a central role in soft set theory, offering a soft structure for addressing uncertainty in data analysis and decision-making. This study investigates the algebraic features of a new soft set operation called complementary extended gamma operation. We also study the distribution of complementary extended gamma over several more soft set operations. Our hope is that this work will provide a foundation for further research on soft set operations. In order to determine what algebraic structures emerge in the collection of soft sets together with complementary extended gamma operations of soft sets, more research may look at different types of complementary extended soft set operations, as well as their distributions and properties.

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Investigation of Groove Width Created by Fiber Laser on ST52 Steel

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Abstract

This paper has presented the results of a study aiming to identify the effects of laser power and two other parameters on groove formation in ST52 steel using a fiber laser and to optimize these parameters so that the groove aspect ratio could be maximized. The Taguchi method has been used to explore the effects of three parameters, namely, laser power, scan speed, and frequency, on the laser grooving characteristics. The analysis has shown that the first parameter has a strong impact of about 69.95 %, while the other two have led to about 15.73 % and 14.31 %, respectively. Each scenario included three arrangements of factors and levels that corresponded to the L9 orthogonal array. Subsequently, a total of nine experiments were conducted, with an extensive variation in the formed grooves observed. The obtained results have shown that the groove's deepness and grooves' width vary substantially when the laser step settings through the developed quantitative range. The subsequent achievement in optimal response elaboration has the following ratio: 100 W laser power, 100 mm/s scan speed, and 20 kHz is a frequency. Moreover, the described ration corresponds to values derived through regression analysis. For that reason, the performed research provides a valuable contribution to furthering knowledge in laser-material interaction during the texturing procedure.

Keywords: Surface texturing, Fiber Laser machining, ST52 Steel, Taguchi Optimization, Surface Groove.

1. INTRODUCTION

Because of its excellent weldability and strong mechanical characteristics, ST52 steel is a low alloy structural steel with high strength that is widely used in a variety of industrial applications (Roodgari et al., 2020). It is covered by DIN 17100, which lists general structural steels suitable for use in engineering and construction (Alloni et al., 1976). With a characteristic yield strength of 355 MPa, the designation "ST52" indicates that it is appropriate for use on structural components that are subjected to heavy loads (Langseth et al., 1991)) Carbon (C), manganese (Mn), silicon (Si), and trace amounts of other alloying elements like phosphorus (P) and sulfur (S) make up the majority of this steel grade (Shahverdi1 and Ravari1, 2021). Its ideal mix of toughness, strength, and machinability is a result of the regulated composition and thermal treatment techniques used during production. The main constituents of ST52 steel are a carefully chosen set of essential alloying elements, each of which is essential to the determination of the steel's mechanical qualities and general performance attributes. ST52 steel is a favored material choice for structural applications needing high strength, durability, and dependability because of its remarkable mechanical qualities, weldability, and corrosion resistance, all of which are a result of the careful selection and management of alloying elements in the steel. Excellent weldability, which allows it to be used in fabrication operations like welding, bending, and forming without sacrificing its structural integrity, is one of ST52 steel's distinguishing qualities. This feature increases its adaptability and simplicity of integration into various structural configurations, from construction frames to heavy gear and equipment. Additionally, ST52 steel has an excellent resistance to corrosion, making it appropriate for applications exposed to corrosive chemicals or harsh environmental conditions (Iroha et al., 2022). However, extra protective coatings can be applied to extend its longevity in corrosive situations. Because of its high yield and tensile strengths, ST52 steel is a good choice for load-bearing structures including bridges, building frameworks, and industrial machinery parts that are subject to both static and dynamic loads (Selamet et al., 2023). For engineered systems where safety and performance are top priorities, its resistance to stress and deformation throughout a range of operating circumstances guarantees their lifespan and dependability. Furthermore, the comparatively low carbon content of ST52 steel makes it easier to machine and process, which allows for more economical manufacturing procedures and shorter lead times for production (Valuev et al., 2014). For structural applications requiring high strength, exceptional weldability, and corrosion resistance, ST52 steel offers a flexible and dependable material option (Ramezani el al., 2018). Because of its well-balanced mechanical qualities and ease of manufacture, it is a desirable material in many different industries, which advances the development of infrastructure and engineering solutions.

ST52 steel is widely used in many different industrial sectors because of its excellent mechanical qualities, weldability, and resistance to corrosion. Because of its high tensile strength, excellent ductility, and toughness, it can be used in a variety of applications where structural integrity and dependability are critical factors. Building and engineering are two notable industries that use ST52 steel extensively. It is a basic material used in the construction of buildings, bridges, and infrastructure projects' structural components. Because of its strength and resilience, ST52 steel is a good material to use when sustaining large loads and enduring dynamic forces. This helps to maintain the structural integrity and long-term functionality of engineered systems. ST52 steel is essential to the automotive industry's production of several parts, such as engine parts, suspension systems, and chassis frames (Okuroğullari et al., 2022). Its excellent formability and weldability, along with its high strength-to-weight ratio, make it the perfect material to use when creating lightweight but structurally sound vehicle chassis (Haghshenas and Gerlich, 2018). Furthermore, automotive components' longevity and service life are improved by ST52 steel's resistance to corrosion, especially in areas with extreme climatic conditions. The production of gear and equipment for industrial operations is another use for ST52 steel. Its superior machinability makes it easier to produce precisely engineered parts for machinery used in a variety of industries, including manufacturing, agriculture, and mining, such as gears, shafts, and structural frames. Because of ST52 steel's exceptional mechanical qualities, heavy-duty equipment can operate effectively in harsh environments, guaranteeing peak efficiency and production. ST52 steel is used in the energy industry to build pressure vessels, storage tanks, and pipelines for the transportation and storage of different gases and liquids (Boydak et al., 2010, Gloc et al., 2016). It is ideally suited to endure the high pressures and corrosive conditions seen in oil and gas exploration, refining, and distribution activities due to its high strength and resistance to corrosion. Furthermore, ST52 steel's weldability makes it easier to construct intricate pipeline systems and infrastructure, which cuts down on building time and expense. Moreover, ST52 steel is used to make harvesting machinery, tractors, and plows, among other agricultural equipment (Nazemosadat et al., 2022). These pieces depend on the steel's strength, resilience, and adaptability to endure the demanding conditions of farming. Because of its resilience to fatigue and wear, agricultural machinery is guaranteed to be dependable and long-lasting, which boosts output and efficiency in food production.

The term "surface treatment" refers to a broad range of procedures and methods used to alter a material's surface features while essentially maintaining its bulk qualities (Silva et al., 2011). These machining techniques are painstakingly customized to fulfill distinct functional goals and performance standards in a range of industrial domains. Mechanical procedures like grinding, polishing, and shot peening are examples of surface treatment techniques that modify the topography and texture of the surface to improve characteristics like fatigue strength and wear resistance (Hashmi et al., 2023). For the purpose of improving surface adhesion and imparting corrosion resistance for later coating applications, chemical treatments like chromate conversion coating, passivation, and anodizing are applied. In order to improve wear resistance and fatigue life, surface hardness, toughness, and metallurgical structure are modified using thermal treatments like heat treatment and surface hardening methods like carburizing and nitriding. Furthermore, surface coatings including paints, platings, and thin films are used to improve surface lubricity, offer barriers against environmental deterioration, or modify surface characteristics for particular functional needs. All things considered, one of the core components of materials engineering is surface treatment, which allows surface qualities to be optimized to satisfy the various performance requirements of contemporary industrial applications.

The term "laser material processing" describes an adaptable and accurate manufacturing process that modifies or transforms the properties of materials using concentrated laser beams. This technique is used in many different industries, including the automotive, aerospace, electronics, and medical device sectors. Its applications include cutting, welding, marking, engraving, and surface modification. When it comes to production, laser material processing is advantageous in a number of ways. Its remarkable accuracy and precision are one of its main benefits, as it allows for the creation of complex geometries and minute details with little waste of material. Because laser processing is non-contact, it reduces mechanical stress and distortion in the workpiece, producing precise dimensional tolerances and high-quality finished products. Furthermore, the power, intensity, and duration of lasers may be accurately adjusted. This feature makes it possible to precisely input heat and deposit concentrated energy, which is very useful for fragile components and heat-sensitive materials. Moreover, laser processing offers versatility in manufacturing processes because it is extremely adaptable and can be used on a variety of materials, such as metals, plastics, ceramics, and composites. Additionally, the quick processing speeds of laser material processing make it ideal for just-in-time manufacturing applications and big volume production. The widespread acceptance and improvement of laser material processing in modern industrial industries can be attributed to its superior qualities, which include precision, adaptability, speed, and minimal thermal effects.

This study investigated the effects of different laser parameters on the texturing process of 420 stainless steel using a Nd:YVO4 fiber laser. Using eighty various combinations of processing parameters (power, scanning speed, and number of passes), the researchers produced 400 textures. According to their research, medium to high laser power values, medium scanning rates, a high number of passes, and larger line spacings were usually needed to produce textures of a high caliber. In particular, scanning speeds between 500 and 2000 mm/s, eight passes or more, and line spacings between 40 and 50 µm were used to develop the most promising textures with laser power levels of 16%, 64%, and 100% (Cunha et al., 2022). The main focus of the research is on the complex dynamics of laser-material interaction, with a focus on absorbed energy density, pulse frequency, scan rate, and overlap coefficients. These factors all have different effects on the parameters of laser surface texturing (LST). Interestingly, the study yields different results depending on these factors. The roughness corresponding to a scan overlap coefficient of -14.3%, for example, is roughly 1.18 times more than that found with a scan overlap coefficient of 28.7%, and about 1.23 times greater than that found with a scan overlap coefficient of 71.4%. Furthermore, the study emphasizes the impact of pulse frequency, showing that the roughness is approximately 1.9 times lower at 50 kHz and around 2.3 times lower at 100 kHz (Lazov et al., 2023). AISI 430 stainless steel was used for surface texturing tests, producing groove-type sections with different depths and recasting material ejected on the hollow's edge. Two ellipses at a 90-degree angle (type B), three concentric octagonal donut patterns (type A), and an assortment of dimples, holes, and craters (type C) were the three design variations that were investigated. It was found that the processing speed and the roughness of the textured surface showed inverse proportionality, with the roughness exhibiting direct proportionality to the spot density (number of repeats). When there was a large amount of recast material present, as was the case in design A with a higher degree of overlapping, rougher surfaces were attained. On the other hand, designs with less overlap and recast material showed less roughness. One example of this is design type C, which has a high crevice depth. Surface roughness values for design B, which exhibits an intermediate degree of overlap, fall between those of designs A and C (Moldovan et al., 2022).

Lately, laser processing has captured much attention due to its high level of accuracy, adaptability, and productivity in fabricating materials. Different research works have addressed the role of laser parameters on surface morphology and material properties. Smith and Johnson (2023) studied how changes in different fiber laser parameters affected the surface morphology of steel alloys with an emphasis on the influence of laser settings on target oriented surface characteristics (Smith & Johnson, 2023). In a similar manner, Liu and Wang (2023) used Response Surface Methodology (RSM) to optimize laser machining parameters and demonstrated the effectiveness of this approach in enhancing process outcomes (Liu & Wang, 2023). Moreover, Thompson and Garcia (2024) used Barkhausen Noise Analysis for assessing steel alloys' surfaces after being subjected to laser treatment leading to valuable information regarding alteration in material properties caused by lasers' effects on them during manufacturing processes (Thompson & Garcia, 2024). Brown and Patel also worked out such things like predicting polymers' erosion behaviors using metaheuristic algorithms which is indicative of newly developing computational techniques employed by scientists investigating materials processing nowadays (Brown & Patel, 2024). Consequently these recent advancements necessitate optimization of laser parameters for controlling material performance.

Materials-based grooves are used in a wide range of industrial industries, where they play a critical role in improving system performance, functionality, and efficiency. One well-known industry that makes substantial use of grooves is manufacturing and mechanical engineering. Grooves are frequently used in machining operations to improve surface finish quality and machining efficiency by managing coolant flow and chip evacuation. Additionally, grooves are essential for sealing applications like pneumatic and hydraulic systems, where they help maintain system integrity by preventing fluid leaks. Tire treads with grooves are used in the automobile industry to increase traction and grip, which improves vehicle stability and safety on various types of roads. Furthermore, in the building industry, grooves are used for jointing and sealing, which speeds up the assembly of structural elements and improves their longevity. Additionally, grooves are employed in drug delivery systems, fluid manipulation, and cell culture in microfluidics and biomedical devices. In conclusion, material grooves have a multitude of uses in a variety of industrial contexts, enhancing usefulness, performance, and dependability in a broad range of applications. In this study, grooves were created using different laser parameters on ST52 steel, which is preferred in many fields in industry. The first aim of the study is to examine the effect of the fiber laser parameters used on the groove geometry and the width of the Heat Affected Zone formed around the groove. The second aim of the study is to determine to what extent the laser parameters used affect the end result.

Taguchi method was used in the optimization study of the process parameters of the fiber laser used for these two purposes. The Taguchi method also saved time and material usage.

2. MATERIAL AND METHODS

2.1 Optimization Method

The Taguchi method is a popular optimization technique that yields results in the domains of social and experimental design. This method was created by Dr. Genichi Taguchi with the goal of enhancing quality in the process of designing products and processes. Making experimental plans, carrying out experiments, and evaluating the outcomes are the steps that make up this process. Ensuring minimal variability can lead to improved process and product quality, which is one of the fundamental tenets of this methodology. In light of this, Taguchi's approach makes use of a technique known as "multiple variance analysis". This technique is used to identify the effects and interactions of factors that affect the quality of a process or product. Taguchi method can be used in continuous improvement and quality control processes to improve product quality, reduce costs and ensure customer satisfaction. Therefore, Taguchi methodology is considered an important tool in industrial and manufacturing fields and has a wide range of applications.
The characteristics "larger the better," "smaller the better," and "nominal is the best" are relevant to the Taguchi Method when it comes to response optimization in experimental design. The Taguchi philosophy of robust design, which strives to reduce unpredictability and improve quality in processes and products, is centered around these ideas. When it comes to response variables, such maximal strength or yield, greater values denote superior performance or quality. This is why the phrase "larger the better" is used. In these situations, the goal is to maximize the response in order to attain the intended result. In response variables, on the other hand, the characteristics "smaller the better" is applicable, as smaller values signify superior performance or quality, such as lowering variability or defect rates. Reducing the reaction is the aim here in order to enhance quality. Last but not least, the characteristics "nominal is the best" applies to response variables when a target value or nominal level is thought to be ideal, such as reaching a certain target value or dimension. In these cases, keeping the answer near the nominal value while reducing deviations and guaranteeing consistency is the goal. Practitioners of the Taguchi Method can efficiently optimize processes and goods to achieve quality targets and improve overall performance by comprehending and putting these concepts into practice in experimental design.

1) The higher is better.
2) The lower is the better
3) The nominal value (target value) is the best.

The measurement results for every trial are represented by the value in these computations. To reduce the error rate, measurements were made at each of the three different region on the groove. The symbol "m" in Eqn. 3 denotes the nominal value or intended value. Since the objectives of the study were to obtain a groove with the largest aspect ratio, the largest S/N ratio was aimed to obtain.

2.2 Material and Experimental

In this study, grooves were formed on a 3 mm thick ST52 steel plate using the fiber laser parameters given in Table 1. According to Taguchi method, L_9 orthogonal array was used for 3 parameters and three different levels of these three parameters. The fiber laser settings that were used to create the proper shaped dimples on ST52 Steel plates are examined. Using the Taguchi approach, the effects of different fiber laser parameters on the width of the generated dimples were investigated. This study looked into three parameters that are listed in Table 1 and the three levels that correspond to them.

Table 1. Fiber laser parameters and their levels.

	Laser Power (Watt)	Scan Speed (mm/s)	Laser Frequency (kHz)
1st level	20	1	20
2nd level	60	50	100
3th level	100	100	200

The laser device used in the study is a fiber laser with a wavelength of 1064 nm operating in pulse mode. The maximum power of the fiber laser is 100 W and the average power value is obtained by changing the duty cycle of the PWM signal between 10% and 100%. In addition, the frequency of the fiber laser can be changed between 20-200 kHz and can reach a speed of 9000 mm/s. The laser processing process was carried out in an air environment.

RESULTS AND DISCUSSION

A 20–100 watts laser power range was examined. The lowest power value was found to be 20 W as, in the first investigations, no groove growth was seen on the steel material at laser power values lower than 20 W. Similar to the previous point, no groove creation was seen at speeds higher than 100 mm/s, so that was the maximum laser scan speed limit set. More heat deformation was seen on the steel material as a result of excessive heat transfer when the laser power was more than 100 W and the laser scan speed was 1 mm/s.

When three parameters with three levels are investigated, 3^3 (=27) experiments are needed according to traditional experimental design. Fewer experiments are needed to produce efficient findings when optimization techniques are used [5]. The Taguchi method, which yields good results in many scientific and engineering domains, was applied in this investigation. The Taguchi approach states that choosing the right orthogonal array should come first in order to minimize the number of experiments [6]. The Taguchi approach states that choosing the right orthogonal array should come first in order to minimize the number of experiments. L_9 orthogonal array was utilized in this investigation because 3 parameters and 3 levels were employed [7]. Experiment sets were constructed, and the laser settings listed in Table 1 were arranged in accordance with the L_9 orthogonal array. Table 2 included the experiment setups.

Table 2. Experiment sets generated using the Taguchi method and grouped in accordance with the L_9 orthogonal array.

	Scan Speed (mm/s)	Laser Power (Watt)	Laser Frequency (kHz)
1	20	1	20
2	20	50	100
3	20	100	200
4	60	1	100
5	60	50	200
6	60	100	20

7	100	1	200
8	100	50	20
9	100	100	100

Table 2 provides experimental sets created using the Taguchi approach. Fig. 1 displays 3D topographic views of the surfaces that were produced by the experiments with the settings in the experimental sets. A laser non-contact optical profilometer was used to obtain all of the images that depict the material surface's detailed topography (Nanovea PS50, USA). Three distinct sections on the groove were selected for measurement in order to lower the experiment's error rate and errors resulting from potential surface inhomogeneity. The utilized laser beam has a diameter of 50 µm.

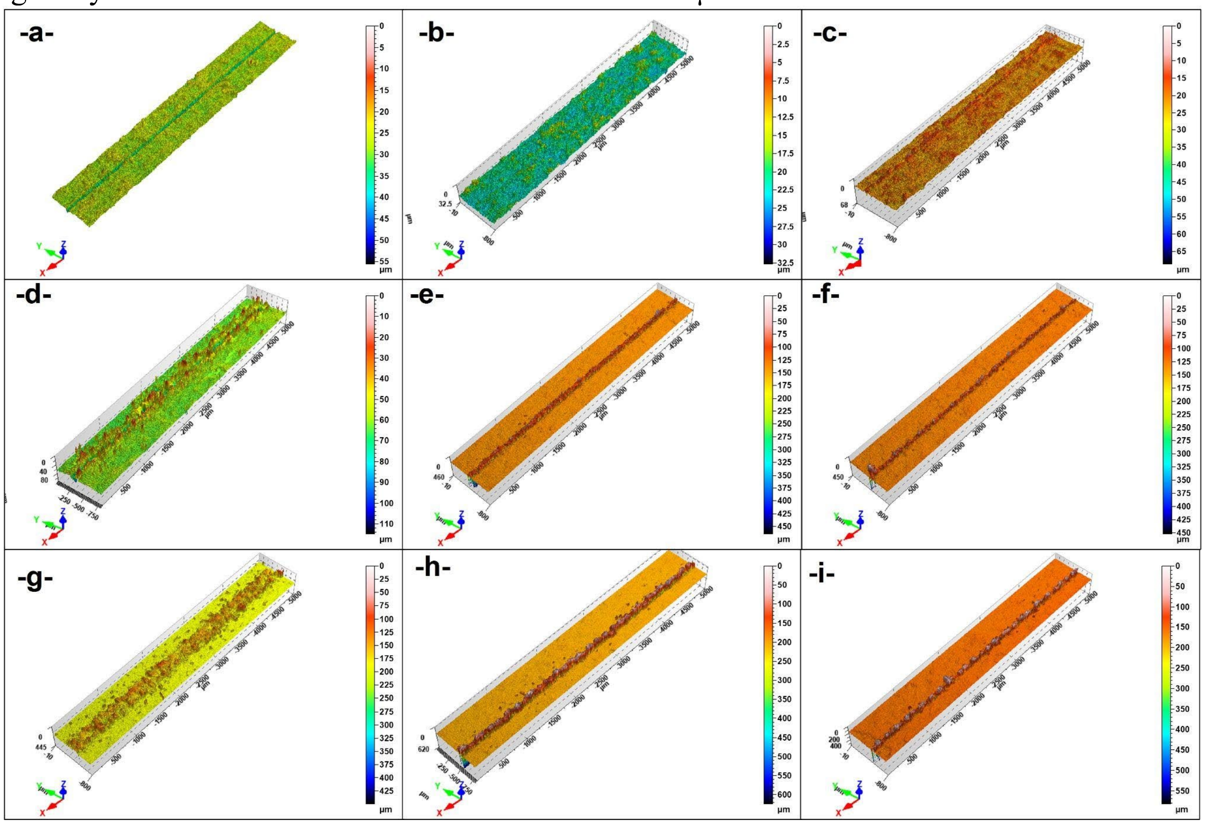

Fig. 1. The grooves that were created using the Taguchi Method from the experiment sets. The experiment sets are: (a) the first set; (b) the second; (c) the third; (d) the fourth; (e) the fifth; (f) the sixth; (g) the seventh; (h) the eighth; and (i) the ninth.

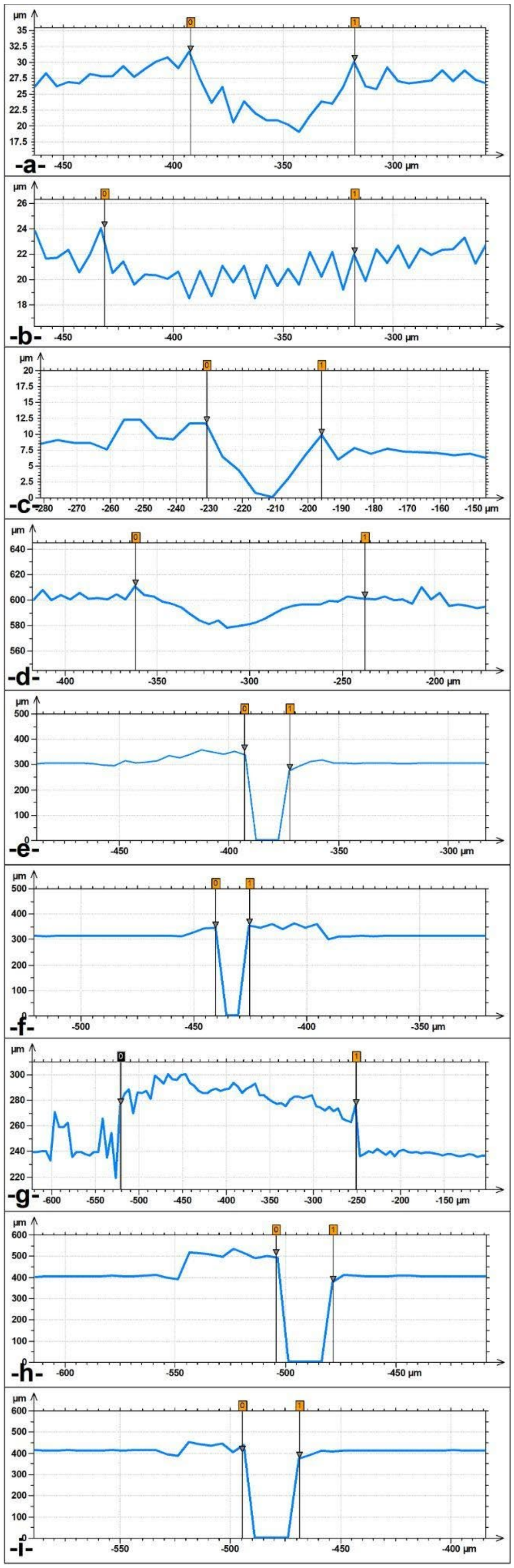

Fig. 2_a. The grooves cross-section width and depth geometry from the experiment sets. The experiment sets are: (a) the first set; (b) the second; (c) the third; (d) the fourth; (e) the fifth; (f) the sixth; (g) the seventh; (h) the eighth; and (i) the ninth.

There are nine sets of experiments, where different grooves were obtained by using various laser machining parameters. Each graph depicts one individual set of experiments, with the depth of the grooves on the Y-axis and their placements on the X-axis. The first set with an average groove depth of 10.65 µm and a width of 74.77 µm has slight variations of the depth. In the second set, the average depth amounted to 4.75 µm, which is relatively low, with the width of 113.86 µm, which is wider compared to others. It means that scan speeds should be higher, or power settings should decrease in this case. The third set with an average depth of 9.83 µm and a width of 34.80 µm has been produced using scan speeds that were lower and power settings that were higher. The fourth set has a groove that is the deepest and the widest, where the average depth is 24.67 µm, and the width is 124.30 µm. Sets 5 and 6 have the deepest grooves, of 309.49 m and 315.23 m, respectively. However, both grooves have very narrow widths, probably because of high power and very slow scan speed. The seventh set reports an average width of 270.27 µm and a height of 60.00 µm. The geometry that emerged with these parameters emerged as an elevation instead of a groove. Lastly, the last two sets, i.e., Set 8 and Set 9, have the deepest grooves, 508.01 µm, and 424.00 µm in-depth, respectively. Such grooves have a width of about 25 µm and demonstrate a highly concentrated energy application. The above findings have shown the drastic impact of varying laser parameters on both the material removal rate and groove dimensions. Therefore, minimizing human factors ensures greater control in micro-machining.

All of the rendered images represent optical microscope views of surfaces undergoing laser treatment at 50x magnification. Three series of images, namely -a-, -d-, and -g-, correspond to the lowest laser power used at different scan speeds and frequencies. Low-energy interactions with the surface result in small, almost unnoticeable surface modifications, and these images depict such processes. When images -b-, -e-, and -h-, captured using the same scan speed and varied frequency settings, are considered, the complexity of the surface structure increases. These two series of images are characterized by more complex and transformed surfaces, which suggest enhanced energy absorption and elaborated interaction with the material. We used the highest power settings under varying scan speeds, and images -c-, -f-, and -i- represent the generated results. These images are associated with the surfaces' extensive morphological and textural changes. The images -f- and -i- exhibit significant melting and re-solidification of the material, which are typical for high-energy laser processing. The -c- image is the least transformed among high-power samples and is most similar to the lowest-energy images. The scan speed of the high-energy laser was the shortest, which means that the duration for energy deposition was insufficient for significant texture alteration. As a result, this variety of images indicates that laser power is the determining factor in surface modification extent and type, and speed and frequency further define the nature of these changes.

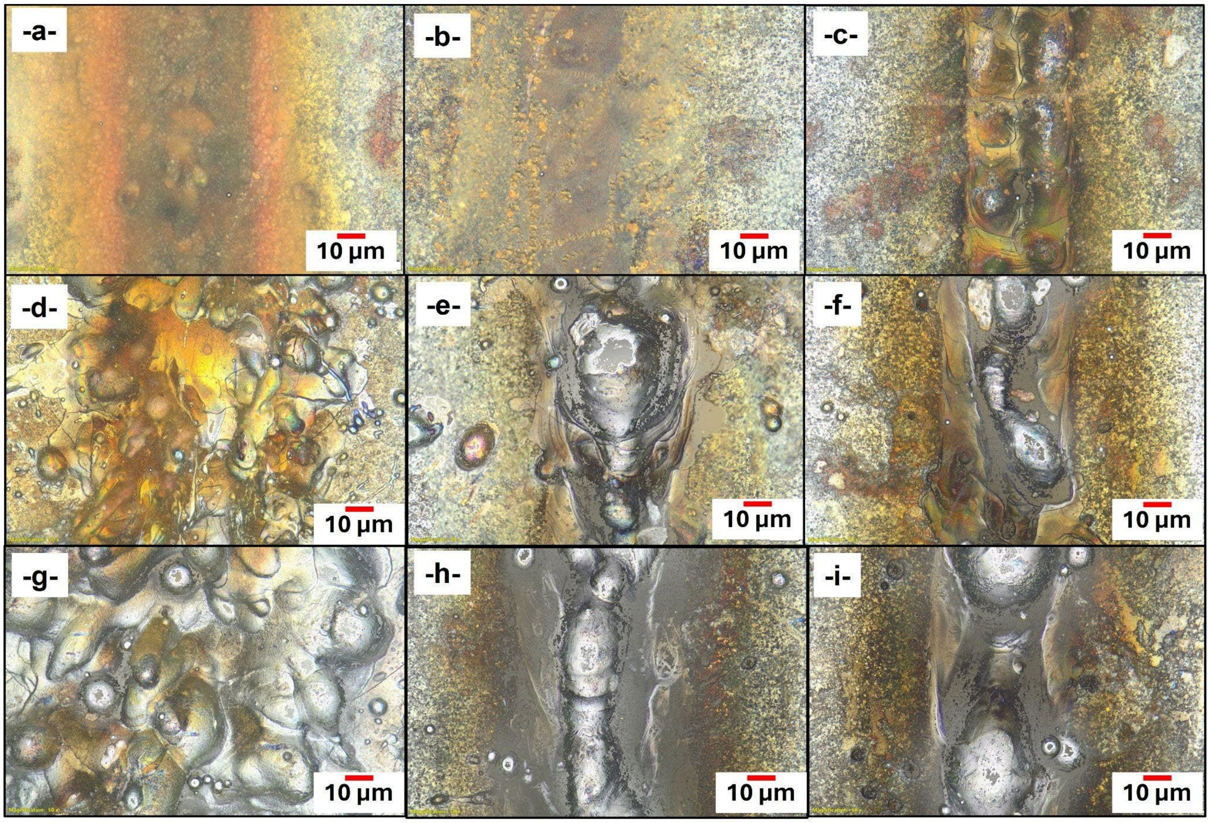

Fig. 3. The optic microscope pictures of laser grooves (a) the first set; (b) the second; (c) the third; (d) the fourth; (e) the fifth; (f) the sixth; (g) the seventh; (h) the eighth; and (i) the ninth.

In Taguchi optimization calculations, the "larger the better" feature was employed to generate a groove with the highest depth/width ratio. A groove with a big aspect ratio will work best for this purpose. Equation 1's "larger the better" characteristic was used to determine S/N for this reason. Table 3 presents the measurement outcomes and computed S/N values.

Table 3. Average values of groove depth, width and aspect ratios measured using the results obtained with the experimental sets. S/N values calculated according to the Taguchi method.

Exp. Set No	Average Depth of groove (µm)	Average Width of groove (µm)	Aspect Ratio (D/W)	Signal to Noise Ratio (S/N)
1	10.65	74.77	0.142	-16,93
2	4.75	113.86	0.042	-27,59
3	9.83	34.80	0.283	-10,98
4	24.67	124.30	0.198	-14,05
5	309.49	20.74	14.924	23,48
6	315.23	14.96.	21.070	26,47
7	270.27	-60.00	-4.505	13,07
8	508.01	25.82	19.672	25,88
9	424.00	26.14	16.220	24,20

In order to optimize the laser parameters that should be used to obtain the largest aspect ratio and to calculate which parameter affects the result and to what extent, the ANOVA table given in Table 4 was prepared. The calculated S/N values of the measurement results given in Table 2 were used to obtain the ANOVA table.

Table 4. ANOVA table prepared using the Taguchi technique. A- Degree of freedom, B- Sum of squares (SSi), C- Average of sum of squares (variance), D- Effect of factors (%), E- Optimum Levels, F- Optimum Values (calculated).

		Average S/N								
Factors	Unit	1st level	2nd level	3rd level	A	B	C	D	E	F
Power	mm/s	-18.50	11.97	21.05	2	3433.45	858.36	69.95	3	100 W
Speed	W	-5.97	7.25	13.23	2	772.28	193.07	15.73	3	100 mm/s
Frequency	Hz	11.81	-5.81	8.52	2	702.38	175.60	14.31	1	20 kHz
	Average		4.84							
	Total					4908.11		100.00		
	Optimum S/N									36.41
	Aspect Ratio of Expected Groove									66.15

As can be seen in the ANOVA table given in Table 4, Sum of squares of S/N values (SSi) and variance of these values were calculated by using the averages of S/N values of each level. As can be seen from the table, according to the Taguchi method, the Aspect ratio value of the groove to be obtained in the experiment to be carried out using laser power 100W, scan speed 100 mm/s and laser frequency 20 kHz parameter values is calculated as 66.15. Again, as can be seen from the ANOVA table, the parameter that affects the Aspect ratio the most is calculated as laser power by 69.95%. Secondly, laser scan speed affects the result by 15.73%. The parameter that affected the result the least with a very small difference was laser frequency with 14.31%.

Main Effect Plots

A vital tool in statistical analysis, main effect plots are especially useful when discussing experimental design and analysis of variance (ANOVA). When all other factors are held constant, these plots offer a visual depiction of the average response of a dependent variable across various values of a single independent variable. Main effect plots are primarily used to investigate and depict the link between the independent and dependent variables, which facilitates the interpretation of experimental data. The capacity of main effect plots to identify the existence and strength of main effects—that is, the influences of distinct independent variables on the dependent variable—is one of their fundamental advantages. Main effect plots let researchers evaluate the overall effect of an independent variable on a dependent variable, independent of the levels of other variables, by charting the mean response for each level of the independent variable. This makes it easier to spot trends, patterns, and variations in the dependent variable at various levels of the independent variable. A useful diagnostic tool for identifying possible interactions between independent variables is the main effect plot. When the amount of one independent variable affects the dependent variable in a different way, this is known as an interaction. Researchers can get insight into the intricate dynamics of the system they are studying by analyzing main effect plots in conjunction with interaction plots to determine whether the connection between the independent and dependent variables varies across levels of other independent variables. Main effect plots are essential for model validation and selection in addition to helping to evaluate experimental results. Researchers can determine whether the linearity and homoscedasticity assumptions that underpin the statistical model are met by visually examining the main effect plots. Additionally, major effect plots can help choose the right model modifications or transformations to better represent the underlying data structure, enhancing the precision and dependability of the statistical analysis.

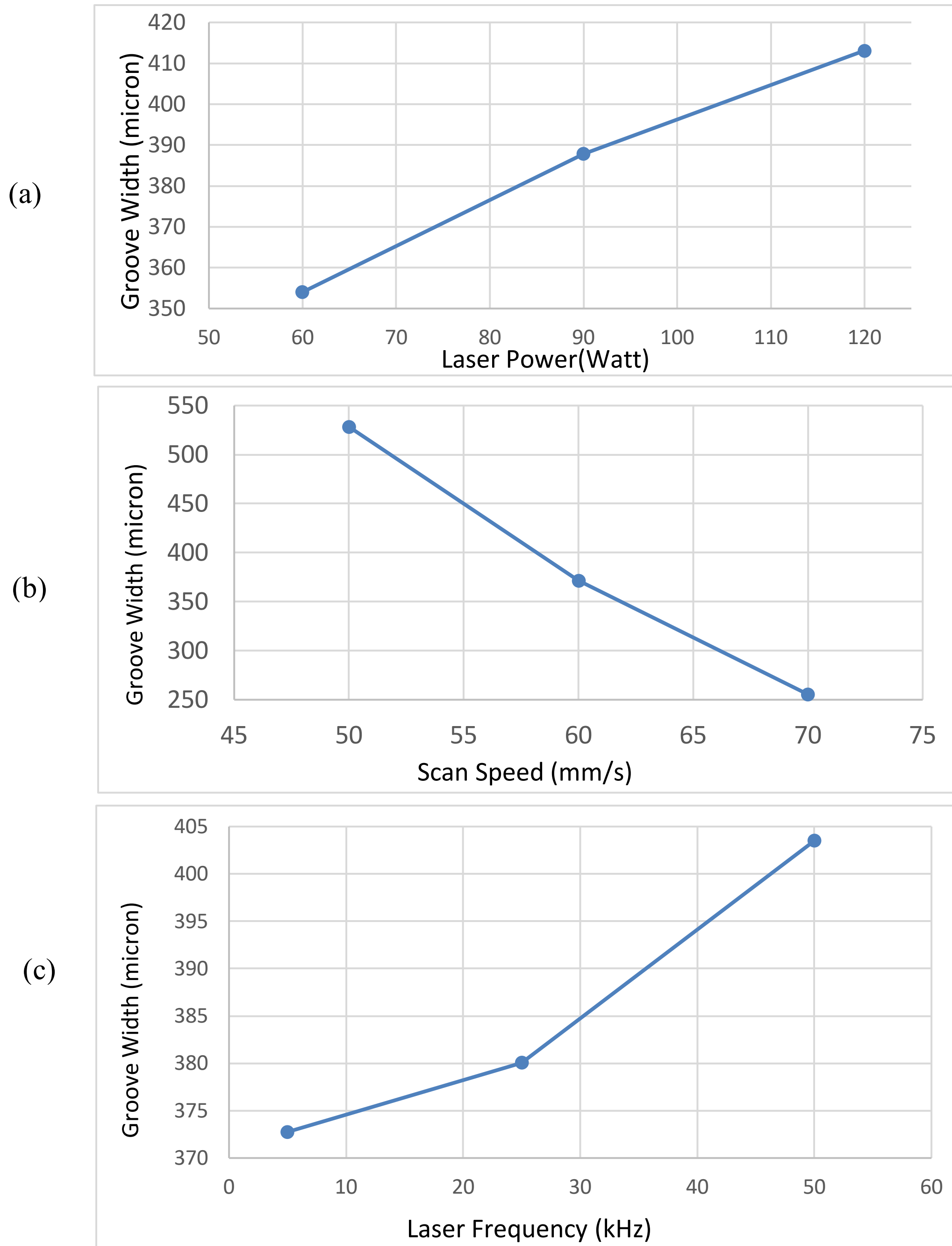

Fig. 4. Main effect plots for; (a) Laser Power, (b) Laser Scan Speed and (c) Laser Frequency.

As can be seen from the main effect plots in Fig. 4, laser power and laser scan speed showed similar behavior. When the laser power was increased from the lowest value of 20 W to the middle value of 60 W, the aspect ratio increased rapidly and reached its maximum value. When the laser power was increased from 60 W to 100 W, a "very slight decrease" was observed in the aspect ratio value, which can be interpreted as "no change". Similarly, when the laser scan speed was increased from its smallest value of 1 mm/s to 50 mm/s, the aspect ratio increased rapidly. When the laser scan speed was increased from 50 mm/s to 100 mm/s, a "very slight increase" in the aspect ratio value was observed, which can be interpreted as "no change". As can be seen from the graph explaining the interaction of aspect ratio with laser frequency, unlike the other two parameters, the aspect ratio decreased rapidly when the laser frequency was increased from its smallest value of 20 kHz to 100 kHz. When the frequency was increased from 100 kHz to 200 kHz, the rate of decrease of the Aspect ratio decreased.

Regression Equation

Mathematical models known as regression equations explain the connection between one or more independent variables and a dependent variable. Depending on the type of relationship that exists between the variables, regression equations can be either linear or nonlinear. Regression equations are primarily used to model and forecast dependent variable values based on independent variable values. Researchers can estimate the regression coefficients that most accurately reflect the relationship between the variables by fitting the regression model to the observed data. Upon estimating the model, predictions for the dependent variable can be made for novel or unknown values of the independent variables. In domains like economics, finance, marketing, and social sciences, where forecasting and comprehending the correlations between variables is crucial for making decisions, this predictive skill is very useful. Regression equations also measure the strength and direction of the associations between variables, which sheds light on the underlying structure and dynamics of the data. Researchers can evaluate the effect of individual independent variables on the dependent variable, find significant predictors, and find patterns and trends in the data by looking at the regression coefficients' magnitude and significance. As a result, theories can be tested, conclusions about the population from which the data were sampled can be drawn, and new insights that guide theory development and empirical study are produced. Regression equations are frequently used for hypothesis testing, model comparison, and causal inference in addition to prediction and inference. Through the statistical significance assessment of the regression coefficients and diagnostic tests, researchers are able to determine the model's goodness-of-fit, find significant observations, and detect model assumption violations. Regression analysis also helps establish causal inference and guide policy decisions by evaluating the causal relationship between variables while accounting for potential biases and confounding variables.

Regression equation obtained as a result of Regression analysis with Minitab®21.1.1 was given in Eqn. 4.

The aspect ratio values calculated with the regression equation and the aspect ratio values and error rates obtained as a result of the measurements are given in Table 5. As can be seen from Table 5, the calculated aspect ratio values are largely compatible with the measurement results.

Table 5. Measured and calculated aspect ratio values and their error rates.

Exp No	P (W)	SS (mm/s)	Frq (kHz)	Measured D/W	Calculated D/W	Error (%)
1	20	1	20	0.142	0.131	8.30
2	20	50	100	0.042	0.032	22.33
3	20	100	200	0.283	0.278	1.48
4	60	1	100	0.198	0.160	19.38
5	60	50	200	14.924	14.894	0.20
6	60	100	20	21.070	21.020	0.24
7	100	1	200	-4.505	-4.563	1.30
8	100	50	20	19.672	19.594	0.40
9	100	100	100	16.220	16.146	0.45

5. CONCLUSION

This research provides a methodical analysis of optimizing fiber laser parameters to create grooves on ST52 steel. Through the utilization of the Taguchi approach, we have identified the most optimal parameters for three key variables using the Taguchi approach: laser power, scan speed, and frequency. These parameters directly affect the aspect ratio of the grooves. Our findings indicate that laser power had the most significant impact on the aspect ratio, accounting for 69.95% of the variation. In comparison, scan speed and frequency contributed 15.73% and 14.31% to the aspect ratio, respectively.

We employed the L_9 orthogonal array to establish our experimental systems. By doing so, we were able to efficiently explore the parameter space and obtain accurate results without wasting resources.

Our regression study has also demonstrated a strong correlation between experimental data and the anticipated model. This proved that the Taguchi method for improving laser processing parameters was correct.

This work adds to the ongoing discourse in the field by presenting convincing evidence that these concepts are well-established, as well as demonstrating current advantages that may be achieved by defining optimal settings.

6. ACKNOWLEDGMENTS

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Tzitzeica Smarandache Curves in Euclidean 3-Space

Research Article

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Abstract

The aim of this study is to examine the relations between Tzitzeica curves and Smarandache curves in Euclidean space. In addition, the necessary and sufficient conditions for Smarandache curves to be Tzitzeica curves in 3-dimensional Euclidean space are investigated and examples are given.

Keywords: Tzitzeica Curves, Smarandache Curves, Planar Tzitzeica Curves

1. INTRODUCTION

Smarandache curves were first described by Turgut and Yılmaz in 2008 [1]. The authors named as the Smarandache curve in Minkowski space, whose position vector is formed by the Frenet frame vectors of the other regular curve. They defined special cases of these curves and expressed the TB_2 curve. In [2], the author studied some special Smarandache curves in Euclidean space. In [3,4,5], the authors studied Smarandache curves according to the Darboux frame in 3-dimensional Euclidean space and Minkowski space. In [6], the author studied Smarandache curves in 4-dimensional Euclidean space. The author obtained Frenet-Serret and Bishop invariants for Smarandache curves and calculated the first, second and third curvatures of the Smarandache curve. In [7], the authors calculated the curvature and torsion of the Smarandache curve when the Frenet vectors of the Anti-Salkowski curve were taken as position vectors. In [8], the author studied Smarandache curves obtained from a curve with by a parallel transport frame in 4-dimensional Euclidean space. In [9], the authors examined families of hypersurfaces with Smarandache curves in 4-dimensional Galilean space. In [10], the authors re-characterized the Smarandache curves with by an alternative frame which is different from the Frenet frame. In [11], the authors defined new conjugate curves by integrating the Smarandache curves and examined the relations between the main curve and the

Frenet vectors of the resulting curve. In [12], the authors studied Smarandache Ruled surfaces. In [13], the authors examined some special Smarandache curves according to the Flc-frame in 3-dimensional Euclidean space.

Romanian Mathematician Gheorghe Tzitzeica defined a class of curves that he called Tzitzeica curves in 1911 [14]. In [15], the authors examined the relations between Tzitzeica curves and surfaces in Minkowski space. In [16], the author showed that elliptic and hyperbolic cylindrical curves in Euclidean space satisfy the Tzitzeica condition. In [17,18], the authors examined hyperbolic and elliptic cylindrical curves in Minkowski space, respectively. In [19], the author gave the necessary and sufficient condition for a space curve to become a Tzitzeica curve. In [20], the authors studied Tzitzeica curves in 3-dimensional Euclidean space. In [21], the authors examined Tzitzeica surfaces in 3-dimensional Euclidean space. In additionally, the planar Tzitzeica curve definition was defined for the first time. In [22,23], the authors studied Tzitzeica curves in 4-dimensional Euclidean space. In [24], the authors studied osculating and rectifiying curve in 4-dimensional Galilean space.

2. BASIC NOTATIONS

For a regular curve $\alpha: I \subset \mathbb{R} \rightarrow \mathbb{E}^3$, if the k_1 curvature $k_1(s)$ and the k_2 curvature $k_2(s)$ of α are constant functions, then α is called a screw curve or helix curve [26]. Since these curves are traces of one-parameter groups of Euclidean transformations, they were named W-curves by F. Klein and S. Lie [27]. If the $\frac{k_1(s)}{k_2(s)}$ ratio of a curve in $\mathbb{E}^3$ is a non-zero constant, this curve is called a general Helix [28]. For a regular space curve $\alpha: I \subset \mathbb{R} \rightarrow \mathbb{E}^3$, the planes at each point of $\alpha(s)$ spanned by $\{T, N_1\}, \{T, N_2\}, \{N_1, N_2\}$ are know as the osculating plane, rectifying plane and normal plane, respectively. If the position vector α lies on its rectifying plane or osculating plane or normal plane then, $\alpha(s)$ is called rectifying curve, osculating curve and normal curve, respectively [29,30].

Let $\alpha: I \subset \mathbb{R} \rightarrow \mathbb{E}^3$ be a unit speed curve in three-dimensional Euclidean space. Let us denote $T(s) = \alpha'(s)$ and call $T(s)$ a unit tangent vector of α at s. We denote the curvature of α by $k_1(s) = \|\alpha''(s)\|$. If $k_1(s) \neq 0$, then the unit principal normal vector $N_1(s)$ of the curve α at s is given by $T'(s) = k_1(s)N_1(s)$. The unit vector $N_2(s) = T(s) \times N_1(s)$ is called the unit binormal vector of α at s.
Then we have the Frenet-Serret formulae

$$\begin{aligned} T'(s) &= k_1(s)N_1(s) \\ N_1'(s) &= -k_1(s)T(s) + k_2(s)N_2(s) \\ N_2'(s) &= -k_2(s)N_1(s) \end{aligned} \tag{1}$$

where $k_2(s)$ is the torsion of the curve α at s [25]

Let $\alpha: I \subset \mathbb{R} \rightarrow \mathbb{E}^3$ be a unit speed curve, with a curvature $k_1(s) > 0$ and $k_2(s) \neq 0$. If the torsion of $\alpha(s)$ satisfies the condition

$$\frac{k_2(s)}{d_{osc}^2} = \mathrm{a} \tag{2}$$

for some real non-zero constant a then $\alpha(s)$ is called the Tzitzeica curve (Tz-curve), where

$$d_{osc} = \langle \alpha(s), N_2(s) \rangle \tag{3}$$

is the orthogonal distance from the origin to the osculating plane of $\alpha(s)$. Here, $N_2(s)$ is the binormal vector field of α [14].

Let $\alpha: I \subset \mathbb{R} \to \mathbb{E}^2$ be a unit speed planar curve $(k_1(s) > 0)$. In this case, if the curvature of $\alpha(s)$ satisfies the condition

$$k_1(s) = \mathrm{a}_1 . d_{osc}^2 \tag{4}$$

for some real non-zero constant a_1 then $\alpha(s)$ is called the planar Tzitzeica curve (planar Tz-curve) where

$$d_{osc} = \langle \alpha(s), N_1(s) \rangle \tag{5}$$

and $N_1(s)$ is the unit normal vector field of α [21].
If the position vector of a regular curve in Minkowski space consists of by the Frenet frame vectors on another regular curve, this curve is called a Smarandache curve [1].

Let $\alpha: I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed regular curve and $\{T(s), N_1(s), N_2(s)\}$ be its Frenet frame. Then we have Smarandache curves of $\alpha(s)$ by with

1) TN_1- Smarandache curve are defined by

$$\beta_{TN_1}(s_\beta) = \frac{1}{\sqrt{2}}\big(T(s) + N_1(s)\big) \tag{6}$$

2) TN_2- Smarandache curve are defined by

$$\beta_{TN_2}(s_\beta) = \frac{1}{\sqrt{2}}\big(T(s) + N_2(s)\big) \tag{7}$$

3) N_1N_2- Smarandache curve are defined by

$$\beta_{N_1N_2}(s_\beta) = \frac{1}{\sqrt{2}}\big(N_1(s) + N_2(s)\big) \tag{8}$$

4) TN_1N_2- Smarandache curve are defined by

$$\beta_{TN_1N_2}(s_\beta) = \frac{1}{\sqrt{3}}\big(T(s) + N_1(s) + N_2(s)\big) \tag{9}$$

[2]. Here s_β is the arc-length parameter of the curve.

3.TZITZEICA SMARANDACHE CURVES IN $\mathbb{E}^3$

3.1 TN_1- Smarandache Curve

Let $\alpha: I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed regular curve in $\mathbb{E}^3$. Let β_{TN_1} Smarandache curve in $\mathbb{E}^3$given with the parametrization (6). If we denote the arc-length parameter of the β_{TN_1} curve with s_β and take the derivative of the β_{TN_1} curve, we obtain

$$\beta'_{TN_1}(s_\beta) = \frac{d_{\beta_{TN_1}}}{d_{s_\beta}} \cdot \frac{d_{s_\beta}}{d_s} = \frac{1}{\sqrt{2}}(-k_1(s)T(s) + k_1(s)N_1(s) + k_2(s)N_2(s)). \tag{10}$$

For the norm of this expression is $\left\|\frac{d_{\beta_{TN_1}}}{d_{s_\beta}}\right\| = 1$, we get

$$\frac{d_{s_\beta}}{d_s} = \frac{\sqrt{2k_1^2 + k_2^2}}{\sqrt{2}} \neq 0. \tag{11}$$

In this case, from the expression (10), the tangent vector field of the β_{TN_1} curve becomes

$$T_{\beta_{TN_1}} = \frac{d_{\beta_{TN_1}}}{d_{s_\beta}} = \frac{1}{\sqrt{2k_1^2 + k_2^2}}(-k_1T + k_1N_1 + k_2N_2). \tag{12}$$

Again, by taking derivative of (12) and using (11), we obtain

$$(T_{\beta_{TN_1}})' = \left(\frac{d_{\beta_{TN_1}}}{d_{s_\beta}}\right)' \cdot \frac{d_{s_\beta}}{d_s} = \frac{\sqrt{2}}{(2k_1^2 + k_2^2)^2}(AT + BN_1 + CN_2) \tag{13}$$

where

$$A(s) = -k_1^2(2k_1^2 + k_2^2) - k_2(k_1'k_2 - k_1k_2')$$

$$B(s) = -k_1^2(2k_1^2 + 3k_2^2) + k_2(k_1'k_2 - k_1k_2' - k_2^3)$$

$$C(s)=k_1k_2(2k_1^2+k_2^2)-2k_1(k_1'k_2-k_1k_2').$$

The curvature of the β_{TN_1} curve is

$$k_{1\beta_{TN_1}}=\left\|T'_{\beta_{TN_1}}\right\|=\frac{\sqrt{2}}{(2k_1^2+k_2^2)^2}\sqrt{A^2+B^2+C^2}. \tag{14}$$

The principal normal vector field of the β_{TN_1} curve is

$$N_{1\beta_{TN_1}}=\frac{T'_{\beta_{TN_1}}}{k_{1\beta_{TN_1}}}=\frac{1}{\sqrt{A^2+B^2+C^2}}(AT+BN_1+CN_2). \tag{15}$$

By using (13) and (14). The binormal vector field of the β_{TN_1} curve is

$$N_{2\beta_{TN_1}}=T_{\beta_{TN_1}}\text{x}\,N_{1\beta_{TN_1}}=\frac{1}{DE}[(Ck_1-Bk_2)T+(Ck_1+Ak_2)N_1+(-Bk_1-Ak_1)N_2]. \tag{16}$$

By using (12) and (15), where

$$D(s)=\sqrt{2k_1^2+k_2^2}$$
$$E(s)=\sqrt{A^2+B^2+C^2}\,.$$

By taking derivative of (16) and using (11), we obtain

$$(N_{2\beta_{TN_1}})'=\frac{\sqrt{2}}{D^3E^2}\left\{\begin{array}{c}[-(D'E+DE')(Ck_1-Bk_2)+DE(C'k_1+Ck_1'-B'k_2-Bk_2'-Ck_1^2-Ak_1k_2)]T\\ +[-(D'E+DE')(Ck_1+Ak_2)+DE(Ck_1^2+C'k_1+Ck_1'+A'k_2+Ak_2'+Ak_1k_2)]N_1\\ +[(D'E+DE')(k_1A+k_1B)+DE(Ck_1k_2+Ak_2^2-Ak_1'-Bk_1'-A'k_1-B'k_1)]N_2\end{array}\right\}. \tag{17}$$

By using (15) and (17), we obtain

$$k_{2\beta_{TN_1}}=-\langle N'_{2\beta_{TN_1}},N_{1\beta_{TN_1}}\rangle=\frac{-\sqrt{2}}{D^2E^2}\begin{bmatrix}k_1\big(C'(A+B)-C(A'+B')\big)+k_2(-AB'+A'B)\\ +k_1^2C(-A+B)+k_2^2AC+k_1k_2(-A^2+AB+C^2)\end{bmatrix}. \tag{18}$$

Theorem 1: Let β_{TN_1} curve be the Smarandache curve of the unit speed curve α(s), where $k_1,k_2\neq 0$ are non-constant curvatures. If β_{TN_1} Smarandache curve is a Tz-curve then, the equation must be

$$\frac{k_{2\beta_{TN_1}}}{d_{osc}^2}=\frac{-2\sqrt{2}}{4C^2k_1^2+4Ck_1k_2(A-B)+k_2^2(A-B)^2}\left\{\begin{array}{l}k_1[C'(A+B)-C(A'+B')]\\ +k_2(-AB'+A'B)\\ +k_1^2C(B-A)+k_2^2AC\\ +k_1k_2(-A^2+AB+C^2)\end{array}\right\}=\text{a} \tag{19}$$

where a is nonzero constant [31].

Proof. By using (6) and (16), we get

$$d_{osc}=\langle\beta_{TN_1},N_{2\beta_{TN_1}}\rangle=\frac{1}{\sqrt{2}DE}\big(2Ck_1+k_2(A-B)\big). \tag{20}$$

Then, using equations (18) and (20), the expression (19) is obtained.

Corollary 2: Let β_{TN_1} curve be the Smarandache curve of the unit speed curve α(s), where $k_1,k_2\neq 0$ are non-constant curvatures. If $k_1=ck_2$ $(c\neq 0\ constant)$, β_{TN_1} Smarandache curve becomes a planar curve.

Theorem 3: Let β_{TN_1} curve be the Smarandache curve of the unit speed curve $\alpha(s)$, where $k_1,k_2\neq 0$ are non-constant curvatures. If $k_1=ck_2$ $(c\neq 0\ constant)$, β_{TN_1} Smarandache curve becomes a planar Tz-curve [31].

Proof: By using (6) and (15), we obtain

$$d_{osc}=\langle\beta_{TN_1},N_{1\beta_{TN_1}}\rangle=\frac{A+B}{\sqrt{2}\sqrt{A^2+B^2+C^2}}\,. \tag{21}$$

Using the expressions (4), (14) and (21), we get

$$\frac{k_{1\beta_{TN_1}}}{d_{osc}^2}=\frac{2\sqrt{2}}{\left(\sqrt{2k_1^2+k_2^2}\right)^9}\begin{bmatrix}(2k_1^2+k_2^2)^2(k_1^2+k_2^2)\\ +2(k_1'k_2-k_1k_2')[k_1'k_2-k_1k_2'-k_2(2k_1^2+k_2^2)]\end{bmatrix}^{\frac{3}{2}}. \tag{22}$$

If $k_1=ck_2$ is used in (22), we obtain

$$\frac{k_{1\beta_{TN_1}}}{d_{osc}^2}=2\sqrt{2}\left(\frac{c^2+1}{2c^2+1}\right)^{\frac{3}{2}}=\text{constant}.$$

Therefore, β_{TN_1} Smarandache curve is a planar Tz-curve.

Theorem 4: Let β_{TN_1} curve be the Smarandache curve of the unit speed curve $\alpha(s)$. If $\alpha(s)$ is W-curve $(i.e.\, k_1, k_2 \neq 0\ constant)$ then, β_{TN_1} Smarandache curve is a planar Tz-curve [31].
Proof: $k_1, k_2 \neq 0\ (constant)$, from equation (17), we obtain $(N_{2\beta_{TN_1}})' = 0$. Then from equation (18), we get $k_{2\beta_{TN_1}} = 0$. This means that the β_{TN_1} Smarandache curve is a planar curve. Then substituting $k_1, k_2 \neq 0\ (constant)$ into (22), we obtain

$$\frac{k_{1\beta_{TN_1}}}{d_{osc}^2} = \frac{2\sqrt{2}(k_1^2+k_2^2)^{\frac{3}{2}}}{(2k_1^2+k_2^2)^{\frac{3}{2}}} \neq 0\ (constant).$$

Therefore, β_{TN_1} Smarandache curve is a planar Tz-curve.

3.2 TN_2- Smarandache Curve

Let $\alpha: I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed regular curve in $\mathbb{E}^3$. Let β_{TN_2} be a Smarandache curve in $\mathbb{E}^3$ given with the parametrization (7). If we denote the arc-length parameter of the β_{TN_2} curve with s_β and take the derivative of the β_{TN_2} curve, we obtain

$$\beta'_{TN_2}(\mathrm{s}_\beta) = \frac{d_{\beta_{TN_2}}}{d_{s_\beta}} \cdot \frac{d_{s_\beta}}{d_s} = \frac{1}{\sqrt{2}}\big(k_1(s) - k_2(s)\big)N_1(s) \tag{23}$$

For the norm of this expression is $\left\|\frac{d_{\beta_{TN_2}}}{d_{s_\beta}}\right\| = 1$, we get

$$\frac{d_{s_\beta}}{d_s} = \frac{\sqrt{(k_1-k_2)^2}}{\sqrt{2}} = \frac{|k_1-k_2|}{\sqrt{2}} \neq 0. \tag{24}$$

In this case, from the expression (23), the tangent vector field of the β_{TN_2} curve becomes

$$T_{\beta_{TN_2}} = \frac{d_{\beta_{TN_2}}}{d_{s_\beta}} = \frac{(k_1-k_2)N_1}{|k_1-k_2|} = \begin{cases} N_1,\ k_1 > k_2 \\ -N_1,\ k_1 < k_2 \end{cases} \tag{25}$$

Again, by taking derivative of (25) and using (24), we obtain

$$(T_{\beta_{TN_2}})' = \left(\frac{d_{\beta_{TN_2}}}{d_{s_\beta}}\right)' \cdot \frac{d_{s_\beta}}{d_s} = \frac{-\sqrt{2}k_1}{(k_1-k_2)}T + \frac{\sqrt{2}k_2}{(k_1-k_2)}N_2. \tag{26}$$

The curvature of the β_{TN_2} curve is

$$k_{1\beta_{TN_2}} = \left\|T'_{\beta_{TN_2}}\right\| = \frac{\sqrt{2}\sqrt{k_1^2+k_2^2}}{|k_1-k_2|}. \tag{27}$$

The principal normal vector field of the β_{TN_2} curve is

$$N_{1\beta_{TN_2}} = \frac{T'_{\beta_{TN_2}}}{k_{1\beta_{TN_2}}} = \begin{cases} \frac{-k_1T+k_2N_2}{\sqrt{k_1^2+k_2^2}},\ k_1 > k_2 \\ \frac{k_1T-k_2N_2}{\sqrt{k_1^2+k_2^2}},\ k_1 < k_2 \end{cases}. \tag{28}$$

By using (26) and (27). The binormal vector field of the β_{TN_2} curve is

$$N_{2\beta_{TN_2}} = T_{\beta_{TN_2}} \mathrm{x}\, N_{1\beta_{TN_2}} = \frac{1}{\sqrt{k_1^2+k_2^2}}(k_2T+k_1N_2). \tag{29}$$

By using (25) and (28). By taking derivative of (29) and using (24), we obtain

$$(N_{2\beta_{TN_2}})' = \begin{cases} \frac{\sqrt{2}(k_1'k_2-k_1k_2')}{(k_1-k_2)(k_1^2+k_2^2)^{\frac{3}{2}}}(-k_1T+k_2N_2),\ k_1 > k_2 \\ \frac{-\sqrt{2}(k_1'k_2-k_1k_2')}{(k_1-k_2)(k_1^2+k_2^2)^{\frac{3}{2}}}(-k_1T+k_2N_2),\ k_1 < k_2 \end{cases}. \tag{30}$$

By using (28) and (30), we obtain

$$k_{2\beta_{TN_2}} = -\langle N'_{2\beta_{TN_2}}, N_{1\beta_{TN_2}} \rangle = \frac{-\sqrt{2}(k'_1 k_2 - k_1 k'_2)}{(k_1-k_2)(k_1^2+k_2^2)}. \tag{31}$$

Theorem 5: Let β_{TN_2} curve be the Smarandache curve of the unit speed curve α(s), where $k_1, k_2 \neq 0$ are non-constant curvatures. If β_{TN_2} Smarandache curve is a Tz-curve then, the equation must be

$$\frac{k_{2\beta_{TN_2}}}{d_{osc}^2} = \frac{-2\sqrt{2}(k'_1 k_2 - k_1 k'_2)}{(k_1-k_2)(k_1+k_2)^2} \neq 0 \ (constant) \ [31]. \tag{32}$$

Proof: By using (7) and (29), we get

$$d_{osc} = \langle \beta_{TN_2}, N_{2\beta_{TN_2}} \rangle = \frac{k_1+k_2}{\sqrt{2}\sqrt{k_1^2+k_2^2}}\,. \tag{33}$$

Then, using equations (31) and (33), the expression (32) is obtained.

Corollary 6: Let β_{TN_2} curve be the Smarandache curve of the unit speed curve α(s), where $k_1, k_2 \neq 0$ are non-constant curvatures. $k_1 = ck_2$ $(c \neq 0 \ constant)$ if and only if β_{TN_2} Smarandache curve becomes a planar curve.

Theorem 7: Let β_{TN_2} curve be the Smarandache curve of the unit speed curve $\alpha(s)$, where $k_1, k_2 \neq 0$ are non-constant curvatures. If $k_1 = ck_2$ $(c \neq 0 \ constant)$, β_{TN_2} Smarandache curve becomes a planar Tz-curve [31].

Proof: By using (7) and (28), we obtain

$$d_{osc} = \langle \beta_{TN_2}, N_{1\beta_{TN_2}} \rangle = \begin{cases} \dfrac{-(k_1-k_2)}{\sqrt{2}\sqrt{(k_1^2+k_2^2)}}, & k_1 > k_2 \\ \dfrac{(k_1-k_2)}{\sqrt{2}\sqrt{(k_1^2+k_2^2)}}, & k_1 < k_2 \end{cases}. \tag{34}$$

Using the expressions (4), (27) and (34), we get

$$\frac{k_{1\beta_{TN_2}}}{d_{osc}^2} = \begin{cases} \dfrac{2\sqrt{2}(k_1^2+k_2^2)^{\frac{3}{2}}}{(k_1-k_2)^3}, & k_1 > k_2 \\ \dfrac{2\sqrt{2}(k_1^2+k_2^2)^{\frac{3}{2}}}{-(k_1-k_2)^3}, & k_1 < k_2 \end{cases}. \tag{35}$$

If $k_1 = ck_2$ is used in (35), we obtain

$$\frac{k_{1\beta_{TN_2}}}{d_{osc}^2} = \mp \frac{2\sqrt{2}(c^2+1)^{\frac{3}{2}}}{(c-1)^3} = \text{constant}.$$

Therefore, β_{TN_2} Smarandache curve is a planar Tz-curve.

Theorem 8: Let β_{TN_2} curve be the Smarandache curve of the unit speed curve $\alpha(s)$. If $\alpha(s)$ is W-curve ($i.e.\ k_1, k_2 \neq 0 \ constant$) then, β_{TN_2} Smarandache curve is a planar Tz-curve [31].

Proof: $k_1, k_2 \neq 0$ $(constant)$, from equation (30), we obtain $(N_{2\beta_{TN_2}})' = 0$. Then from equation (31), we get $k_{2\beta_{TN_2}} = 0$. This means that the β_{TN_2} Smarandache curve is a planar curve. Then substituting $k_1, k_2 \neq 0$ $(constant)$ into (35), we obtain

$$\frac{k_{1\beta_{TN_2}}}{d_{osc}^2} = \begin{cases} \dfrac{2\sqrt{2}(k_1^2+k_2^2)^{\frac{3}{2}}}{(k_1-k_2)^3}, & k_1 > k_2 \\ \dfrac{2\sqrt{2}(k_1^2+k_2^2)^{\frac{3}{2}}}{-(k_1-k_2)^3}, & k_1 < k_2 \end{cases} \neq 0 \ (constant).$$

Therefore, β_{TN_2} Smarandache curve is a planar Tz-curve.

3.3 N_1N_2- Smarandache Curve

Let $\alpha: I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed regular curve in $\mathbb{E}^3$. Let $\beta_{N_1N_2}$ Smarandache curve in $\mathbb{E}^3$ given with the parametrization (8). If we denote the arc-length parameter of the $\beta_{N_1N_2}$ curve with s_β and take the derivative of the $\beta_{N_1N_2}$ curve, we obtain

$$\beta'_{N_1N_2}(s_\beta) = \frac{d_{\beta_{N_1N_2}}}{d_{s_\beta}} \cdot \frac{d_{s_\beta}}{d_s} = \frac{1}{\sqrt{2}}\big(-k_1(s)T(s) - k_2(s)N_1(s) + k_2(s)N_2(s)\big). \tag{36}$$

For the norm of this expression is $\left\|\frac{d_{\beta_{N_1N_2}}}{d_{s_\beta}}\right\| = 1$, we get

$$\frac{d_{s_\beta}}{d_s} = \frac{\sqrt{k_1^2+2k_2^2}}{\sqrt{2}} \neq 0. \tag{37}$$

In this case, from the expression (36), the tangent vector field of the $\beta_{N_1N_2}$ curve becomes

$$T_{\beta_{N_1N_2}} = \frac{d_{\beta_{N_1N_2}}}{d_{s_\beta}} = \frac{1}{\sqrt{k_1^2+2k_2^2}}(-k_1T - k_2N_1 + k_2N_2). \tag{38}$$

Again, by taking derivative of (38) and using (37), we obtain

$$(T_{\beta_{N_1N_2}})' = \left(\frac{d_{\beta_{N_1N_2}}}{d_{s_\beta}}\right)' \cdot \frac{d_{s_\beta}}{d_s} = \frac{\sqrt{2}}{B^2}\begin{bmatrix} (-2k_2A + k_1k_2B)T \\ +(k_1A - (k_1^2 + k_2^2)B)N_1 \\ +(-k_1A - k_2^2B)N_2 \end{bmatrix} \tag{39}$$

where

$A(s) = k_1'k_2 - k_1k_2'$

$B(s) = k_1^2 + 2k_2^2$.

The curvature of the $\beta_{N_1N_2}$ curve is

$$k_{1\beta_{N_1N_2}} = \left\|T'_{\beta_{N_1N_2}}\right\| = \frac{\sqrt{2}}{B^{\frac{3}{2}}}C \tag{40}$$

where

$C(s) = \sqrt{2A^2 + (k_1^2 + k_2^2)B^2 - 2k_1AB}$.

The principal normal vector field of the $\beta_{N_1N_2}$ curve is

$$N_{1\beta_{N_1N_2}} = \frac{T'_{\beta_{N_1N_2}}}{k_{1\beta_{N_1N_2}}} = \frac{1}{\sqrt{B}C}\begin{bmatrix} (-2k_2A + k_1k_2B)T \\ +(k_1A - (k_1^2 + k_2^2)B)N_1 \\ +(-k_1A - k_2^2B)N_2 \end{bmatrix}. \tag{41}$$

By using (39) and (40). The binormal vector field of the $\beta_{N_1N_2}$ curve is

$$N_{2\beta_{N_1N_2}} = T_{\beta_{N_1N_2}} \text{x}\, N_{1\beta_{N_1N_2}} = \frac{1}{C}[(k_2B)T - AN_1 + (-A + k_1B)N_2]. \tag{42}$$

By using (38) and (41). By taking derivative of (42) and using (37), we obtain

$$(N_{2\beta_{N_1N_2}})' = \frac{\sqrt{2}}{\sqrt{B}C^2}\begin{Bmatrix} [-k_2BC' + C(k_2'B + k_2B' + k_1A)]T \\ +[AC' + C(k_2A - A')]N_1 \\ +[C'(A - k_1B) + C(-k_2A - A' + k_1'B + k_1B')]N_2 \end{Bmatrix}. \tag{43}$$

By using (41) and (43), we obtain

$$k_{2\beta_{N_1N_2}} = -\langle N'_{2\beta_{N_1N_2}}, N_{1\beta_{N_1N_2}}\rangle = \frac{-1}{\sqrt{2}C^2}[-3AB' + 2B(A' - k_2A)] \tag{44}$$

Theorem 9: Let $\beta_{N_1N_2}$ curve be the Smarandache curve of the unit speed curve α(s), where $k_1, k_2 \neq 0$ are non-constant curvatures. If $\beta_{N_1N_2}$ Smarandache curve is a Tz-curve then, the equation must be

$$\frac{k_{2\beta_{N_1N_2}}}{d_{osc}^2} = \frac{-\sqrt{2}[-3AB' + 2B(A' - k_2A)]}{(-2A + k_1B)^2} \neq 0\ (constant)\ [31]. \tag{45}$$

Proof: By using (8) and (42), we get

$$d_{osc} = \langle \beta_{N_1N_2}, N_{2\beta_{N_1N_2}} \rangle = \frac{1}{\sqrt{2C}}(-2A + k_1B). \tag{46}$$

Then, using equations (44) and (46), the expression (45) is obtained.

Corollary 10: Let $\beta_{N_1N_2}$ curve be the Smarandache curve of the unit speed curve α(s), where $k_1, k_2 \neq 0$ are non-constant curvatures. If $k_1 = ck_2$ $(c \neq 0\ constant)$, $\beta_{N_1N_2}$ Smarandache curve becomes a planar curve.

Theorem 11: Let $\beta_{N_1N_2}$ curve be the Smarandache curve of the unit speed curve $\alpha(s)$, where $k_1, k_2 \neq 0$ are non-constant curvatures. If $k_1 = ck_2$ $(c \neq 0\ constant)$, $\beta_{N_1N_2}$ Smarandache curve becomes a planar Tz-curve [31].

Proof: By using (8) and (41), we obtain

$$d_{osc} = \langle \beta_{N_1N_2}, N_{1\beta_{N_1N_2}} \rangle = \frac{-B\sqrt{B}}{\sqrt{2C}}. \tag{47}$$

Using the expressions (4), (40) and (47), we get

$$\frac{k_{1\beta_{N_1N_2}}}{d_{osc}^2} = \frac{2\sqrt{2}C^3}{B^{\frac{9}{2}}}. \tag{48}$$

If $k_1 = ck_2$ is used in (48), we obtain

$$\frac{k_{1\beta_{N_1N_2}}}{d_{osc}^2} = 2\sqrt{2}\left(\frac{c^2+1}{c^2+2}\right)^{\frac{3}{2}} = \text{constant}.$$

Therefore, $\beta_{N_1N_2}$ Smarandache curve is a planar Tz-curve.

Theorem 12: Let $\beta_{N_1N_2}$ curve be the Smarandache curve of the unit speed curve $\alpha(s)$. If $\alpha(s)$ is W-curve ($i.e.\ k_1, k_2 \neq 0\ constant$) then, $\beta_{N_1N_2}$ Smarandache curve is a planar Tz-curve [31].

Proof: $k_1, k_2 \neq 0\ (constant)$, from equation (43), we obtain $(N_{2\beta_{N_1N_2}})' = 0$. Then from equation (44), we get $k_{2\beta_{N_1N_2}} = 0$. This means that the $\beta_{N_1N_2}$ Smarandache curve is a planar curve. Then substituting $k_1, k_2 \neq 0\ (constant)$ into (48), we obtain

$$\frac{k_{1\beta_{N_1N_2}}}{d_{osc}^2} = \frac{2\sqrt{2}(k_1^2+k_2^2)^{\frac{3}{2}}}{(k_1^2+2k_2^2)^{\frac{3}{2}}} \neq 0\ (constant).$$

Therefore, $\beta_{N_1N_2}$ Smarandache curve is a planar Tz-curve.

3.4 TN_1N_2- Smarandache Curve

Let $\alpha: I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed regular curve in $\mathbb{E}^3$. Let $\beta_{TN_1N_2}$ Smarandache curve in $\mathbb{E}^3$ given with the parametrization (9). If we denote the arc-length parameter of the $\beta_{TN_1N_2}$ curve with s_β and take the derivative of the $\beta_{TN_1N_2}$ curve, we obtain

$$\beta'_{TN_1N_2}(s_\beta) = \frac{d\beta_{TN_1N_2}}{ds_\beta} \cdot \frac{ds_\beta}{ds} = \frac{1}{\sqrt{3}}(-k_1(s)T(s) + (k_1(s) - k_2(s))N_1(s) + k_2(s)N_2(s). \tag{49}$$

For the norm of this expression is $\left\| \frac{d\beta_{TN_1N_2}}{ds_\beta} \right\| = 1$, we get

$$\frac{ds_\beta}{ds} = \frac{\sqrt{2}\sqrt{k_1^2+k_2^2-k_1k_2}}{\sqrt{3}} \neq 0. \tag{50}$$

In this case, from the expression (49), the tangent vector field of the $\beta_{TN_1N_2}$ curve becomes

$$T_{\beta_{TN_1N_2}} = \frac{d\beta_{TN_1N_2}}{ds_\beta} = \frac{1}{\sqrt{2A}}(-k_1T + (k_1 - k_2)N_1 + k_2N_2) \tag{51}$$

Again, by taking derivative of (51) and using (50), we obtain

$$(T_{\beta_{TN_1N_2}})' = \left(\frac{d\beta_{TN_1N_2}}{ds_\beta}\right)' \cdot \frac{ds_\beta}{ds} = \frac{\sqrt{3}}{4(A)^4}(BT + CN_1 + DN_2) \tag{52}$$

where
$A(s) = \sqrt{k_1^2 + k_2^2 - k_1 k_2}$
$B(s) = (k_1' k_2 - k_1 k_2')(k_1 - 2k_2) - 2k_1(k_1^2 + k_2^2 - k_1 k_2)(k_1 - k_2)$
$C(s) = (k_1 + k_2)(k_1' k_2 - k_1 k_2') - 2(k_1^2 + k_2^2 - k_1 k_2)(k_1^2 + k_2^2)$
$D(s) = -(k_1' k_2 - k_1 k_2')(2k_1 - k_2) + 2k_2(k_1^2 + k_2^2 - k_1 k_2)(k_1 - k_2)$.
The curvature of the $\beta_{TN_1N_2}$ curve is

$$k_{1\beta_{TN_1N_2}} = \left\| T'_{\beta_{TN_1N_2}} \right\| = \frac{\sqrt{3}}{4(A)^4}\sqrt{B^2 + C^2 + D^2} \tag{53}$$

The principal normal vector field of the $\beta_{TN_1N_2}$ curve is

$$N_{1\beta_{TN_1N_2}} = \frac{T'_{\beta_{TN_1N_2}}}{k_{1\beta_{TN_1N_2}}} = \frac{1}{\sqrt{B^2+C^2+D^2}}(BT + CN_1 + DN_2). \tag{54}$$

By using (52) and (53). The binormal vector field of the $\beta_{TN_1N_2}$ curve is

$$N_{2\beta_{TN_1N_2}} = T_{\beta_{TN_1N_2}} \text{x}\, N_{1\beta_{TN_1N_2}} = \frac{1}{\sqrt{2}A\sqrt{B^2+C^2+D^2}}\begin{Bmatrix} (Dk_1 - (D + C)k_2)T \\ +(Dk_1 + Bk_2)N_1 \\ +(-(B + C)k_1 + Bk_2)N_2 \end{Bmatrix}. \tag{55}$$

By using (51) and (54). By taking derivative of (55) and using (50), we obtain

$$(N_{2\beta_{TN_1N_2}})' = \frac{\sqrt{3}}{2A^3(B^2+C^2+D^2)^{\frac{3}{2}}}\begin{Bmatrix} \begin{bmatrix} EDk_1 - (D + C)Ek_2 \\ +A(B^2 + C^2 + D^2)[D(k_1' - k_2' - k_1^2) + D'(k_1 - k_2) - Ck_2' - C'k_2 - k_1k_2B] \end{bmatrix} T \\ \\ +\begin{bmatrix} EDk_1 + EBk_2 \\ +A(B^2 + C^2 + D^2)[D(k_1^2 - k_1k_2 + k_1') + B(-k_2^2 + k_1k_2 + k_2') + D'k_1 + B'k_2] \end{bmatrix} N_1 \\ \\ +\begin{bmatrix} -E(B + C)k_1 + EBk_2 \\ +A(B^2 + C^2 + D^2)[D(k_1k_2) + B(k_2^2 - k_1' + k_2') + B'(-k_1 + k_2) - Ck_1' - C'k_1] \end{bmatrix} N_2 \end{Bmatrix} \tag{56}$$

where
$E(s) = -A'(B^2 + C^2 + D^2) - A(BB' + CC' + DD')$.
By using (54) and (56), we obtain

$$k_{2\beta_{TN_1N_2}} = -\langle N'_{2\beta_{TN_1N_2}}, N_{1\beta_{TN_1N_2}} \rangle = \frac{-\sqrt{3}}{2A^2(B^2+C^2+D^2)}\begin{bmatrix} k_1\big(D'(B + C) - D(B' + C')\big) \\ +k_2\big(-B(C' + D') + B'(C + D)\big) \\ +k_1^2 D(C - B) + k_2^2 B(D - C) \\ +k_1k_2(-B^2 + BC - DC + D^2) \end{bmatrix}. \tag{57}$$

Theorem 13: Let $\beta_{TN_1N_2}$ curve be the Smarandache curve of the unit speed curve α(s), where $k_1, k_2 \neq 0$ are non-constant curvatures. If $\beta_{TN_1N_2}$ Smarandache curve is a Tz-curve then, the equation must be

$$\frac{k_{2\beta_{TN_1N_2}}}{d_{osc}^2} = \frac{-3\sqrt{3}}{\big(D(2k_1 - k_2) - C(k_1 + k_2) + B(2k_2 - k_1)\big)^2}\begin{Bmatrix} k_1\big(D'(B + C) - D(B' + C')\big) \\ +k_2\big(-B(C' + D') + B'(C + D)\big) \\ +k_1^2 D(C - B) + k_2^2 B(D - C) \\ +k_1k_2(-B^2 + BC - DC + D^2) \end{Bmatrix} \neq 0\ (constant)\ [31]. \tag{58}$$

Proof: By using (9) and (55), we get

$$d_{osc} = \langle \beta_{TN_1N_2}, N_{2\beta_{TN_1N_2}} \rangle = \frac{1}{\sqrt{6}A\sqrt{B^2+C^2+D^2}}\big(D(2k_1 - k_2) - C(k_1 + k_2) + B(2k_2 - k_1)\big). \tag{59}$$

Then, using equations (57) and (59), the expression (58) is obtained.
Corollary 14: Let $\beta_{TN_1N_2}$ curve be the Smarandache curve of the unit speed curve α(s), where $k_1, k_2 \neq 0$ are non-constant curvatures. If $k_1 = ck_2$ $(c \neq 0\ constant)$, $\beta_{TN_1N_2}$ Smarandache curve becomes a planar curve.

Theorem 15: Let $\beta_{TN_1N_2}$ curve be the Smarandache curve of the unit speed curve $\alpha(s)$, where $k_1, k_2 \neq 0$ are non-constant curvatures. If $k_1 = ck_2$ $(c \neq 0\ constant)$, $\beta_{TN_1N_2}$ Smarandache curve becomes a planar Tz-curve [31].

Proof: By using (9) and (54), we obtain

$$d_{osc} = \langle \beta_{TN_1N_2}, N_{1\beta_{TN_1N_2}} \rangle = \frac{1}{\sqrt{3}\sqrt{B^2+C^2+D^2}}(B+C+D). \tag{60}$$

Using the expressions (4), (53) and (60), we get

$$\frac{k_{1\beta_{TN_1N_2}}}{d_{osc}^2} = \frac{3\sqrt{3}(B^2+C^2+D^2)^{\frac{3}{2}}}{4A^4(B+C+D)^2}. \tag{61}$$

If $k_1 = ck_2$ is used in (61), we obtain

$$\frac{k_{1\beta_{TN_1N_2}}}{d_{osc}^2} = \frac{3\sqrt{3}}{2\sqrt{2}}\left(\frac{c^2+1}{c^2-c+1}\right)^{\frac{3}{2}} = \text{constant}.$$

Therefore, $\beta_{TN_1N_2}$ Smarandache curve is a planar Tz-curve.

Theorem 16: Let $\beta_{TN_1N_2}$ curve be the Smarandache curve of the unit speed curve $\alpha(s)$. If $\alpha(s)$ is W-curve $(i.e.\ k_1, k_2 \neq 0\ constant)$ then, $\beta_{TN_1N_2}$ Smarandache curve is a planar Tz-curve [31].

Proof: $k_1, k_2 \neq 0\ (constant)$, from equation (56), we obtain $(N_{2\beta_{TN_1N_2}})' = 0$. Then from equation (57), we get $k_{2\beta_{TN_1N_2}} = 0$. This means that the $\beta_{TN_1N_2}$ Smarandache curve is a planar curve. Then substituting $k_1, k_2 \neq 0\ (constant)$ into (61), we obtain

$$\frac{k_{1\beta_{TN_1N_2}}}{d_{osc}^2} = \frac{3\sqrt{3}(k_1^2+k_2^2)^{\frac{3}{2}}}{2\sqrt{2}(k_1^2+k_2^2-k_1k_2)^{\frac{3}{2}}} \neq 0\ (constant).$$

Therefore, $\beta_{TN_1N_2}$ Smarandache curve is a planar Tz-curve.

Example 17: Let $\alpha_1(s)$ be a unit speed helix curve (W-curve) given with the parametrization

$$\alpha_1(s) = \left(cos\left(\frac{s}{\sqrt{2}}\right), sin\left(\frac{s}{\sqrt{2}}\right), \frac{s}{\sqrt{2}}\right)$$

(Fig. 1). Frenet vectors, curvature and torsion of α_1 curve can be given by

$$T_{\alpha_1}(s) = \left(\frac{-1}{\sqrt{2}}sin\left(\frac{s}{\sqrt{2}}\right), \frac{1}{\sqrt{2}}cos\left(\frac{s}{\sqrt{2}}\right), \frac{1}{\sqrt{2}}\right)$$

$$N_{1\alpha_1}(s) = \left(-cos\left(\frac{s}{\sqrt{2}}\right), -sin\left(\frac{s}{\sqrt{2}}\right), 0\right)$$

$$N_{2\alpha_1}(s) = \left(\frac{1}{\sqrt{2}}sin\left(\frac{s}{\sqrt{2}}\right), \frac{-1}{\sqrt{2}}cos\left(\frac{s}{\sqrt{2}}\right), \frac{1}{\sqrt{2}}\right)$$

$k_{1\alpha_1}(s) = \frac{1}{2}$ and $k_{2\alpha_1}(s) = \frac{1}{2}$.

a) TN_1 -Smarandache curve of $\alpha_1(s)$ curve

$$\beta_{TN_1} = \frac{1}{\sqrt{2}}\left(T_{\alpha_1} + N_{1\alpha_1}\right) = \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}sin\left(\frac{s}{\sqrt{2}}\right) - cos\left(\frac{s}{\sqrt{2}}\right), \frac{1}{\sqrt{2}}cos\left(\frac{s}{\sqrt{2}}\right) - sin\left(\frac{s}{\sqrt{2}}\right), \frac{1}{\sqrt{2}}\right)$$

(Fig. 2). Frenet vectors, curvature and torsion of β_{TN_1} curve using equations (11), (12), (14), (15), (16) and (18), we obtain

$$T_{\beta_{TN_1}}(s_\beta) = \frac{1}{\sqrt{3}}\left(-cos\left(\frac{2}{\sqrt{3}}s_\beta\right) + \sqrt{2}sin\left(\frac{2}{\sqrt{3}}s_\beta\right), -sin\left(\frac{2}{\sqrt{3}}s_\beta\right) - \sqrt{2}cos\left(\frac{2}{\sqrt{3}}s_\beta\right), 0\right)$$

$$N_{1\beta_{TN_1}}(s_\beta) = \frac{1}{\sqrt{3}}\left(sin\left(\frac{2}{\sqrt{3}}s_\beta\right) + \sqrt{2}\,cos\left(\frac{2}{\sqrt{3}}s_\beta\right), -cos\left(\frac{2}{\sqrt{3}}s_\beta\right) + \sqrt{2}sin\left(\frac{2}{\sqrt{3}}s_\beta\right), 0\right)$$

$$N_{2\beta_{TN_1}}(s_\beta) = (0,0,1)$$

$k_{1\beta_{TN_1}}(s_\beta) = \frac{2}{\sqrt{3}}$ and $k_{2\beta_{TN_1}}(s_\beta) = 0$.

In this case, since $k_{2\beta_{TN_1}}(s_\beta) = 0$, the β_{TN_1} curve becomes a planar curve. By using (21), we get
$d_{osc} = \langle \beta_{TN_1}(s_\beta), N_{1\beta_{TN_1}}(s_\beta) \rangle = \frac{-\sqrt{3}}{2}$.
By using (22), we obtain
$\frac{k_{1\beta_{TN_1}}}{d_{osc}^2} = \frac{8}{3\sqrt{3}} \neq 0\ (constant)$.

Therefore, β_{TN_1} curve becomes a planar Tz-curve.

b) N_1N_2 -Smarandache curve of $\alpha_1(s)$ curve

$\beta_{N_1N_2} = \frac{1}{\sqrt{2}}\left(N_{1\alpha_1} + N_{2\alpha_1}\right) = \frac{1}{\sqrt{2}}\left(-\cos\left(\frac{s}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\sin\left(\frac{s}{\sqrt{2}}\right), -\sin\left(\frac{s}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}\cos\left(\frac{s}{\sqrt{2}}\right), \frac{1}{\sqrt{2}}\right)$

(Fig. 3). Frenet vectors, curvature and torsion of $\beta_{N_1N_2}$ curve using equations (37), (38), (40), (41), (42) and (44), we obtain

$T_{\beta_{N_1N_2}}(s_\beta) = \frac{1}{\sqrt{3}}\left(\sqrt{2}\sin\left(\frac{2}{\sqrt{3}}s_\beta\right) + \cos\left(\frac{2}{\sqrt{3}}s_\beta\right), -\sqrt{2}\cos\left(\frac{2}{\sqrt{3}}s_\beta\right) + \sin\left(\frac{2}{\sqrt{3}}s_\beta\right), 0\right)$

$N_{1\beta_{N_1N_2}}(s_\beta) = \frac{1}{\sqrt{3}}\left(\sqrt{2}\cos\left(\frac{2}{\sqrt{3}}s_\beta\right) - \sin\left(\frac{2}{\sqrt{3}}s_\beta\right), \sqrt{2}\sin\left(\frac{2}{\sqrt{3}}s_\beta\right) + \cos\left(\frac{2}{\sqrt{3}}s_\beta\right), 0\right)$

$N_{2\beta_{N_1N_2}}(s_\beta) = (0,0,1)$

$k_{1\beta_{N_1N_2}}(s_\beta) = \frac{2}{\sqrt{3}}$ and $k_{2\beta_{N_1N_2}}(s_\beta) = 0$.

In this case, since $k_{2\beta_{N_1N_2}}(s_\beta) = 0$, the $\beta_{N_1N_2}$ curve becomes a planar curve. By using (47), we get
$d_{osc} = \langle \beta_{N_1N_2}(s_\beta), N_{1\beta_{N_1N_2}}(s_\beta) \rangle = \frac{-\sqrt{3}}{2}$.
By using equation (48)
$\frac{k_{1\beta_{N_1N_2}}}{d_{osc}^2} = \frac{8}{3\sqrt{3}} \neq 0\ (constant)$.
Therefore, $\beta_{N_1N_2}$ curve becomes a planar Tz-curve.

c) TN_1N_2 -Smarandache curve of $\alpha_1(s)$ curve

$\beta_{TN_1N_2} = \frac{1}{\sqrt{3}}\left(T_{\alpha_1} + N_{1\alpha_1} + N_{2\alpha_1})\right) = \frac{1}{\sqrt{3}}\left(-\cos\left(\frac{s}{\sqrt{2}}\right), -\sin\left(\frac{s}{\sqrt{2}}\right), \sqrt{2}\right)$

(Fig. 4). Frenet vectors, curvature and torsion of curve $\beta_{TN_1N_2}$ using equations (50), (51), (53), (54), (55) and (57), we obtain

$T_{\beta_{TN_1N_2}}(s_\beta) = \left(\sin(\sqrt{3}\, s_\beta), -\cos(\sqrt{3}\, s_\beta), 0\right)$

$N_{1\beta_{TN_1N_2}}(s_\beta) = \left(-\cos(\sqrt{3}\, s_\beta), -\sin(\sqrt{3}\, s_\beta), 0\right)$

$N_{2\beta_{TN_1N_2}}(s_\beta) = (0,0,1)$

$k_{1\beta_{TN_1N_2}}(s_\beta) = \sqrt{3}$ and $k_{2\beta_{TN_1N_2}}(s_\beta) = 0$.

In this case, since $k_{2\beta_{TN_1N_2}}(s_\beta) = 0$, the $\beta_{TN_1N_2}$ curve becomes a planar curve. By using (60), we get
$d_{osc} = \langle \beta_{TN_1N_2}(s_\beta), N_{1\beta_{TN_1N_2}}(s_\beta) \rangle = \frac{-1}{\sqrt{3}}$.
By using (61), we obtain
$\frac{k_{1\beta_{TN_1N_2}}}{d_{osc}^2} = 3\sqrt{3} \neq 0\ (constant)$.
Therefore, $\beta_{TN_1N_2}$ curve becomes a planar Tz-curve

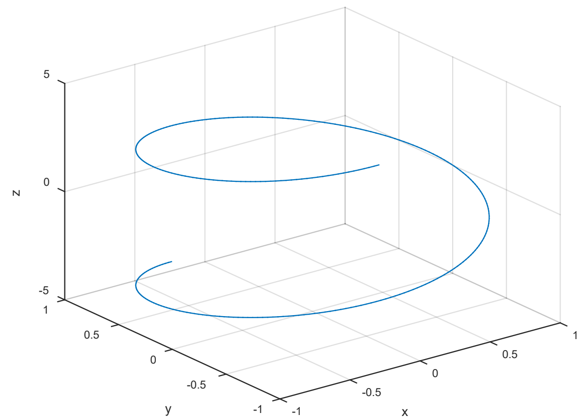

Fig. 1. $\alpha_1(s)$ curve

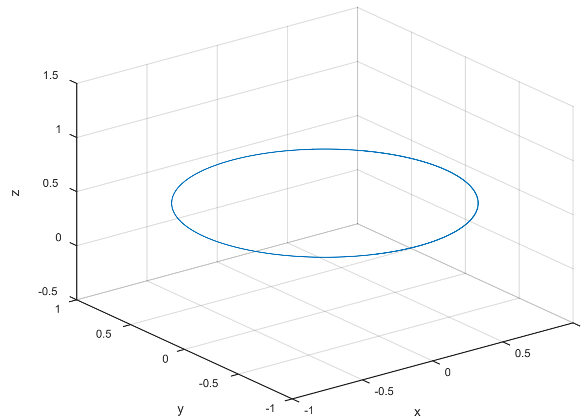

Fig. 2. β_{TN_1} curve of $\alpha_1(s)$ curve

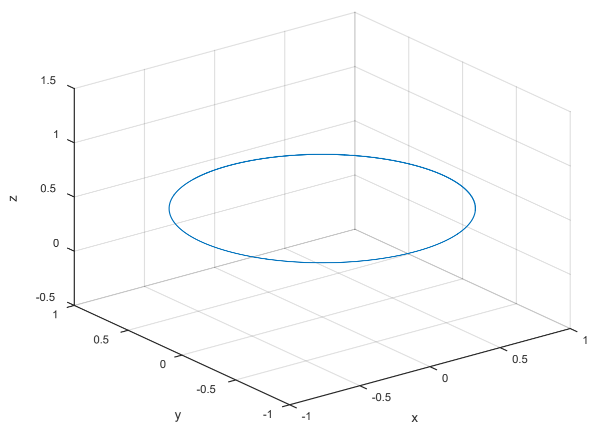

Fig. 3. $\beta_{N_1N_2}$ curve of $\alpha_1(s)$ curve

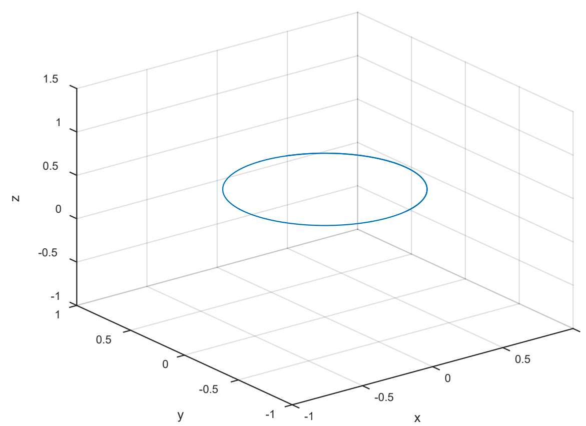

Fig. 4. $\beta_{TN_1N_2}$ curve of $\alpha_1(s)$ curve

Example 18: Let $\alpha_2(s)$ be a unit speed cylindrical helix curve given with the parametrization

$$\alpha_2(s) = \frac{1}{\sqrt{5}}\left(\sqrt{1+s^2}, 2s, \ln\left(s+\sqrt{1+s^2}\right)\right)$$

(Fig. 5). Frenet vectors, curvature and torsion of α_2 curve can be given by

$$T_{\alpha_2}(s) = \frac{1}{\sqrt{5}}\left(\frac{s}{\sqrt{1+s^2}}, 2, \frac{1}{\sqrt{1+s^2}}\right)$$

$$N_{1\alpha_2}(s) = \left(\frac{1}{\sqrt{1+s^2}}, 0, \frac{-s}{\sqrt{1+s^2}}\right)$$

$$N_{2\alpha_2}(s) = \frac{1}{\sqrt{5}}\left(\frac{-2s}{\sqrt{1+s^2}}, 1, \frac{-2}{\sqrt{1+s^2}}\right)$$

$$k_{1\alpha_2}(s) = \frac{1}{\sqrt{5}(1+s^2)} \text{ and } k_{2\alpha_2}(s) = \frac{2}{\sqrt{5}(1+s^2)}.$$

a) TN_1 -Smarandache curve of $\alpha_2(s)$ curve

$\beta_{TN_1} = \frac{1}{\sqrt{2}}\left(T_{\alpha_2} + N_{1\alpha_2}\right) = \frac{1}{\sqrt{10}}\left(\frac{s+\sqrt{5}}{\sqrt{1+s^2}}, 2, \frac{1-\sqrt{5}s}{\sqrt{1+s^2}}\right)$

(Fig. 6). Frenet vectors, curvature and torsion of β_{TN_1} curve using equations (11), (12), (14), (15), (16) and (18), we obtain

$T_{\beta_{TN_1}}(s_\beta)=\frac{1}{\sqrt{6}}\left(cos\left(\frac{\sqrt{5}}{\sqrt{3}}s_\beta\right) - \sqrt{5}sin\left(\frac{\sqrt{5}}{\sqrt{3}}s_\beta\right), 0, -\sqrt{5}cos\left(\frac{\sqrt{5}}{\sqrt{3}}s_\beta\right) - sin\left(\frac{\sqrt{5}}{\sqrt{3}}s_\beta\right)\right)$

$N_{1\beta_{TN_1}}(s_\beta) = \frac{1}{\sqrt{6}}\left(-sin\left(\frac{\sqrt{5}}{\sqrt{3}}s_\beta\right) - \sqrt{5}cos\left(\frac{\sqrt{5}}{\sqrt{3}}s_\beta\right), 0, \sqrt{5}sin\left(\frac{\sqrt{5}}{\sqrt{3}}s_\beta\right) - cos\left(\frac{\sqrt{5}}{\sqrt{3}}s_\beta\right)\right)$

$N_{2\beta_{TN_1}}(s_\beta) = (0,1,0)$

$k_{1\beta_{TN_1}}(s_\beta) = \frac{\sqrt{5}}{\sqrt{3}}$ and $k_{2\beta_{TN_1}}(s_\beta) = 0.$

In this case, since $k_{2\beta_{TN_1}}(s_\beta) = 0$, the β_{TN_1} curve becomes a planar curve. By using (21), we get

$d_{osc} = \langle\beta_{TN_1}(s_\beta), N_{1\beta_{TN_1}}(s_\beta)\rangle = \frac{-\sqrt{3}}{\sqrt{5}}.$

By using (22), we obtain

$\frac{k_{1\beta_{TN_1}}}{d_{osc}^2} = \frac{5\sqrt{5}}{3\sqrt{3}} \neq 0\ (constant)$.

Therefore, β_{TN_1} curve becomes a planar Tz-curve.

b) TN_2 -Smarandache curve of $\alpha_2(s)$ curve

$\beta_{TN_2} = \frac{1}{\sqrt{2}}\left(T_{\alpha_2} + N_{2\alpha_2}\right) = \frac{1}{\sqrt{2}}\left(\frac{-s}{\sqrt{5}\sqrt{1+s^2}}, \frac{3}{\sqrt{5}}, \frac{-1}{\sqrt{5}\sqrt{1+s^2}}\right)$

(Fig. 7). Frenet vectors, curvature and torsion of β_{TN_2} curve using equations (24), (25), (27), (28), (29) and (31), we obtain

$T_{\beta_{TN_2}}(s_\beta) = \left(-cos\left(\sqrt{10}s_\beta\right), 0, sin\left(\sqrt{10}s_\beta\right)\right)$

$N_{1\beta_{TN_2}}(s_\beta) = \left(sin\left(\sqrt{10}s_\beta\right), 0, cos\left(\sqrt{10}s_\beta\right)\right)$

$N_{2\beta_{TN_2}}(s_\beta) = (0,1,0)$

$k_{1\beta_{TN_2}}(s_\beta) = \sqrt{10}$ and $k_{2\beta_{TN_2}}(s_\beta) = 0.$

In this case, since $k_{2\beta_{TN_2}}(s_\beta) = 0$, the β_{TN_2} curve becomes a planar curve. By using (34), we get

$d_{osc} = \langle\beta_{TN_2}(s_\beta), N_{1\beta_{TN_2}}(s_\beta)\rangle = \frac{-1}{\sqrt{10}}.$

By using (35), we obtain

$\frac{k_{1\beta_{TN_2}}}{d_{osc}^2} = 10\sqrt{10} \neq 0\ (constant).$

Therefore, β_{TN_2} curve becomes a planar Tz-curve.

c) N_1N_2 -Smarandache curve of $\alpha_2(s)$ curve

$\beta_{N_1N_2} = \frac{1}{\sqrt{2}}\left(N_{1\alpha_2} + N_{2\alpha_2}\right) = \frac{1}{\sqrt{10}}\left(\frac{\sqrt{5}-2s}{\sqrt{1+s^2}}, 1, \frac{-2-\sqrt{5}s}{\sqrt{1+s^2}}\right)$

(Fig. 8). Frenet vectors, curvature and torsion of $\beta_{N_1N_2}$ curve using equations (37), (38), (40), (41), (42) and (44), we obtain

$T_{\beta_{N_1N_2}}(s_\beta) = \frac{1}{3}\left(-\sqrt{5}sin\left(\frac{\sqrt{10}}{3}s_\beta\right) - 2cos\left(\frac{\sqrt{10}}{3}s_\beta\right), 0, -\sqrt{5}cos\left(\frac{\sqrt{10}}{3}s_\beta\right) + 2sin\left(\frac{\sqrt{10}}{3}s_\beta\right)\right)$

$N_{1\beta_{N_1N_2}}(s_\beta) = \frac{1}{3}\left(-\sqrt{5}cos\left(\frac{\sqrt{10}}{3}s_\beta\right) + 2sin\left(\frac{\sqrt{10}}{3}s_\beta\right), 0, \sqrt{5}sin\left(\frac{\sqrt{10}}{3}s_\beta\right) + 2cos\left(\frac{\sqrt{10}}{3}s_\beta\right)\right)$

$N_{2\beta_{N_1N_2}}(s_\beta) = (0,1,0)$

$k_{1\beta_{N_1N_2}}(s_\beta) = \frac{\sqrt{10}}{3}$ and $k_{2\beta_{N_1N_2}}(s_\beta) = 0.$

In this case, since $k_{2\beta_{N_1N_2}}(s_\beta) = 0$, the $\beta_{N_1N_2}$ curve becomes a planar curve. By using (47), we get

$d_{osc} = \langle \beta_{N_1N_2}(s_\beta), N_{1\beta_{N_1N_2}}(s_\beta) \rangle = \frac{-3}{\sqrt{10}}.$

By using equation (48)

$\frac{k_{1\beta_{N_1N_2}}}{d_{osc}^2} = \frac{10\sqrt{10}}{27} \neq 0\ (constant).$

Therefore, $\beta_{N_1N_2}$ curve becomes a planar Tz-curve.

d) TN_1N_2 -Smarandache curve of $\alpha_2(s)$ curve

$\beta_{TN_1N_2} = \frac{1}{\sqrt{3}}\left(T_{\alpha_2} + N_{1\alpha_2} + N_{2\alpha_2}\right) = \frac{1}{\sqrt{15}}\left(\frac{\sqrt{5}-s}{\sqrt{1+s^2}}, 3, \frac{-1-\sqrt{5}s}{\sqrt{1+s^2}}\right)$

(Fig. 9). Frenet vectors, curvature and torsion of curve $\beta_{TN_1N_2}$ using equations (50), (51), (53), (54), (55) and (57), we obtain

$T_{\beta_{TN_1N_2}}(s_\beta)=\frac{1}{\sqrt{6}}\left(-cos\left(\frac{\sqrt{5}}{\sqrt{2}}s_\beta\right) - \sqrt{5}sin\left(\frac{\sqrt{5}}{\sqrt{2}}s_\beta\right), 0, -\sqrt{5}cos\left(\frac{\sqrt{5}}{\sqrt{2}}s_\beta\right) + sin\left(\frac{\sqrt{5}}{\sqrt{2}}s_\beta\right)\right)$

$N_{1\beta_{TN_1N_2}}(s_\beta) = \frac{1}{\sqrt{6}}\left(sin\left(\frac{\sqrt{5}}{\sqrt{2}}s_\beta\right) - \sqrt{5}cos\left(\frac{\sqrt{5}}{\sqrt{2}}s_\beta\right), 0, \sqrt{5}sin\left(\frac{\sqrt{5}}{\sqrt{2}}s_\beta\right) + cos\left(\frac{\sqrt{5}}{\sqrt{2}}s_\beta\right)\right)$

$N_{2\beta_{TN_1N_2}}(s_\beta) = (0,1,0)$

$k_{1\beta_{TN_1N_2}}(s_\beta) = \frac{\sqrt{5}}{\sqrt{2}}$ and $k_{2\beta_{TN_1N_2}}(s_\beta) = 0.$

In this case, since $k_{2\beta_{TN_1N_2}}(s_\beta) = 0$, the $\beta_{TN_1N_2}$ curve becomes a planar curve. By using (60), we get

$d_{osc} = \langle \beta_{TN_1N_2}(s_\beta), N_{1\beta_{TN_1N_2}}(s_\beta) \rangle = \frac{-\sqrt{2}}{\sqrt{5}}.$

By using (61), we obtain

$\frac{k_{1\beta_{TN_1N_2}}}{d_{osc}^2} = \frac{5\sqrt{5}}{2\sqrt{2}} \neq 0\ (constant).$

Therefore, $\beta_{TN_1N_2}$ curve becomes a planar Tz-curve

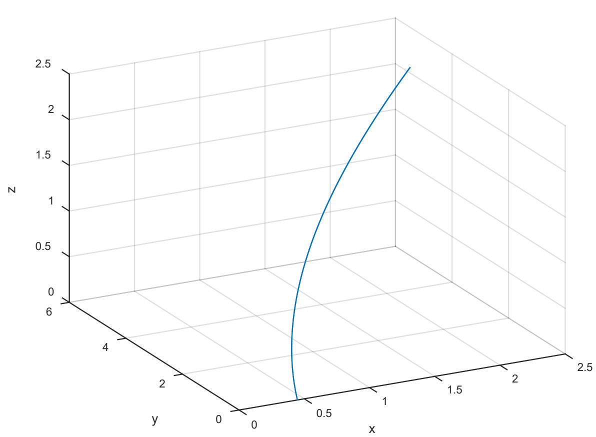

Fig. 5. $\alpha_2(s)$ curve

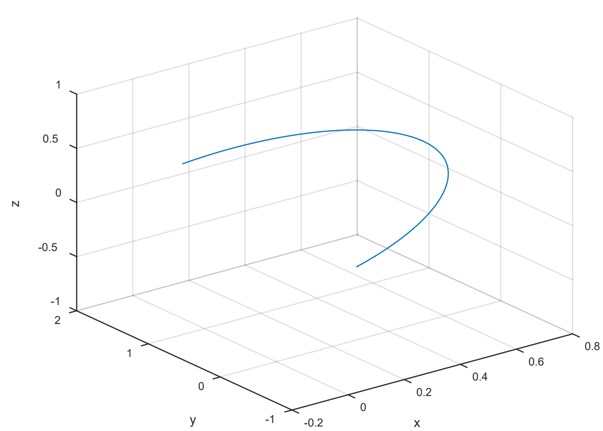

Fig. 6. β_{TN_1} curve of $\alpha_2(s)$ curve

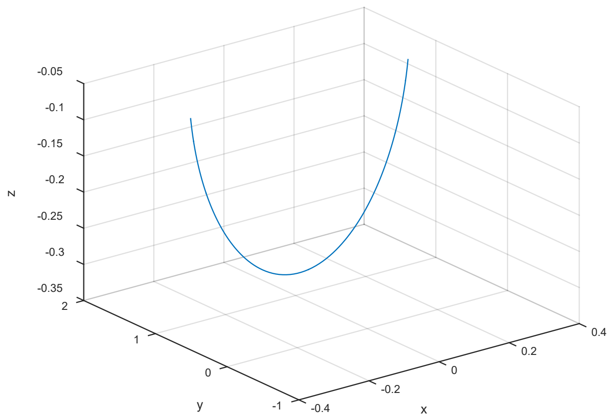

Fig. 7. β_{TN_2} curve of $\alpha_2(s)$ curve

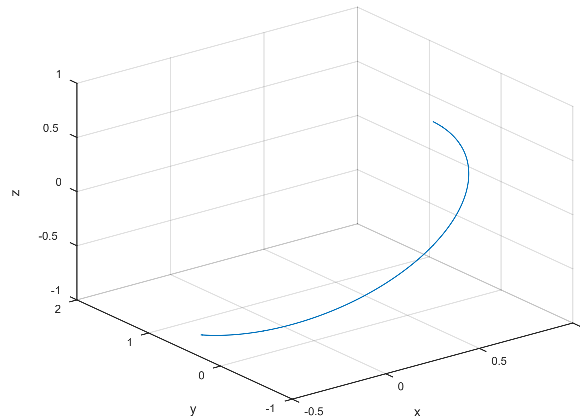

Fig. 8. $\beta_{N_1N_2}$ curve of $\alpha_2(s)$ curve

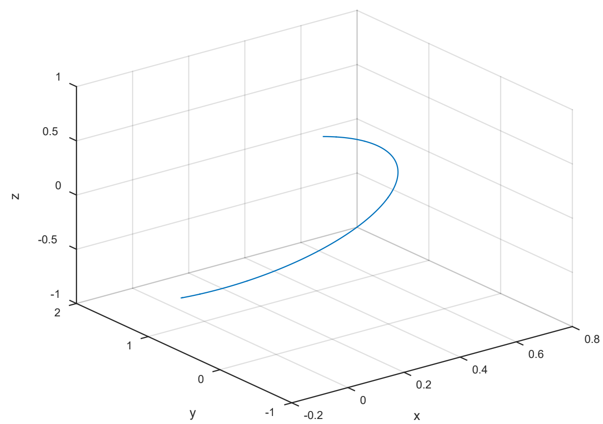

Fig. 9. $\beta_{TN_1N_2}$ curve of $\alpha_2(s)$ curve

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Prevention of Urban Heat Waves by Using Planning Tools: Torbalı Example

Research Article

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Abstract

Climate crisis is a modern era problem that is mainly attributed to human activities. It alludes to severe and abrupt changes in climate brought on by an increase in different gases in the atmosphere, such as drought, desertification, and unequal precipitation. The main factors causing this crisis include greenhouse gases, fossil fuels, agriculture and farming activities, population growth and construction. Rapid urban development caused by population growth can increase climatic threats by accelerating the loss of green space. At the same time, carbon emissions, which increase in proportion to the growing population, are also a part of this crisis. Increasing carbon emissions and ecosystem weakness in urban areas trigger an environment where heat waves are frequently experienced. Urban heat waves usually occur in densely populated cities, where hot and humid weather conditions are prolonged and intense. This is defined as an increase in the daily maximum temperature by 5°C or more above the average temperature recorded for a year for 5 consecutive days. Increasing temperatures under the influence of urban heat waves can bring serious health risks for people. High temperatures cause health problems such as dehydration, dehydration, sunstroke and are especially dangerous for vulnerable groups such as cardiovascular elderly and children. In addition, the environmental impacts of these air waves cannot be ignored. Warming air can reduce air quality and lead to increased air pollution. The increase in energy demand can increase the risk of power outages and increase energy consumption, further deepening environmental impacts.

Urban heat waves are triggered by environmental factors, climate change and human interactions in urban areas. This becomes more pronounced as urban areas are covered with thermally absorbent surfaces, especially materials such as asphalt and concrete, which increase temperatures more, and this effect is defined as "urban heat island". Heat waves are experienced more intensely especially in metropolitan areas. It is important for cities and local governments to take measures to combat climate change and protect public health. Protecting green areas, increasing open spaces, and integrating sustainability principles into urban planning are critical in mitigating the effects of urban heat waves. In this context, among the effective factors to prevent heat waves, providing air flow throughout the city and creating sufficient shade, reducing carbon emissions, turning to renewable energy sources, recycling hard-to-degrade wastes, and promoting public transport systems are the most important tools to help reduce the effects of urban heat waves. The aim of this study is to examine whether planning tools can be used to build resilience against urban heat waves, a common problem in metropolises. For this purpose, the province of Izmir, the metropolis with the third largest population in Turkey, has been selected. It is aimed to create an urban planning approach that provides resistance to heat waves in Torbalı Central district, which attracts attention with its organised industrial zones and high-speed rail system connections in İzmir province. Future research is expected to be able to benefit from the study's model application. It is foreseen that this model can be used in more detailed and realistic studies with full-time data and can contribute to urban heat waves-oriented planning processes.

In order to develop a strategic planning approach for urban settlements, it is proposed to work at the Master Plan scale. In this planning process, the functional identities of the land pieces, zone types and the sizes and development directions of the settlement areas were emphasised. The Master Plan has been accepted as a principled document that determines the basic decisions for the Torbalı settlement to have a resilient structure against urban heat waves. Spatial analytical data compiled through geographical information systems were used to create the problem definition at this scale. Torbalı Central district has been rebuilt with heat wave resistance in mind, in accordance with the Principles on the Construction of Spatial Plans of the Regulation on the Construction of Spatial Plans (Mekânsal Plan Yapım Yönetmeliği Mekânsal Planların Yapımına Dair Esasları). The new urban design is based on three main components: green areas, transport circulation system and population density. This strategy aims to provide protection against heat waves while meeting the requirements of the Master PlanThe study's findings will provide the Torbalı/Merkez district with measures to avert heat waves and strengthen its resistance to them. The new urban configuration includes holistic social facilities and urban infrastructures with balanced population densities. This holistic strategic spatial plan demonstrates that urban planning tools can be an effective instrument for building adaptation and resilience to heat waves.

Keywords: *Master Plan, Urban Heat Waves, Planning Tools, İzmir, Torbalı*

1. INTRODUCTION

With the effect of the accumulating population, cities are becoming more important and confronting settlements with sudden situations and stresses. Today, the new challenge of cities is shaped by the focus on taking rapid action against unusual situations, producing quick solutions and adapting to the new normal order. Preventing the decline in the quality of life in the process of uncontrolled population growth, changes in the ecosystem, decrease in biodiversity, negative effects of climate change, and inadequacy of cities despite various needs are considered as important topics today. With this perspective, urban planners and decision-makers focus on strategic and spatial plans that are suitable for the subjective structure of the city, that recognise the risks and crises of the city well, that observe the balance between protection and utilisation of resources, that offer alternatives and flexibility in the implementation process. All plans that do not define the subjective risks of the city well, do not offer alternative solutions and are not flexible are plans with low implementation capacity, weak in terms of resource utilisation and high consumption. Adaptation to urban stress and disasters, solving problems and transformation vary depending on the subjective capacities of cities. In resilient cities, the system has a flexible, changeable and transformable structure that can adapt to sudden internal or external shocks (Ersavaş, 2020: 1015). Regardless of the cause of stress or disaster, every city needs to plan well in order to increase its resilience against multiple shock and stress sources that may be experienced. At this point, the best solution is to construct a spatial urban plan with a resilient city vision.

The concept of resistance is synonymously defined as endurance, resistance, strength (TDK, 2021). Cities are living spaces built on systems. Urban systems have a complex structure consisting of dynamic interactions between interconnected components (Koren et al., 2017: 1). The fact that cities are built on systems brings with it the need for these systems to be as durable and flexible as possible. Resilience is the capacity of an exposed system, community, or society to withstand, absorb, adapt, and recover from the consequences of the danger in a timely and effective manner to safeguard its essential structures and functions (UNISDR, 2009: 24; Koren et al., 2017: 1). Based on this definition, urban resilience can be defined as cities that can transform, change and develop themselves according to the new situation in the face of changing conditions and maintain their functions in economic, institutional, environmental and social fields (Öztürk & Demirel, 2021). While the OECD defines resilient cities as cities that can positively transform their structures and means in the face of future economic, environmental, social or institutional shocks, absorb, improve, prepare for and recover from shocks (OECD, 2018), the Resilient Cities Network defines resilient cities as cities that have the capacity to cope, adapt and grow with any risk, hazard or crisis that arises, regardless of the source, whether expected or unexpected, by all stakeholders in the cities (Resilient City, 2021). The ability of the urban system and all the socio-ecological and socio-technical networks that comprise it to either quickly recover from a threat or maintain desired functions, adapt to change, and quickly transform systems that limit current or future adaptive capacity is referred to as urban resilience. (Meerow, Newell, Stults, 2016.) In the simplest terms, the expected reactions of cities to problems and threats that may be experienced can be defined as urban resilience.

The focus of resilient cities is on man-made disasters like nuclear, biological, and chemical accidents, transportation accidents, industrial accidents, accidents brought on by overcrowding, migrants and displaced persons, etc., as well as sudden natural disasters like earthquakes, floods, water floods, landslides, rock falls, avalanches, storms, tornadoes, volcanoes, fires, etc. In a sense, climate crisis, which constitutes the top heading of slow-developing disasters, has also taken its place among the issues that are in the focus of resilient cities. "Climate crisis" refers to sudden and extreme changes in the climate, such as drought, desertification, and uneven precipitation, caused by an increase in the rates of various atmospheric gases. It is acknowledged that the unchecked use of non-renewable resources worldwide is the root cause of the climate catastrophe, and it is expected to create irreversible harm. The global catastrophes of 2020 and 2021 demonstrate just how severe the effects of the climate problem will be. Human activity is clearly the main cause of the climate crisis. Greenhouse gases, fossil fuels, agriculture and allied industries, population growth, and the built environment are the main contributors to the climate issue. The rapid loss of natural spaces, the effects of population growth, and the sophisticated development that is taking place could all be contributing factors to climate dangers. Climate hazards could be sparked by the sophisticated building that is occurring, the impact of population increase, and the quick disappearance of green spaces. The need for action to achieve zero carbon and greenhouse gas emissions is frequently stressed. Heat waves are commonly noticed as a result of rising carbon emissions in constructed metropolitan environments and the slow deterioration of the ecological system. A heat wave is a heat wave when the daily maximum temperature rises 5°C or more above the average temperature recorded for 1 year for 5 consecutive days. The main factors that will prevent heat waves are ensuring air flow in cities and creating sufficient shade, developing strategies to reduce carbon emissions to zero throughout the city, increasing the use of renewable resources to be used in urban systems, recycling wastes that take a long time to decompose in nature and directing the transport system to public transport.

The carbon emission rate, which rises in direct proportion to population growth, is one of the reasons contributing to the climate problem. Heat waves are commonly noticed due to the rise in carbon emissions in urban developed environments and the debilitation of the ecological system. (Figure 1). Heat waves affect human health, living comfort, productivity, cause fatalities and death, forest fires, poor air quality, excessive consumption of electricity and water. Heat waves affect urban resilience positively and negatively in terms of social, economic and ecological aspects.

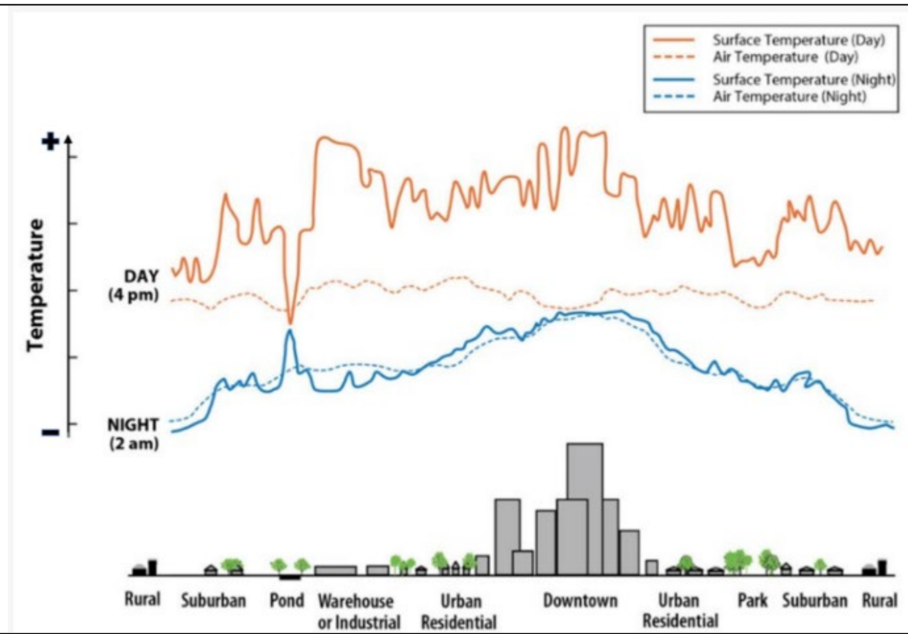


Figure 1. Effect of air temperature in different urban textures (Source : URL-1, 2022)

Urban heat island effect reduction and the creation of more livable cities can be achieved by reducing heat waves in urban areas, integrating public transportation and bicycle routes into transportation, implementing green roof applications, and building social facilities and urban infrastructures that function holistically with balanced population densities in urban morphology. In order to reduce the heat wave, green areas that will absorb carbon should be protected and increased. Urban gaps and open spaces should be increased and designed with local planting as holistic and accessible recreational activity areas. Development areas for settlements should be designed by considering meteorological parameters. It should be the responsibility of decision makers to produce green and blue strategies of cities with this focus.

The purpose of this research is to determine whether it is possible to use planning methods to prevent climate change, which is acknowledged as a global catastrophe. In order to achieve this goal, the plan uses planning techniques specific to the Torbalı Central neighborhood of Izmir in order to reduce the effects of the expected dangers to a manageable level and take the necessary precautions against heat waves. Two hypotheses were put forward for the study:

H1- An Urban Heat Wave Resilient City plan can be created.

H2- Planning tools can be used to create a city resistant to urban heat wave.

In this regard, it has been acknowledged that working at the Master Plan scale is necessary. The principal zone kinds, development orientations, settlement area sizes, and general land use for urban settlements are all highlighted in this scaleIt is also the place where the tenets that guide the settlement are decided. Geographic information systems have been used to collect the geographical analytical data required by the relevant scale for the Torbalı settlement and to clearly define the problem. Within the framework of the spatial plan making regulation and the principles on the making of spatial plans, Torbalı Central district has been reconstructed within the framework of climate resilience.

2. MATERIALS & METHODS

Within the scope of the study, literature research on resilient cities and prevention of heat waves was conducted in the first stage. In the second part, the data (land use, topography, prevailing wind, temperature, population density, social facilities availability and accessibility, green areas availability and accessibility, etc.) required to be collected at the scale of Master Development Plan presented by the Zoning Law No. 3194 and Spatial Plans Construction Regulation for the study area determined within the borders of the central district of Torbalı province were evaluated and a problem definition was put forward with a resilient city vision focused on the prevention of heat waves. Afterwards, a problem definition was put forward by approaching Torbalı campus through spatial analytical data and geographical information systems. In the third part; recommendations on resilient urban planning for Torbalı Centre district are presented within the framework of the principles for the construction of spatial plans.

3. ANALYSIS AND FINDINGS

The study area Torbalı is located in the south-east of Izmir province in the west of the Aegean region. Torbalı is surrounded by Gaziemir, Buca, Kemalpaşa, Bayındır, Tire, Selçuk and Menderes (Figure 2). Torbalı district is located in the west of the Aegean Region and has an area of 600 km² in the southeast of Izmir.

Figure 2. Torbalı urban settlement satellite image

In the district, where the Mediterranean climate is dominant, summers are hot and dry and winters are mild and rainy. In this direction, when we look at the average temperature and precipitation graph of Torbalı district for the last 30 years, the temperature is 36 degrees in June, July and August. The average precipitation in those months is at minimum level. The highest amount of precipitation is observed in November-December and January (Figure 3).

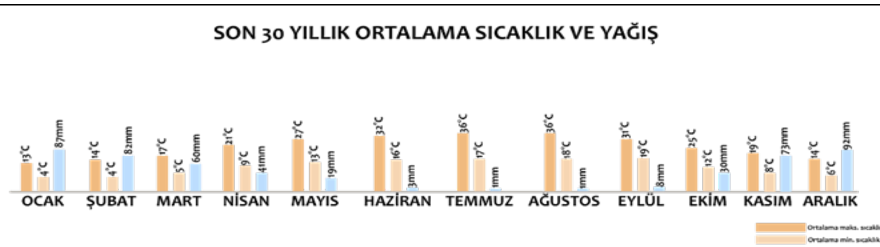


Figure 3. Torbalı 30-year Temperature and Precipitation Graph (Source: Meteorojoji Genel Müdürlüğü, 2021)

The 2021 population of the settlement is 201,476 people. Looking at the population graph of Torbalı, there is a 60% increase from 120 thousand people to approximately 200 thousand people in the period from 2006 to 2020. As a result of the increasing population, access to social facilities and urban infrastructure in the settlement has been insufficient. Along with the problems caused by the increasing population, these deficiencies increase the vulnerability of the central district of Torbalı with dangers such as natural disasters and climate change (Figure 4).

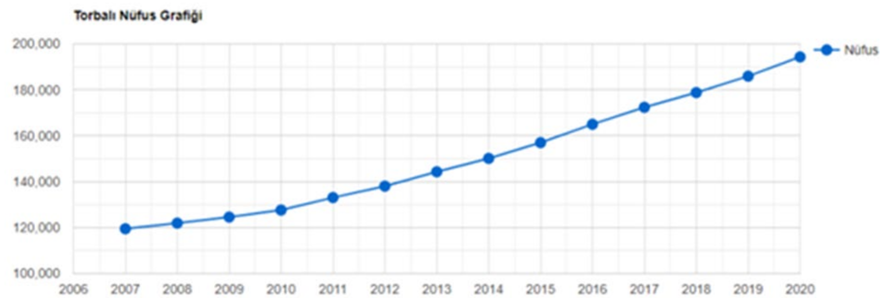


Figure 4. Population graph of Torbalı 2006-2020 (Source: www.tuik.gov.tr , www.endeksa.com/tr)

When the current land use of the study area is analysed, the city periphery of Torbalı is surrounded by agricultural lands and there is a developed industry within the settlement. Having agricultural lands working with industry makes Torbalı open to development. There is an industrial zone in the north of the settlement, a residential area in the centre, a mass trade area in the south and agricultural land surrounding the settlement. The district generally has a flat land structure. It is seen that it is an agriculture and industry orientated settlement. There is a hydrogeological asset in the northeast of the site, but this hydrogeological asset is not active (Figure 5).

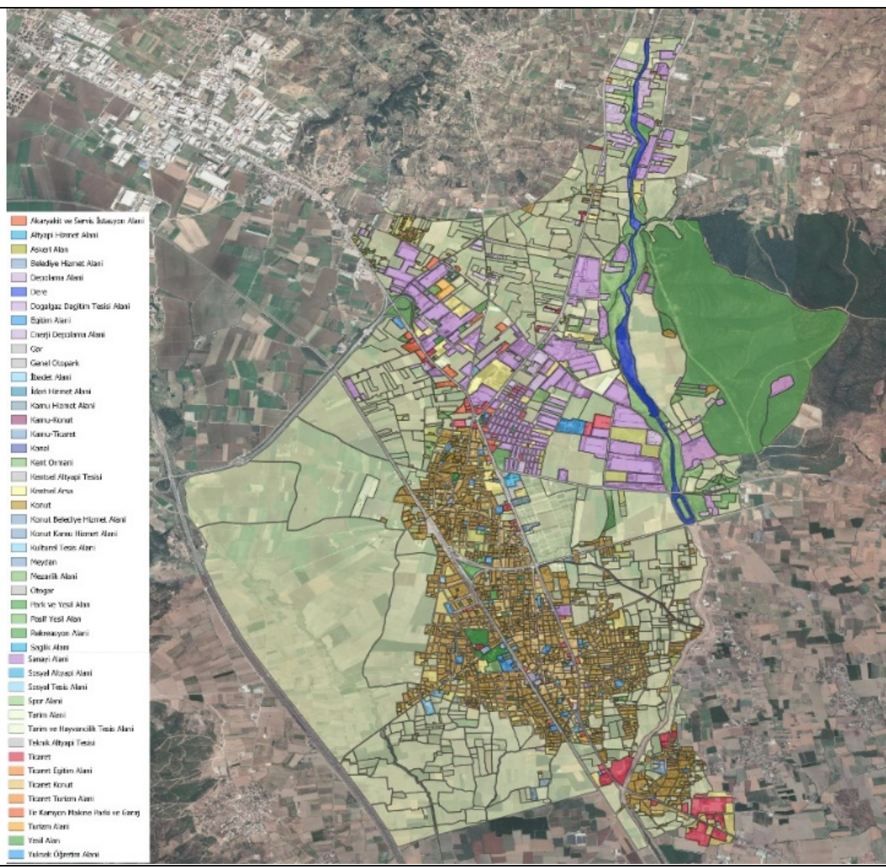


Figure 5. Torbalı Land use map

When the transport structure of Torbalı central settlement is examined, İzmir-Aydın motorway constitutes the main road connection of the settlement. The settlement provides İzmir connection with the light rail railway line following the north-south-east artery. However, it is seen that the transport hierarchy is not properly connected and that İZBAN and İzmir-Aydın motorway divide the city into three. At this point, it is possible to say that both pedestrian and road access is restricted and the city is divided into three main parts (Figure 6).

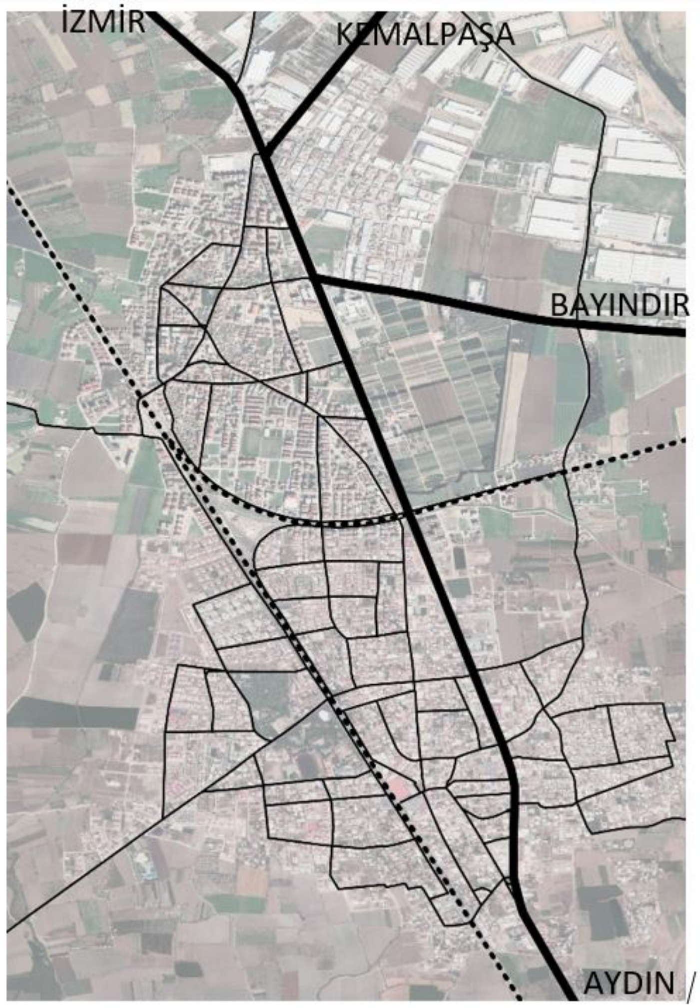


Figure 6. Torbalı's current transport system

In line with the walking distances accepted within the framework of the Spatial Plans Construction Regulation (Mekansal Planlar Yapım Yönetmeliği), the existence and accessibility of social reinforcement areas were examined through Quantum GIS programme. According to the analyses, there are 7 kindergartens, 36 primary schools, 15 secondary schools and 16 high schools in the central district of Torbalı. Educational areas are concentrated in the centre of the district. There are areas where access to primary and secondary school education areas in Torbalı district is insufficient (Figure 7).

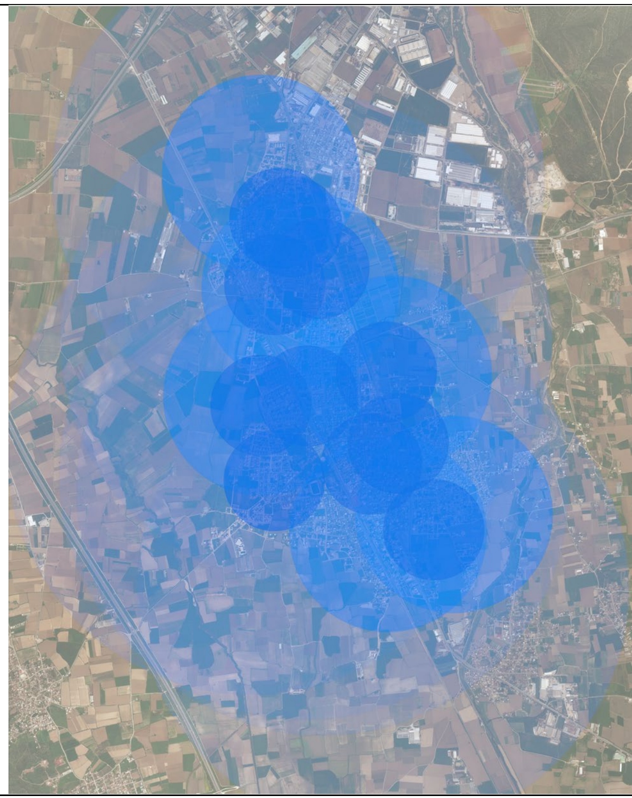


Figure 7. Accessibility of Education Areas

Similarly, in line with the service radius analysis of primary health care areas, it is seen that there are neighborhoods that cannot receive services (Figure 8).

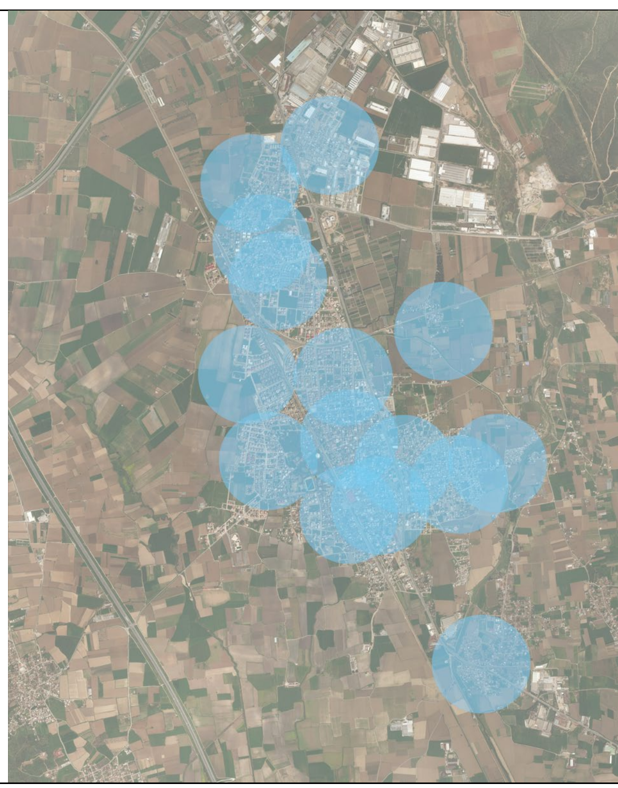


Figure 8. Accessibility of Worship Areas

When the areas of worship are evaluated; it is seen that access is insufficient in the south and north of the settlement. The fact that access to social reinforcement areas is limited shows that daily life needs are met by car or public transport, not on foot.

When the existence and accessibility of green areas, which are of primary importance in mitigating urban heat waves, are examined, it is seen that there is no access in the central region, which forms the built environment of the area, and the amount of green areas per capita is insufficient. This inadequacy shows that both in the city as a whole and at the neighbourhood scale, urbanites cannot access recreational activities and lack the minimum amount of green space required for healthy environments.

In terms of social reinforcement, in summary, the settlement has not been able to develop spatially despite intensive migration, and the standards of access to social reinforcements and urban infrastructure have been insufficient.

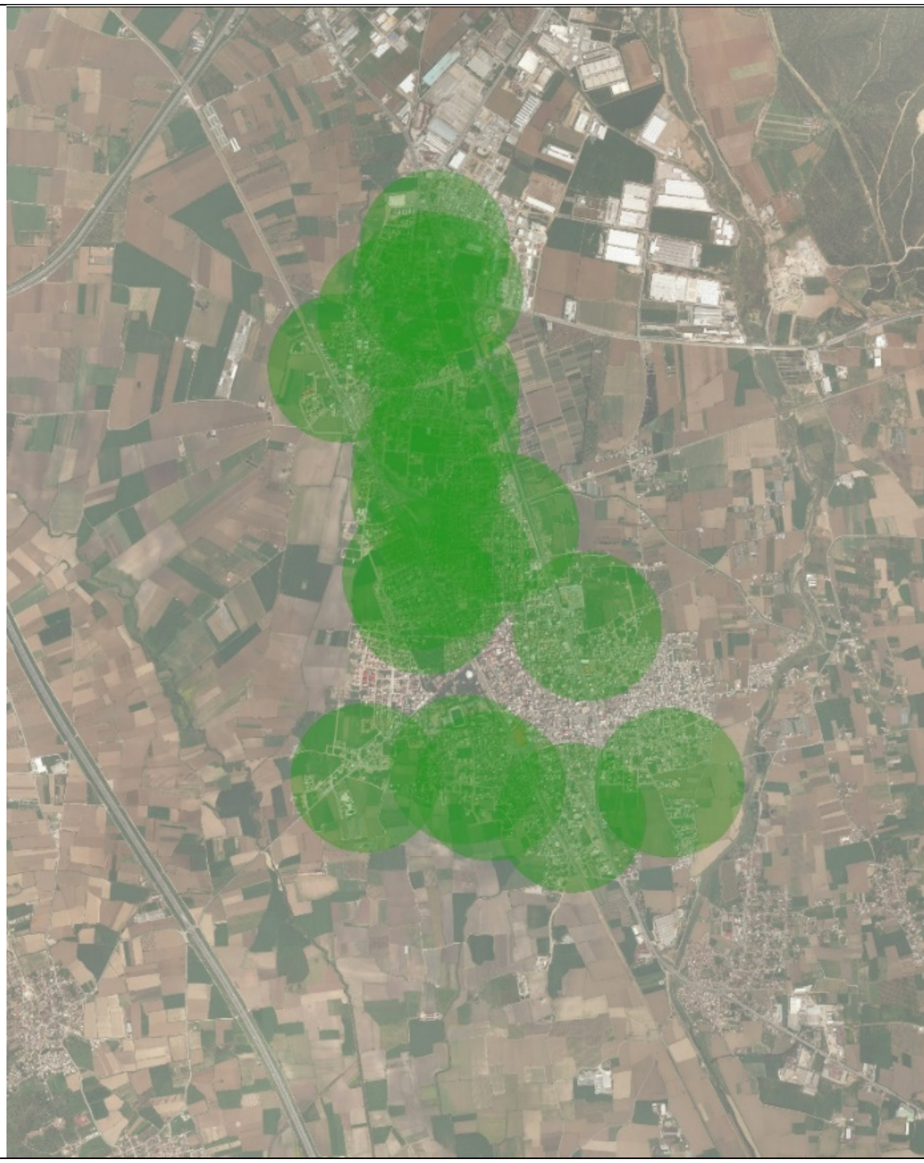


Figure 9. Green Space Accessibility

A temperature analysis was carried out to measure the urban heat for the study area. Looking at the temperature in the afternoon, it is seen that the temperature in the north of the settlement is approximately 35 degrees in the afternoon, similarly, the temperature in the centre of the settlement is 35 degrees in the afternoon. In addition, the temperature in the built-up area is approximately 30 degrees. On the other hand, as expected in agricultural areas, the temperature in the area forming the natural structure is 29 degrees.

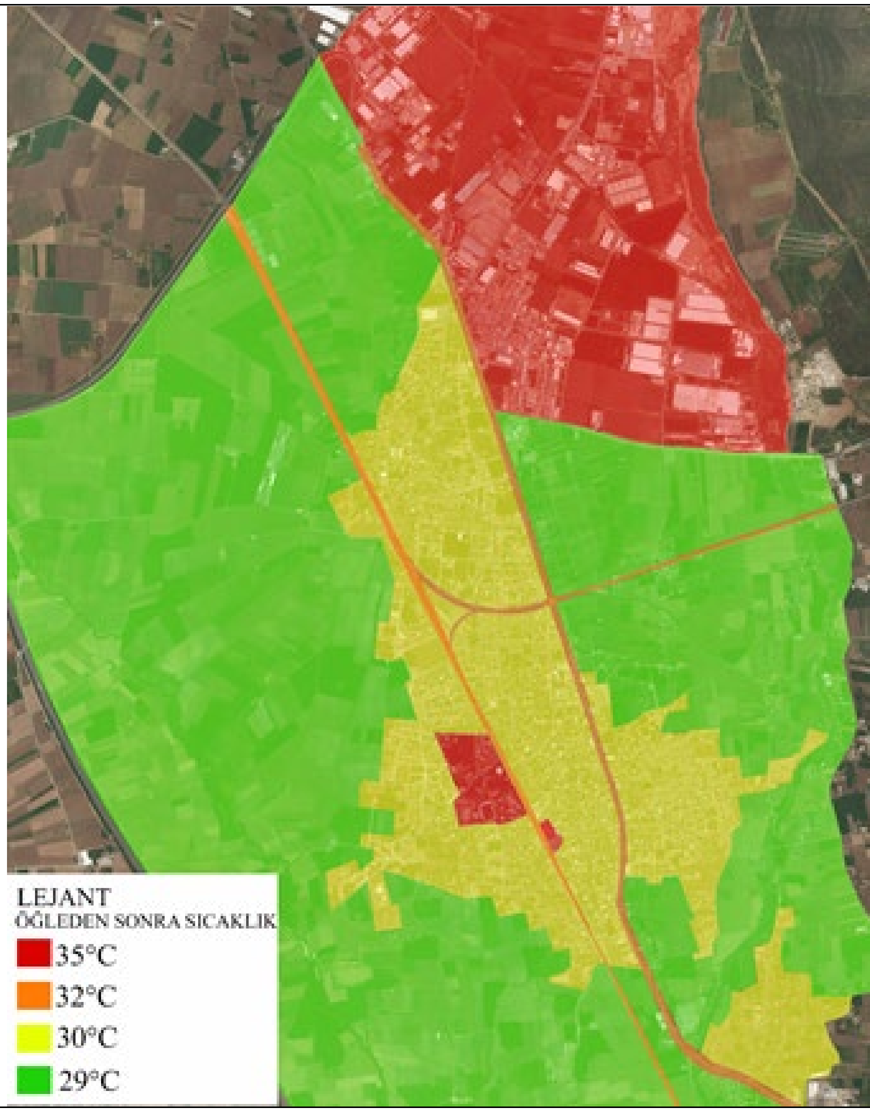


Figure 10. 2020 Torbalı Temperature Analysis

It is seen that the general air quality of Torbalı is generally at a medium level. It has been determined that the air quality is poor in the north due to harmful gases emitted from industrial facilities. In the regions where agricultural areas are located, air quality is generally good (Figure 11).

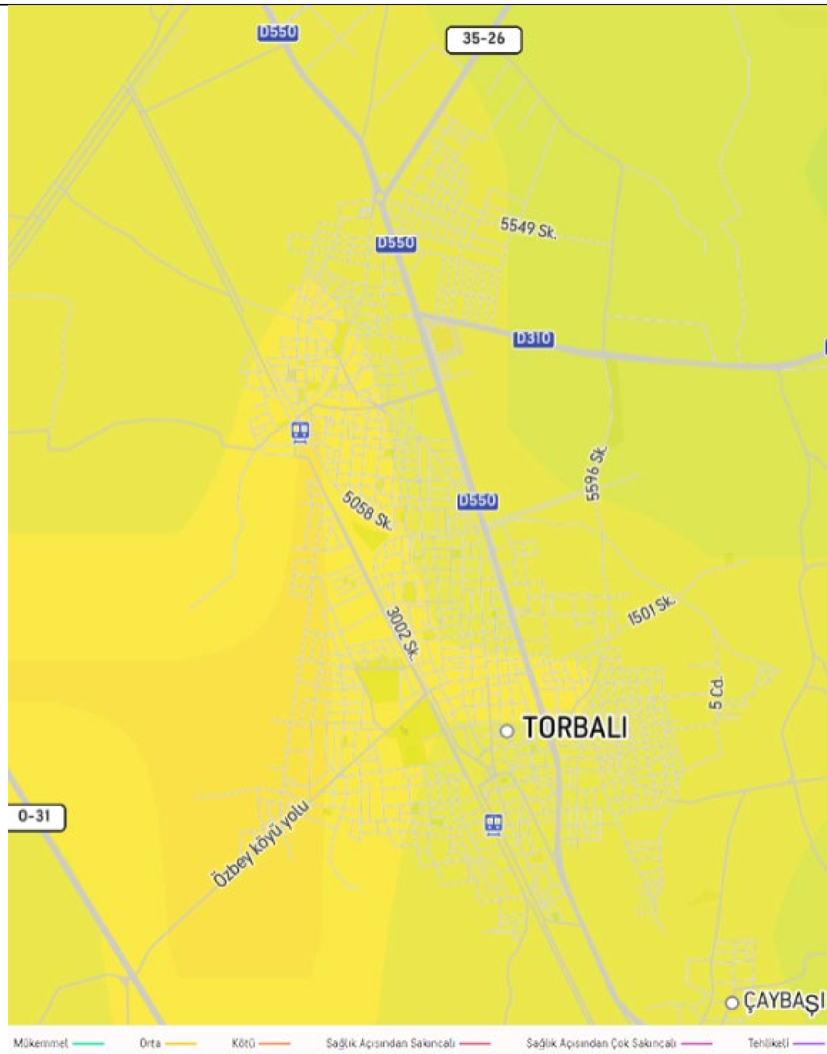


Figure 11. 2020 Torbalı Air Quality Analysis

4. ASSESSMENT

By evaluating the analytical data, two alternative master development plans were presented for Torbalı centre settlement within the framework of the Spatial Plans Construction Regulation (Figure 12-13). In the 1st master development plan, the borders of the existing industrial area of 307.5155 hectares were preserved and food industry facilities were selected to serve the industrial zone by adhering to the targets within the industrial area. In addition, a recovery facility to serve in the industrial area has been planned in order to store and recycle the solid wastes in the industrial area. The existing boundaries of the urban forest within the borders of the study area have been preserved. A recreational area has been planned within the borders of the urban forest. By preserving the existing agricultural areas, it is aimed to improve soil health with ecological agriculture system. The problems of the settlement areas in the existing area have been identified and determined as renovation areas. In the residential areas to be renovated, construction provisions will be applied. It has been decided to make analyses against disaster risk before building in the development housing areas to be constructed and to construct the buildings in accordance with the disaster risk. The park planned in Torbalı neighbourhood shown in the plan will be designed as a recycling park by selecting the materials to be used from recycled materials or materials that require less energy both in transport and production. Likewise, the parks planned in Atatürk Neighbourhood and Ertuğrul Neighbourhood will be designed as sustainable parks and water collection parks by focusing on seasonal changes, maintaining the vitality of the park throughout the year, and aiming to retain and reuse the water available throughout the park.

In the second alternative plan, the industrial zone is kept fixed in the north and the mass trade area is connected to the green circulation within the built-up area. Since there is an access problem to the area, social facilities, service areas and trade areas have been made accessible together with the green circulation integrated with transportation. In order to prevent the heat wave, agricultural areas have been protected and green areas have been increased. The prevailing wind direction blows from the north-east of the city. In order to prevent the effect of heat waves and the effect of poor quality air from the industry on residential areas, the area to be afforested is planned to surround the periphery of the city. In both plans, housing groups are planned gradually in terms of population density.

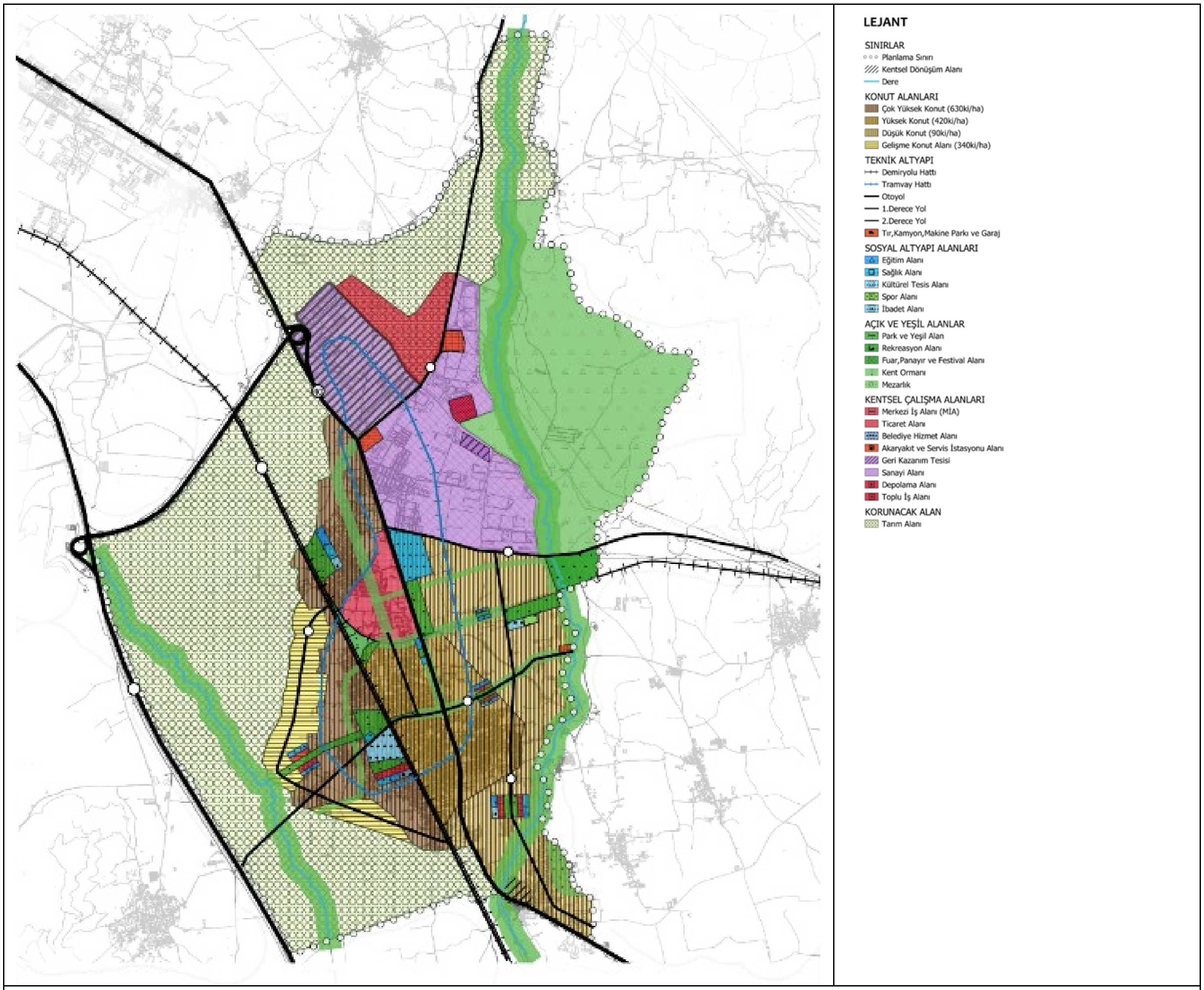


Figure 12. 2042 Torbalı Centre District Master Plans Alternative 1

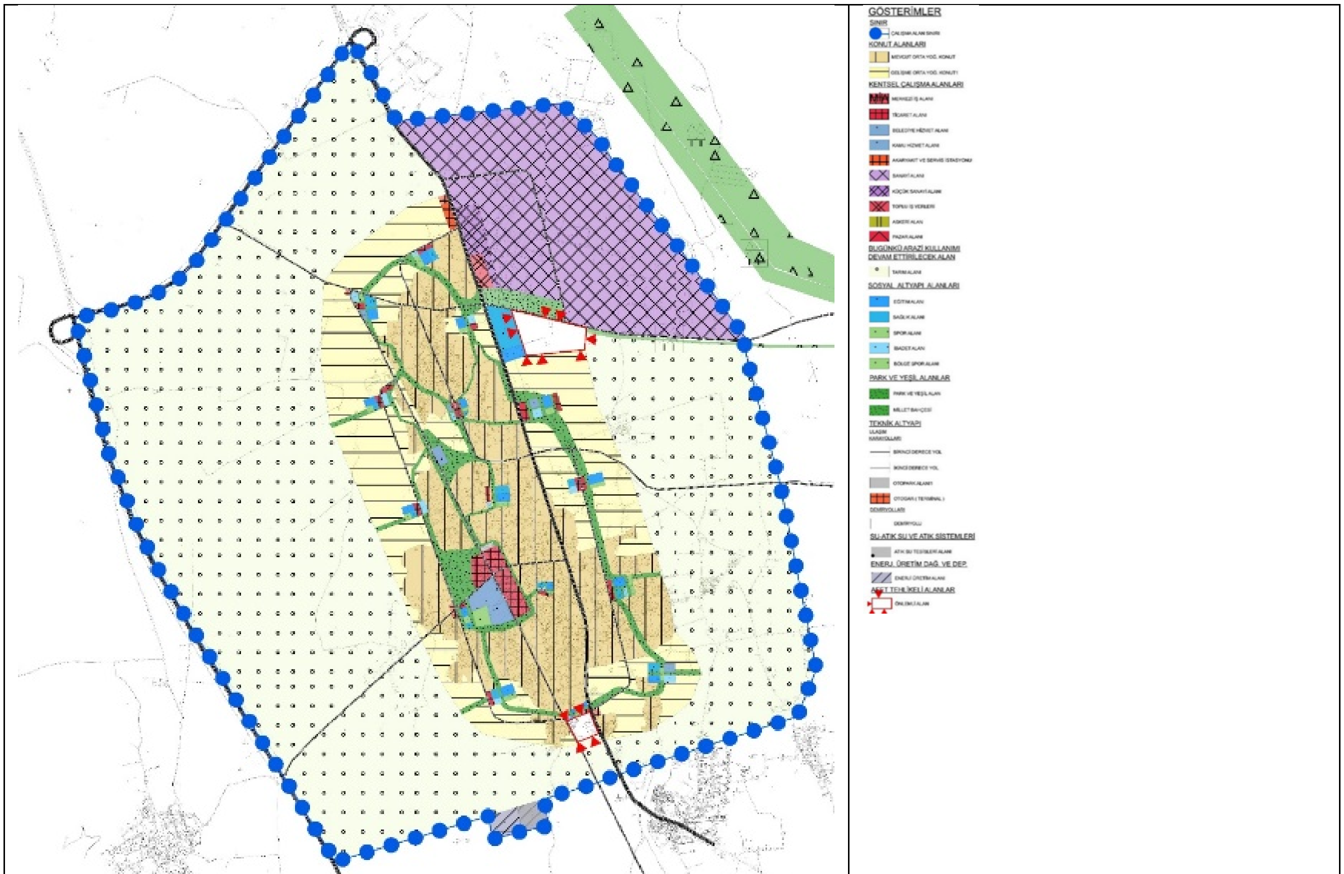
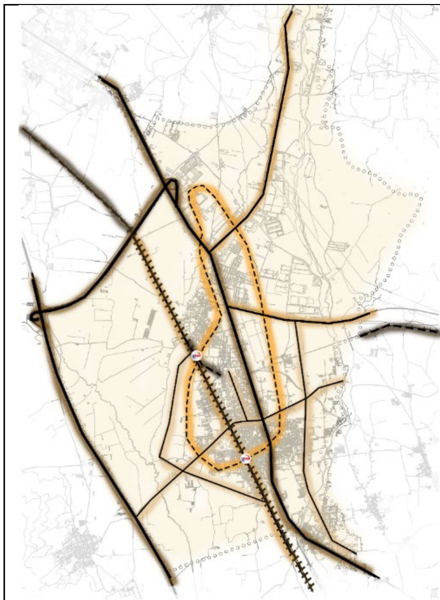


Figure 13. 2042 Torbalı Centre District Master Plans Alternative 2

Spatial and policy strategies to prevent heat waves were also evaluated. Spatial strategies were evaluated in the focus of morphological view, transportation planning, green circulation system, and policy objectives were evaluated in the focus of economic, management, community and ecological. When we look at the transport planning proposals; public transport, bicycle and tram types are evaluated. In both plans, it is aimed to reduce the use of private vehicles and provide accessibility with a focus on public transport in the transport plan (Figure 14).

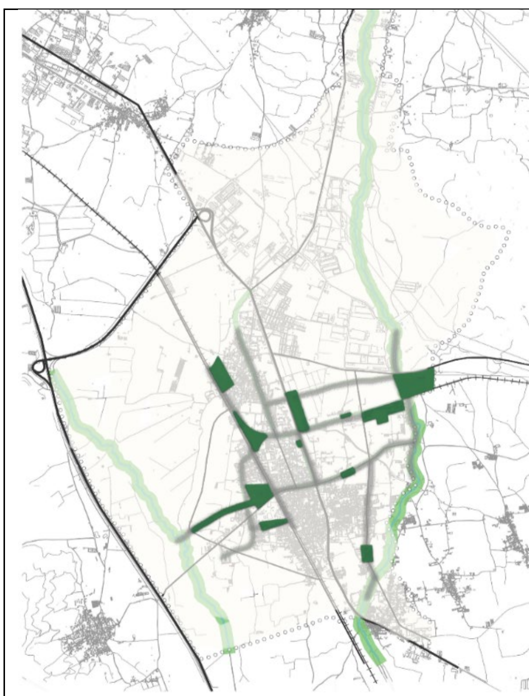
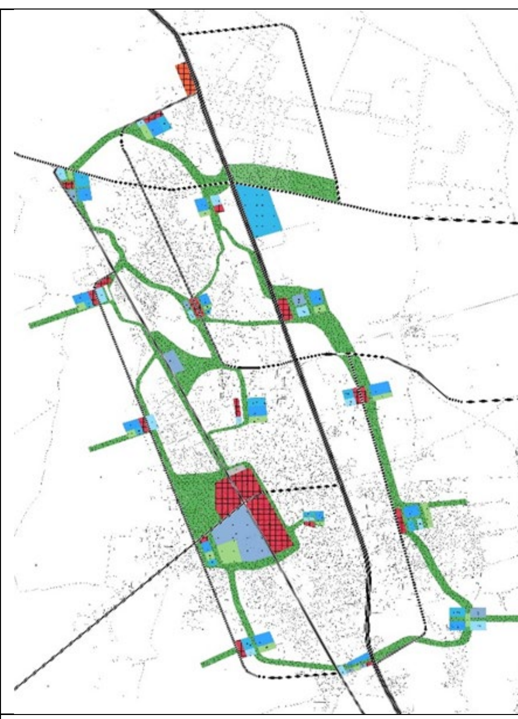



Masterplan Alternative 1	Masterplan Alternative 2

Figure 14. Transport proposals for the plans

In order to reduce carbon emissions, cycling is considered topographically appropriate and an integrated network of public transport and cycling is planned.

In order to prevent heat wave effects, it is also important to eliminate the insufficiency of existing green areas and to provide an integrated green circulation. In order to reduce the urban heat island effect, green circulation system green corridors covering neighbourhood units have been planned and non-motorised vehicle access in the city has been facilitated. These green corridors also form a holistic transport system with bicycle and pedestrian routes (Figure 15).

Master Plan Alternative 1	Master Plan Alternative 2

Figure 15. Green circulation system related to the plans

The amount of green space per capita required by the Regulation on the Construction of Spatial Plans has been increased above the qualifications (Table 1), the effect of urban heat waves has been tried to be reduced, and green foci have been selected within walking distance on the basis of accessibility criteria. In this way, it is ensured that the city dwellers can fulfil their recreational activity needs on foot.

Table 1. Green area assets and sizes of Master Plan 1 and Master Plan 2

PLAN	*YEAR*	*NUMBER*	*AREA (HA)*
MASTER PLAN-1	*2020- GREEN FIELD*	*20*	*8 HA*
	2045- GREEN FIELD	*18*	*12 HA*
MASTER PLAN-2	*2020- GREEN FIELD*	*20*	*8 HA*
	2045- GREEN FIELD	*21*	*18 HA*

Within the framework of the resilient city implementation guide for Torbalı district, policies have also been developed in terms of economy, governance, community and ecological resilience (Figure 15).

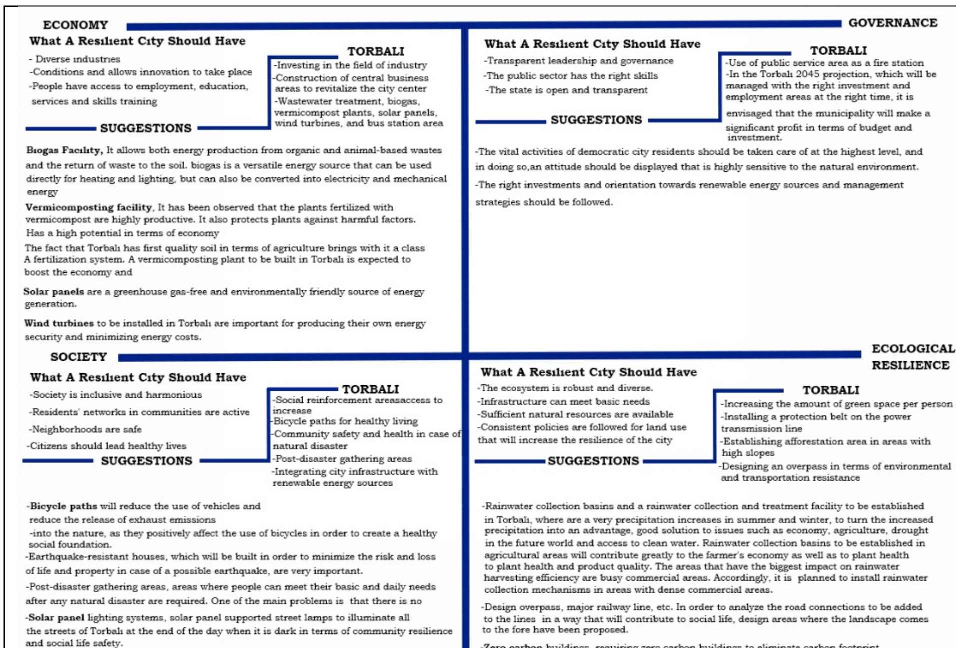


Figure 15. 2042 Hot weather resilient city Torbalı political strategies

5. RESULTS

Considering the analyses made and the development potential of the central district of Torbalı, the spatial strategy plan targeted for 2042 was designed within the whole of the resilient city vision and components focused on preventing heat waves. The purpose of this research is to determine whether the heat waves caused by climate change, which are acknowledged as a global emergency, may be mitigated in accordance with planning instruments. In line with this purpose; the hypotheses of the study were verified and a city resistant to urban heat waves could be created and planning tools could be used to create a city resistant to urban heat waves.

In line with the vision of Torbalı resilient to heat waves, expert transformation in the field of industry in the economic focus, creation of a central business area to revive the city centre, wastewater treatment, biogas and vermicomposting facilities, solar panels, wind turbines are planned in the environmental focus. In the management focus, orientation to renewable energy resources with the right investments and management strategy should be followed. In the community focus, suggestions have been made for Torbalı to increase access to social facilities, to build bicycle paths for a healthy and sustainable life, and to integrate the city infrastructure with renewable energy sources. In order to provide an ecological focus in the city, it has been suggested to increase the amount of green space per capita, to make areas to be afforested in areas with high slope and to make a protection belt for the energy transmission line. In this study, it has been analyzed how to create resilient cities in cities experiencing heat waves against climate change and how planning tools can be used against climate change. This examination was carried out through the city of Torbalı, and it is thought that all the strategies discussed through the settlement can also be used for other cities to apply against climate change.

6. CONFLICTS OF INTEREST

The authors declare no conflict of interest.

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